Section 4.5 Integration by Substitution

$\int f(g(x))g'(x)\,dx$	$u = g(x)$	$du = g'(x)\,dx$
1. $\int (8x^2+1)^2(16x)dx$	$8x^2+1$	$16x\,dx$
2. $\int x^2\sqrt{x^3+1}\,dx$	x^3+1	$3x^2\,dx$
3. $\int \tan^2 x \sec^2 x\,dx$	$\tan x$	$\sec^2 x\,dx$
4. $\int \frac{\cos x}{\sin^2 x}\,dx$	$\sin x$	$\cos x\,dx$

5. $\int (1+6x)^4(6)\,dx = \frac{(1+6x)^5}{5} + C$

Check: $\frac{d}{dx}\left[\frac{(1+6x)^5}{5} + C\right] = 6(1+6x)^4$

6. $\int (x^2-9)^3(2x)\,dx = \frac{(x^2-9)^4}{4} + C$

Check: $\frac{d}{dx}\left[\frac{(x^2-9)^4}{4} + C\right] = \frac{4(x^2-9)^3}{4}(2x) = (x^2-9)^3(2x)$

7. $\int \sqrt{25-x^2}(-2x)\,dx = \frac{(25-x^2)^{3/2}}{3/2} + C = \frac{2}{3}(25-x^2)^{3/2} + C$

Check: $\frac{d}{dx}\left[\frac{2}{3}(25-x^2)^{3/2} + C\right] = \frac{2}{3}\left(\frac{3}{2}\right)(25-x^2)^{1/2}(-2x) = \sqrt{25-x^2}(-2x)$

8. $\int \sqrt[3]{3-4x^2}(-8x)\,dx = \int (3-4x^2)^{1/3}(-8x)\,dx = \frac{(3-4x^2)^{4/3}}{4/3} + C = \frac{3}{4}(3-4x^2)^{4/3} + C$

Check: $\frac{d}{dx}\left[\frac{3}{4}(3-4x^2)^{4/3} + C\right] = \frac{3}{4}\left(\frac{4}{3}\right)(3-4x^2)^{1/3}(-8x) = (3-4x^2)^{1/3}(-8x)$

9. $\int x^3(x^4+3)^2\,dx = \frac{1}{4}\int (x^4+3)^2(4x^3)\,dx = \frac{1}{4}\frac{(x^4+3)^3}{3} + C = \frac{(x^4+3)^3}{12} + C$

Check: $\frac{d}{dx}\left[\frac{(x^4+3)^3}{12} + C\right] = \frac{3(x^4+3)^2}{12}(4x^3) = (x^4+3)^2(x^3)$

10. $\int x^2(6-x^3)\,dx = -\frac{1}{3}\int (6-x^3)^5(-3x^2)\,dx = -\frac{1}{3}\cdot\frac{(6-x^3)^6}{6} + C = -\frac{(6-x^3)^6}{18} + C$

Check: $\frac{d}{dx}\left[-\frac{(6-x^3)^6}{18} + C\right] = \frac{-6(6-x^3)^5(-3x^2)}{18} = x^2(6-x^3)^5$

11. $\int x^2(x^3 - 1)^4\,dx = \frac{1}{3}\int (x^3 - 1)^4(3x^2)\,dx = \frac{1}{3}\left[\frac{(x^3 - 1)^5}{5}\right] + C = \frac{(x^3 - 1)^5}{15} + C$

Check: $\frac{d}{dx}\left[\frac{(x^3 - 1)^5}{15} + C\right] = \frac{5(x^3 - 1)^4(3x^2)}{15} = x^2(x^3 - 1)^4$

12. $\int x(5x^2 + 4)^3\,dx = \frac{1}{10}\int (5x^2 + 4)^3(10x)\,dx = \frac{1}{10}\left[\frac{(5x^2 + 4)^4}{4}\right] + C = \frac{(5x^2 + 4)^4}{40} + C$

Check: $\frac{d}{dx}\left[\frac{(5x^2 + 4)^4}{40} + C\right] = \frac{4(5x^2 + 4)^3(10x)}{40} = x(5x^2 + 4)^3$

13. $\int t\sqrt{t^2 + 2}\,dt = \frac{1}{2}\int (t^2 + 2)^{1/2}(2t)\,dt = \frac{1}{2}\frac{(t^2 + 2)^{3/2}}{3/2} + C = \frac{(t^2 + 2)^{3/2}}{3} + C$

Check: $\frac{d}{dt}\left[\frac{(t^2 + 2)^{3/2}}{3} + C\right] = \frac{3/2(t^2 + 2)^{1/2}(2t)}{3} = (t^2 + 2)^{1/2}t$

14. $\int t^3\sqrt{2t^4 + 3}\,dt = \frac{1}{8}\int (2t^4 + 3)^{1/2}(8t^3)\,dt = \frac{1}{8}\cdot\frac{(2t^4 + 3)^{3/2}}{(3/2)} + C = \frac{(2t^4 + 3)^{3/2}}{12} + C$

Check: $\frac{d}{dt}\left[\frac{(2t^4 + 3)^{3/2}}{12} + C\right] = \frac{\frac{3}{2}(2t^4 + 3)^{1/2}(8t^3)}{12} = t^3\sqrt{2t^4 + 3}$

15. $\int 5x(1 - x^2)^{1/3}\,dx = -\frac{5}{2}\int (1 - x^2)^{1/3}(-2x)\,dx = -\frac{5}{2}\cdot\frac{(1 - x^2)^{4/3}}{4/3} + C = -\frac{15}{8}(1 - x^2)^{4/3} + C$

Check: $\frac{d}{dx}\left[-\frac{15}{8}(1 - x^2)^{4/3} + C\right] = -\frac{15}{8}\cdot\frac{4}{3}(1 - x^2)^{1/3}(-2x) = 5x(1 - x^2)^{1/3} = 5x\sqrt[3]{1 - x^2}$

16. $\int u^2\sqrt{u^3 + 2}\,du = \frac{1}{3}\int (u^3 + 2)^{1/2}(3u^2)\,du = \frac{1}{3}\frac{(u^3 + 2)^{3/2}}{3/2} + C = \frac{2(u^3 + 2)^{3/2}}{9} + C$

Check: $\frac{d}{du}\left[\frac{2(u^3 + 2)^{3/2}}{9} + C\right] = \frac{2}{9}\cdot\frac{3}{2}(u^3 + 2)^{1/2}(3u^2) = (u^3 + 2)^{1/2}(u^2)$

17. $\int \frac{x}{(1 - x^2)^3}\,dx = -\frac{1}{2}\int (1 - x^2)^{-3}(-2x)\,dx = -\frac{1}{2}\frac{(1 - x^2)^{-2}}{-2} + C = \frac{1}{4(1 - x^2)^2} + C$

Check: $\frac{d}{dx}\left[\frac{1}{4(1 - x^2)^2} + C\right] = \frac{1}{4}(-2)(1 - x^2)^{-3}(-2x) = \frac{x}{(1 - x^2)^3}$

18. $\int \frac{x^3}{(1+x^4)^2}\,dx = \frac{1}{4}\int (1+x^4)^{-2}(4x^3)\,dx = -\frac{1}{4}(1+x^4)^{-1} + C = \frac{-1}{4(1+x^4)} + C$

Check: $\frac{d}{dx}\left[\frac{-1}{4(1+x^4)} + C\right] = \frac{1}{4}(1+x^4)^{-2}(4x^3) = \frac{x^3}{(1+x^4)^2}$

19. $\int \frac{x^2}{(1+x^3)^2}\,dx = \frac{1}{3}\int (1+x^3)^{-2}(3x^2)\,dx = \frac{1}{3}\left[\frac{(1+x^3)^{-1}}{-1}\right] + C = -\frac{1}{3(1+x^3)} + C$

Check: $\frac{d}{dx}\left[-\frac{1}{3(1+x^3)} + C\right] = -\frac{1}{3}(-1)(1+x^3)^{-2}(3x^2) = \frac{x^2}{(1+x^3)^2}$

20. $\int \frac{6x^2}{(4x^3-9)^3}\,dx = \frac{1}{2}\int (4x^3-9)^{-3}(12x^2)\,dx = \frac{1}{2}\cdot\frac{(4x^3-9)^{-2}}{-2} + C = -\frac{1}{4(4x^3-9)^{-2}} + C$

Check: $\frac{d}{dx}\left[\frac{-1}{4(4x^3-9)^2} + C\right] = \frac{d}{dx}\left[-\frac{1}{4}(4x^3-9)^{-2} + C\right] = -\frac{1}{4}(-2)(4x^3-9)^{-3}(12x^2) = \frac{6x^2}{(4x^3-9)^3}$

21. $\int \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int (1-x^2)^{-1/2}(-2x)\,dx = -\frac{1}{2}\frac{(1-x^2)^{1/2}}{1/2} + C = -\sqrt{1-x^2} + C$

Check: $\frac{d}{dx}\left[-(1-x^2)^{1/2} + C\right] = -\frac{1}{2}(1-x^2)^{-1/2}(-2x) = \frac{x}{\sqrt{1-x^2}}$

22. $\int \frac{x^3}{\sqrt{1+x^4}}\,dx = \frac{1}{4}\int (1+x^4)^{-1/2}(4x^3)\,dx = \frac{1}{4}\frac{(1+x^4)^{1/2}}{1/2} + C = \frac{\sqrt{1+x^4}}{2} + C$

Check: $\frac{d}{dx}\left[\frac{\sqrt{1+x^4}}{2} + C\right] = \frac{1}{2}\cdot\frac{1}{2}(1+x^4)^{-1/2}(4x^3) = \frac{x^3}{\sqrt{1+x^4}}$

23. $\int \left(1+\frac{1}{t}\right)^3\left(\frac{1}{t^2}\right)dt = -\int \left(1+\frac{1}{t}\right)^3\left(-\frac{1}{t^2}\right)dt = -\frac{\left[1+\left(\frac{1}{t}\right)\right]^4}{4} + C$

Check: $\frac{d}{dt}\left[-\frac{[1+(1/t)]^4}{4} + C\right] = -\frac{1}{4}(4)\left(1+\frac{1}{t}\right)^3\left(-\frac{1}{t^2}\right) = \frac{1}{t^2}\left(1+\frac{1}{t}\right)^3$

24. $\int \left[x^2 + \frac{1}{(3x)^2}\right]dx = \int \left(x^2 + \frac{1}{9}x^{-2}\right)dx = \frac{x^3}{3} + \frac{1}{9}\left(\frac{x^{-1}}{-1}\right) + C = \frac{x^3}{3} - \frac{1}{9x} + C = \frac{3x^4-1}{9x} + C$

Check: $\frac{d}{dx}\left[\frac{1}{3}x^3 - \frac{1}{9}x^{-1} + C\right] = x^2 + \frac{1}{9}x^{-2} = x^2 + \frac{1}{(3x)^2}$

25. $\int \frac{1}{\sqrt{2x}}\,dx = \frac{1}{2}\int (2x)^{-1/2} 2\,dx = \frac{1}{2}\left[\frac{(2x)^{1/2}}{1/2}\right] + C = \sqrt{2x} + C$

Alternate Solution: $\int \frac{1}{\sqrt{2x}}\,dx = \frac{1}{\sqrt{2}}\int x^{-1/2}\,dx = \frac{1}{\sqrt{2}}\frac{x^{1/2}}{(1/2)} + C = \sqrt{2x} + C$

Check: $\frac{d}{dx}\left[\sqrt{2x} + C\right] = \frac{1}{2}(2x)^{-1/2}(2) = \frac{1}{\sqrt{2x}}$

26. $\int \frac{x}{\sqrt[3]{5x^2}}\,dx = \int \frac{1}{\sqrt[3]{5}} x^{1/3}\,dx = \frac{1}{\sqrt[3]{5}} \cdot \frac{3}{4}x^{4/3} + C = \frac{3}{20}\sqrt[3]{25x^4} + C$

Alternate Solution:

$$\int \frac{x}{\sqrt[3]{5x^2}}\,dx = \int \left(5x^2\right)^{-1/3} x\,dx = \frac{1}{10}\int \left(5x^2\right)^{-1/3}(10x)\,dx = \frac{1}{10}\cdot\frac{\left(5x^2\right)^{2/3}}{(2/3)} + C = \frac{3}{20}\left(5x^2\right)^{2/3} + C = \frac{3}{4}\cdot\frac{1}{\sqrt[3]{5}}x^{4/3} + C$$

Check: $\frac{d}{dx}\left[\frac{1}{\sqrt[3]{5}}\cdot\frac{3}{4}x^{4/3} + C\right] = \frac{1}{\sqrt[3]{5}}\cdot\frac{3}{4}\cdot\frac{4}{3}x^{1/3} = \frac{x}{\sqrt[3]{5x^2}}$

27. $y = \int\left[4x + \frac{4x}{\sqrt{16 - x^2}}\right]dx = 4\int x\,dx - 2\int\left(16 - x^2\right)^{-1/2}(-2x)\,dx = 4\left(\frac{x^2}{2}\right) - 2\left[\frac{\left(16 - x^2\right)^{1/2}}{1/2}\right] + C = 2x^2 - 4\sqrt{16 - x^2} + C$

28. $y = \int \frac{10x^2}{\sqrt{1 + x^3}}\,dx$

$= \frac{10}{3}\int\left(1 + x^3\right)^{-1/2}\left(3x^2\right)dx$

$= \frac{10}{3}\left[\frac{\left(1 + x^3\right)^{1/2}}{1/2}\right] + C$

$= \frac{20}{3}\sqrt{1 + x^3} + C$

29. $y = \int \frac{x + 1}{\left(x^2 + 2x - 3\right)^2}\,dx$

$= \frac{1}{2}\int\left(x^2 + 2x - 3\right)^{-2}(2x + 2)\,dx$

$= \frac{1}{2}\left[\frac{\left(x^2 + 2x - 3\right)^{-1}}{-1}\right] + C$

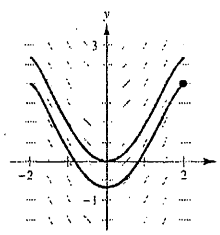
$= -\frac{1}{2\left(x^2 + 2x - 3\right)} + C$

30. $y = \int \frac{x - 4}{\sqrt{x^2 - 8x + 1}}\,dx$

$= \frac{1}{2}\int\left(x^2 - 8x + 1\right)^{-1/2}(2x - 8)\,dx$

$= \frac{1}{2}\left[\frac{\left(x^2 - 8x + 1\right)^{1/2}}{1/2}\right] + C = \sqrt{x^2 - 8x + 1} + C$

31. (a) Answers will vary. *Sample answer:*



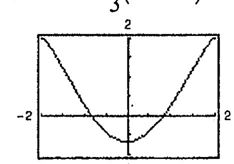
(b) $\frac{dy}{dx} = x\sqrt{4 - x^2}$, $(2, 2)$

$y = \int x\sqrt{4 - x^2}\,dx = -\frac{1}{2}\int\left(4 - x^2\right)^{1/2}(-2x\,dx)$

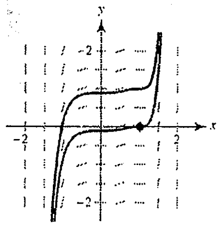
$= -\frac{1}{2}\cdot\frac{2}{3}\left(4 - x^2\right)^{3/2} + C = -\frac{1}{3}\left(4 - x^2\right)^{3/2} + C$

$(2, 2)$: $2 = -\frac{1}{3}\left(4 - 2^2\right)^{3/2} + C \Rightarrow C = 2$

$y = -\frac{1}{3}\left(4 - x^2\right)^{3/2} + 2$



32. (a) Answers will vary: *Sample answer:*



(b) $$\frac{dy}{dx} = x^2\left(x^3 - 1\right)^2, \ (1, 0)$$

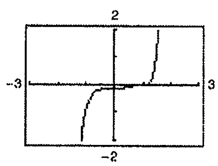
$$y = \int x^2\left(x^3 - 1\right)^2 dx$$

$$\left(u = x^3 - 1\right) = \frac{1}{3}\int\left(x^3 - 1\right)^2\left(3x^2\, dx\right)$$

$$= \frac{1}{3}\frac{\left(x^3 - 1\right)^3}{3} + C = \frac{1}{9}\left(x^3 - 1\right)^3 + C$$

$$0 = C$$

$$y = \frac{1}{9}\left(x^3 - 1\right)^3$$



33. $\int \pi \sin \pi x\, dx = -\cos \pi x + C$

34. $\int \sin 4x\, dx = \frac{1}{4}\int(\sin 4x)(4)\, dx = -\frac{1}{4}\cos 4x + C$

35. $\int \cos 8x\, dx = \frac{1}{8}\int(\cos 8x)(8)\, dx = \frac{1}{8}\sin 8x + C$

36. $\int \csc^2\left(\frac{x}{2}\right) dx = 2\int \csc^2\left(\frac{x}{2}\right)\left(\frac{1}{2}\right) dx = -2\cot\left(\frac{x}{2}\right) + C$

37. $\int \frac{1}{\theta^2}\cos\frac{1}{\theta}\, d\theta = -\int \cos\frac{1}{\theta}\left(-\frac{1}{\theta^2}\right) d\theta = -\sin\frac{1}{\theta} + C$

38. $\int x \sin x^2\, dx = \frac{1}{2}\int\left(\sin x^2\right)(2x)\, dx = -\frac{1}{2}\cos x^2 + C$

39. $\int \sin 2x \cos 2x\, dx = \frac{1}{2}\int(\sin 2x)(2\cos 2x)\, dx = \frac{1}{2}\frac{(\sin 2x)^2}{2} + C = \frac{1}{4}\sin^2 2x + C$ OR

$\int \sin 2x \cos 2x\, dx = -\frac{1}{2}\int(\cos 2x)(-2\sin 2x)\, dx = -\frac{1}{2}\frac{(\cos 2x)^2}{2} + C_1 = -\frac{1}{4}\cos^2 2x + C_1$ OR

$\int \sin 2x \cos 2x\, dx = \frac{1}{2}\int 2\sin 2x \cos 2x\, dx = \frac{1}{2}\int \sin 4x\, dx = -\frac{1}{8}\cos 4x + C_2$

40. $\int \sqrt{\tan x}\, \sec^2 x\, dx = \frac{(\tan x)^{3/2}}{3/2} + C = \frac{2}{3}(\tan x)^{3/2} + C$

41. $$\int \frac{\csc^2 x}{\cot^3 x}\, dx = -\int(\cot x)^{-3}\left(-\csc^2 x\right) dx$$

$$= -\frac{(\cot x)^{-2}}{-2} + C = \frac{1}{2\cot^2 x} + C = \frac{1}{2}\tan^2 x + C = \frac{1}{2}\left(\sec^2 x - 1\right) + C = \frac{1}{2}\sec^2 x + C_1$$

42. $$\int \frac{\sin x}{\cos^3 x}\, dx = -\int(\cos x)^{-3}(-\sin x)\, dx$$

$$= -\frac{(\cos x)^{-2}}{-2} + C$$

$$= \frac{1}{2\cos^2 x} + C = \frac{1}{2}\sec^2 x + C$$

43. $f(x) = \int -\sin\frac{x}{2}\, dx = 2\cos\frac{x}{2} + C$

Because $f(0) = 6 = 2\cos\left(\frac{0}{2}\right) + C$, $C = 4$. So,

$f(x) = 2\cos\frac{x}{2} + 4.$

44. $f'(x) = \sec^2(2x), \left(\frac{\pi}{2}, 2\right)$

$$f(x) = \frac{1}{2}\tan(2x) + C$$

$$f\left(\frac{\pi}{2}\right) = \frac{1}{2}\tan\left(2\left(\frac{\pi}{2}\right)\right) + C = 2$$

$$\frac{1}{2}(0) + C = 2$$

$$C = 2$$

$$f(x) = \frac{1}{2}\tan(2x) + 2$$

45. $f'(x) = 2x(4x^2 - 10)^2, (2, 10)$

$$f(x) = \frac{(4x^2 - 10)^3}{12} + C = \frac{2(2x^2 - 5)^3}{3} + C$$

$$f(2) = \frac{2(8 - 5)^3}{3} + C = 18 + C = 10 \Rightarrow C = -8$$

$$f(x) = \frac{2}{3}(2x^2 - 5)^3 - 8$$

46. $f'(x) = -2x\sqrt{8 - x^2}, (2, 7)$

$$f(x) = \frac{2(8 - x^2)^{3/2}}{3} + C$$

$$f(2) = \frac{2(4)^{3/2}}{3} + C = \frac{16}{3} + C = 7 \Rightarrow C = \frac{5}{3}$$

$$f(x) = \frac{2(8 - x^2)^{3/2}}{3} + \frac{5}{3}$$

47. $u = x + 6, x = u - 6, dx = du$

$$\begin{aligned}\int x\sqrt{x + 6}\,dx &= \int (u - 6)\sqrt{u}\,du \\ &= \int \left(u^{3/2} - 6u^{1/2}\right) du \\ &= \frac{2}{5}u^{5/2} - 4u^{3/2} + C \\ &= \frac{2u^{3/2}}{5}(u - 10) + C \\ &= \frac{2}{5}(x + 6)^{3/2}\left[(x + 6) - 10\right] + C \\ &= \frac{2}{5}(x + 6)^{3/2}(x - 4) + C\end{aligned}$$

48. $u = 3x - 4, x = \frac{u + 4}{3}, dx = \frac{1}{3}du$

$$\begin{aligned}\int x\sqrt{3x - 4}\,dx &= \int \frac{u + 4}{3} \cdot \sqrt{u} \cdot \frac{1}{3}du \\ &= \frac{1}{9}\int \left(u^{3/2} + 4u^{1/2}\right) du \\ &= \frac{1}{9}\left(\frac{2}{5}u^{5/2} + \frac{8}{3}u^{3/2}\right) + C \\ &= \frac{2}{45}(3x - 4)^{5/2} + \frac{8}{27}(3x - 4)^{3/2} + C \\ &= \frac{2}{135}(3x - 4)^{3/2}\left[3(3x - 4) + 20\right] + C \\ &= \frac{2}{135}(3x - 4)^{3/2}(9x + 8) + C\end{aligned}$$

49. $u = 1 - x, x = 1 - u, dx = -du$

$$\begin{aligned}\int x^2\sqrt{1 - x}\,dx &= -\int (1 - u)^2\sqrt{u}\,du \\ &= -\int \left(u^{1/2} - 2u^{3/2} + u^{5/2}\right) du \\ &= -\left(\frac{2}{3}u^{3/2} - \frac{4}{5}u^{5/2} + \frac{2}{7}u^{7/2}\right) + C \\ &= -\frac{2u^{3/2}}{105}\left(35 - 42u + 15u^2\right) + C \\ &= -\frac{2}{105}(1 - x)^{3/2}\left[35 - 42(1 - x) + 15(1 - x)^2\right] + C \\ &= -\frac{2}{105}(1 - x)^{3/2}\left(15x^2 + 12x + 8\right) + C\end{aligned}$$

50. $u = 2 - x,\ x = 2 - u,\ dx = -du$

$$\begin{aligned}\int (x+1)\sqrt{2-x}\,dx &= -\int (3-u)\sqrt{u}\,du \\ &= -\int \left(3u^{1/2} - u^{3/2}\right) du \\ &= -\left(2u^{3/2} - \frac{2}{5}u^{5/2}\right) + C \\ &= -\frac{2u^{3/2}}{5}(5-u) + C \\ &= -\frac{2}{5}(2-x)^{3/2}\left[5-(2-x)\right] + C \\ &= -\frac{2}{5}(2-x)^{3/2}(x+3) + C\end{aligned}$$

51. $u = 2x - 1,\ x = \frac{1}{2}(u+1),\ dx = \frac{1}{2}\,du$

$$\begin{aligned}\int \frac{x^2-1}{\sqrt{2x-1}}\,dx &= \int \frac{\left[(1/2)(u+1)\right]^2 - 1}{\sqrt{u}}\frac{1}{2}\,du \\ &= \frac{1}{8}\int u^{-1/2}\left[\left(u^2+2u+1\right) - 4\right] du \\ &= \frac{1}{8}\int \left(u^{3/2} + 2u^{1/2} - 3u^{-1/2}\right) du \\ &= \frac{1}{8}\left(\frac{2}{5}u^{5/2} + \frac{4}{3}u^{3/2} - 6u^{1/2}\right) + C \\ &= \frac{u^{1/2}}{60}\left(3u^2 + 10u - 45\right) + C \\ &= \frac{\sqrt{2x-1}}{60}\left[3(2x-1)^2 + 10(2x-1) - 45\right] + C \\ &= \frac{1}{60}\sqrt{2x-1}\left(12x^2 + 8x - 52\right) + C \\ &= \frac{1}{15}\sqrt{2x-1}\left(3x^2 + 2x - 13\right) + C\end{aligned}$$

52. $u = x + 4,\ x = u - 4,\ du = dx$

$$\begin{aligned}\int \frac{2x+1}{\sqrt{x+4}}\,dx &= \int \frac{2(u-4)+1}{\sqrt{u}}\,du \\ &= \int \left(2u^{1/2} - 7u^{-1/2}\right) du \\ &= \frac{4}{3}u^{3/2} - 14u^{1/2} + C \\ &= \frac{2}{3}u^{1/2}(2u - 21) + C \\ &= \frac{2}{3}\sqrt{x+4}\left[2(x+4) - 21\right] + C \\ &= \frac{2}{3}\sqrt{x+4}(2x-13) + C\end{aligned}$$

53. $u = x + 1,\ x = u - 1,\ dx = du$

$$\begin{aligned}\int \frac{-x}{(x+1) - \sqrt{x+1}}\,dx &= \int \frac{-(u-1)}{u - \sqrt{u}}\,du \\ &= -\int \frac{\left(\sqrt{u}+1\right)\left(\sqrt{u}-1\right)}{\sqrt{u}\left(\sqrt{u}-1\right)}\,du \\ &= -\int \left(1 + u^{-1/2}\right) du \\ &= -\left(u + 2u^{1/2}\right) + C \\ &= -u - 2\sqrt{u} + C \\ &= -(x+1) - 2\sqrt{x+1} + C \\ &= -x - 2\sqrt{x+1} - 1 + C \\ &= -\left(x + 2\sqrt{x+1}\right) + C_1\end{aligned}$$

where $C_1 = -1 + C$.

54. $u = t + 10,\ t = u - 10,\ du = dt$

$$\begin{aligned}\int t(t + 10)^{1/3}\,dt &= \int (u - 10)u^{1/3}\,du\\ &= \int \left(u^{4/3} - 10u^{1/3}\right)du\\ &= \frac{3}{7}u^{7/3} - \frac{15}{2}u^{4/3} + C\\ &= \frac{3}{14}u^{4/3}(2u - 35) + C\\ &= \frac{3}{14}(t + 10)^{4/3}\left[2(t + 10) - 35\right] + C\\ &= \frac{3}{14}(t + 10)^{4/3}(2t - 15) + C\end{aligned}$$

55. Let $u = x^2 + 1,\ du = 2x\,dx$.

$$\int_{-1}^{1} x\left(x^2 + 1\right)^3 dx = \frac{1}{2}\int_{-1}^{1}\left(x^2 + 1\right)^3(2x)\,dx = \left[\frac{1}{8}\left(x^2 + 1\right)^4\right]_{-1}^{1} = 0$$

56. Let $u = 2x^4 + 1,\ du = 8x^3\,dx$.

$$\int_0^1 x^3\left(2x^4 + 1\right)^2 dx = \frac{1}{8}\int_0^1\left(2x^4 + 1\right)^2\left(8x^3\right)dx = \left[\frac{1}{8}\cdot\frac{\left(2x^4 + 1\right)^3}{3}\right]_0^1 = \frac{1}{24}\left(3^3 - 1^3\right) = \frac{13}{12}$$

57. Let $u = x^3 + 1, du = 3x^2\,dx$.

$$\int_1^2 2x^2\sqrt{x^3 + 1}\,dx = 2\cdot\frac{1}{3}\int_1^2\left(x^3 + 1\right)^{1/2}\left(3x^2\right)dx = \left[\frac{\left(x^3 + 1\right)^{3/2}}{3/2}\right]_1^2 = \frac{4}{9}\left[\left(x^3 + 1\right)^{3/2}\right]_1^2 = \frac{4}{9}\left[27 - 2\sqrt{2}\right] = 12 - \frac{8}{9}\sqrt{2}$$

58. Let $u = 1 - x^2,\ du = -2x\,dx$.

$$\int_0^1 x\sqrt{1 - x^2}\,dx = -\frac{1}{2}\int_0^1\left(1 - x^2\right)^{1/2}(-2x)\,dx = \left[-\frac{1}{3}\left(1 - x^2\right)^{3/2}\right]_0^1 = 0 + \frac{1}{3} = \frac{1}{3}$$

59. Let $u = 2x + 1,\ du = 2\,dx$.

$$\int_0^4 \frac{1}{\sqrt{2x + 1}}\,dx = \frac{1}{2}\int_0^4 (2x + 1)^{-1/2}(2)\,dx = \left[\sqrt{2x + 1}\right]_0^4 = \sqrt{9} - \sqrt{1} = 2$$

60. Let $u = 1 + 2x^2,\ du = 4x\,dx$.

$$\int_0^2 \frac{x}{\sqrt{1 + 2x^2}}\,dx = \frac{1}{4}\int_0^2\left(1 + 2x^2\right)^{-1/2}(4x)\,dx = \left[\frac{1}{2}\sqrt{1 + 2x^2}\right]_0^2 = \frac{3}{2} - \frac{1}{2} = 1$$

61. Let $u = 1 + \sqrt{x}, du = \dfrac{1}{2\sqrt{x}}\,dx$.

$$\int_1^9 \frac{1}{\sqrt{x}\left(1 + \sqrt{x}\right)^2}\,dx = 2\int_1^9\left(1 + \sqrt{x}\right)^{-2}\left(\frac{1}{2\sqrt{x}}\right)dx = \left[-\frac{2}{1 + \sqrt{x}}\right]_1^9 = -\frac{1}{2} + 1 = \frac{1}{2}$$

62. Let $u = 2x - 1$, $du = 2\,dx$, $x = \frac{1}{2}(u + 1)$.

When $x = 1, u = 1$. When $x = 5, u = 9$.

$$\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx = \int_1^9 \frac{1/2(u+1)}{\sqrt{u}}\frac{1}{2}\,du = \frac{1}{4}\int_1^9 \left(u^{1/2} + u^{-1/2}\right)du$$
$$= \frac{1}{4}\left[\frac{2}{3}u^{3/2} + 2u^{1/2}\right]_1^9$$
$$= \frac{1}{4}\left[\left(\frac{2}{3}(27) + 2(3)\right) - \left(\frac{2}{3} + 2\right)\right]$$
$$= \frac{16}{3}$$

63. $\frac{dy}{dx} = 18x^2\left(2x^3 + 1\right)^2, (0, 4)$

$$y = 3\int\left(2x^3 + 1\right)^2\left(6x^2\right)dx \quad \left(u = 2x^3 + 1\right)$$
$$y = 3\frac{\left(2x^3+1\right)^3}{3} + C = \left(2x^3 + 1\right)^3 + C$$
$$4 = 1^3 + C \Rightarrow C = 3$$
$$y = \left(2x^3 + 1\right)^3 + 3$$

64. $\frac{dy}{dx} = \frac{-48}{(3x+5)^3}, (-1, 3)$

$$y = -48\int(3x + 5)^{-3}dx$$
$$= (-48)\frac{1}{3}\int(3x+5)^{-3}3\,dx$$
$$= \frac{-16(3x+5)^{-2}}{-2} + C$$
$$= \frac{8}{(3x+5)^2} + C$$
$$3 = \frac{8}{(3(-1)+5)^2} + C = \frac{8}{4} + C \Rightarrow C = 1$$
$$y = \frac{8}{(3x+5)^2} + 1$$

65. $u = x + 1$, $x = u - 1$, $dx = du$

When $x = 0, u = 1$. When $x = 7$, $u = 8$.

$$\text{Area} = \int_0^7 x\sqrt[3]{x+1}\,dx = \int_1^8 (u-1)\sqrt[3]{u}\,du$$
$$= \int_1^8 \left(u^{4/3} - u^{1/3}\right)du$$
$$= \left[\frac{3}{7}u^{7/3} - \frac{3}{4}u^{4/3}\right]_1^8$$
$$= \left(\frac{384}{7} - 12\right) - \left(\frac{3}{7} - \frac{3}{4}\right)$$
$$= \frac{1209}{28}$$

66. $u = x + 2$, $x = u - 2$, $dx = du$

When $x = -2$, $u = 0$. When $x = 6$, $u = 8$.

$$\text{Area} = \int_{-2}^6 x^2\sqrt[3]{x+2}\,dx$$
$$= \int_0^8 (u-2)^2\sqrt[3]{u}\,du$$
$$= \int_0^8 \left(u^{7/3} - 4u^{4/3} + 4u^{1/3}\right)du$$
$$= \left[\frac{3}{10}u^{10/3} - \frac{12}{7}u^{7/3} + 3u^{4/3}\right]_0^8 = \frac{4752}{35}$$

67. $$\text{Area} = \int_{\pi/2}^{2\pi/3} \sec^2\left(\frac{x}{2}\right)dx$$
$$= 2\int_{\pi/2}^{2\pi/3} \sec^2\left(\frac{x}{2}\right)\left(\frac{1}{2}\right)dx$$
$$= \left[2\tan\left(\frac{x}{2}\right)\right]_{\pi/2}^{2\pi/3} = 2\left(\sqrt{3} - 1\right)$$

68. Let $u = 2x$, $du = 2\,dx$.

$$\text{Area} = \int_{\pi/12}^{\pi/4} \csc 2x \cot 2x\,dx$$
$$= \frac{1}{2}\int_{\pi/12}^{\pi/4} \csc 2x \cot 2x(2)\,dx$$
$$= \left[-\frac{1}{2}\csc 2x\right]_{\pi/12}^{\pi/4} = \frac{1}{2}$$

69. $f(x) = x^2(x^2 + 1)$ is even.

$$\int_{-2}^{2} x^2(x^2+1)\,dx = 2\int_0^2 (x^4 + x^2)\,dx = 2\left[\frac{x^5}{5} + \frac{x^3}{3}\right]_0^2$$
$$= 2\left[\frac{32}{5} + \frac{8}{3}\right] = \frac{272}{15}$$

70. $f(x) = x(x^2 + 1)^3$ is odd.

$$\int_{-2}^{2} x(x^2+1)^3\,dx = 0$$

71. $f(x) = \sin^2 x \cos x$ is even.

$$\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x\,dx = 2\int_0^{\pi/2} \sin^2 x(\cos x)\,dx$$
$$= 2\left[\frac{\sin^3 x}{3}\right]_0^{\pi/2}$$
$$= \frac{2}{3}$$

72. $f(x) = \sin x \cos x$ is odd.

$$\int_{-\pi/2}^{\pi/2} \sin x \cos x\,dx = 0$$

73. $\int_0^4 x^2\,dx = \left[\frac{x^3}{3}\right]_0^4 = \frac{64}{3}$; the function x^2 is an even function.

(a) $\int_{-4}^{0} x^2\,dx = \int_0^4 x^2\,dx = \frac{64}{3}$

(b) $\int_{-4}^{4} x^2\,dx = 2\int_0^4 x^2\,dx = \frac{128}{3}$

(c) $\int_0^4 (-x^2)\,dx = -\int_0^4 x^2\,dx = -\frac{64}{3}$

(d) $\int_{-4}^{0} 3x^2\,dx = 3\int_0^4 x^2\,dx = 64$

74. (a) $\int_{-\pi/4}^{\pi/4} \sin x\,dx = 0$ because $\sin x$ is symmetric to the origin.

(b) $\int_{-\pi/4}^{\pi/4} \cos x\,dx = 2\int_0^{\pi/4} \cos x\,dx = [2\sin x]_0^{\pi/4} = \sqrt{2}$ because $\cos x$ is symmetric to the y-axis.

(c) $\int_{-\pi/2}^{\pi/2} \cos x\,dx = 2\int_0^{\pi/2} \cos x\,dx = [2\sin x]_0^{\pi/2} = 2$

(d) $\int_{-\pi/2}^{\pi/2} \sin x \cos x\,dx = 0$ because $\sin(-x)\cos(-x) = -\sin x \cos x$ and so, is symmetric to the origin.

75. $\int_{-3}^{3} (x^3 + 4x^2 - 3x - 6)\,dx = \int_{-3}^{3} (x^3 - 3x)\,dx + \int_{-3}^{3} (4x^2 - 6)\,dx = 0 + 2\int_0^3 (4x^2 - 6)\,dx = 2\left[\frac{4}{3}x^3 - 6x\right]_0^3 = 36$

76. $\int_{-\pi/2}^{\pi/2} (\sin 4x + \cos 4x)\,dx = \int_{-\pi/2}^{\pi/2} \sin 4x\,dx + \int_{-\pi/2}^{\pi/2} \cos 4x\,dx = 0 + 2\int_0^{\pi/2} \cos 4x\,dx = \left[\frac{2}{4}\sin 4x\right]_0^{\pi/2} = 0$

77. If $u = 5 - x^2$, then $du = -2x\,dx$ and $\int x(5 - x^2)^3\,dx = -\frac{1}{2}\int (5 - x^2)^3(-2x)\,dx = -\frac{1}{2}\int u^3\,du$.

78. $f(x) = x(x^2 + 1)^2$ is odd. So, $\int_{-2}^{2} x(x^2+1)^2\,dx = 0$.

79. (a) The second integral is easier. Use substitution with $u = x^3 + 1$ and $du = 3x^2dx$. The answer is

$$\int x^2\sqrt{x^3+1}\,dx = \frac{1}{3}\int(x^3+1)^{1/2}3x^2dx = \frac{2}{9}(x^3+1)^{3/2} + C.$$

(b) The first integral is easier. Use substitution with $u = \tan 3x$ and $du = 3\sec^2(3x)dx$. The answer is

$$\int \tan(3x)\sec^2(3x)dx = \frac{1}{3}\int \tan(3x)3\sec^2(3x)dx = \frac{1}{6}\tan^2 3x + C.$$

80. (a) $\int(2x-1)^2\,dx = \frac{1}{2}\int(2x-1)^2 2\,dx = \frac{1}{6}(2x-1)^3 + C_1 = \frac{4}{3}x^3 - 2x^2 + x - \frac{1}{6} + C_1$

$\int(2x-1)^2\,dx = \int(4x^2-4x+1)\,dx = \frac{4}{3}x^3 - 2x^2 + x + C_2$

They differ by constant: $C_2 = C_1 - \frac{1}{6}$.

(b) $\int \tan x \sec^2 x\,dx = \frac{\tan^2 x}{2} + C_1$

$\int \tan x \sec^2 x\,dx = \int \sec x(\sec x \tan x)\,dx = \frac{\sec^2 x}{2} + C_2$

$\frac{\tan^2 x}{2} + C_1 = \frac{\sec^2 x - 1}{2} + C_1 = \frac{\sec^2 x}{2} - \frac{1}{2} + C_1$

They differ by a constant: $C_2 = C_1 - \frac{1}{2}$.

81. $\frac{dV}{dt} = \frac{k}{(t+1)^2}$

$V(t) = \int \frac{k}{(t+1)^2}\,dt = -\frac{k}{t+1} + C$

$V(0) = -k + C = 500{,}000$

$V(1) = -\frac{1}{2}k + C = 400{,}000$

Solving this system yields $k = -200{,}000$ and $C = 300{,}000$. So, $V(t) = \frac{200{,}000}{t+1} + 300{,}000$.

When $t = 4$, $V(4) = \$340{,}000$.

82. (a) The maximum flow is approximately $R \approx 62$ thousand gallons at 9:00 A.M. $(t \approx 9)$.

(b) The volume of water used during the day is the area under the curve for $0 \le t \le 24$. That is, $V = \int_0^{24} R(t)dt$.

(c) The least amount of water is used approximately from 1 A.M. to 3 A.M. $(1 \le t \le 3)$.

83. $\frac{1}{b-a}\int_a^b\left[74.50 + 43.75\sin\frac{\pi t}{6}\right]dt = \frac{1}{b-a}\left[74.50t - \frac{262.5}{\pi}\cos\frac{\pi t}{6}\right]_a^b$

(a) $\frac{1}{3}\left[74.50t - \frac{262.5}{\pi}\cos\frac{\pi t}{6}\right]_0^3 = \frac{1}{3}\left(223.5 + \frac{262.5}{\pi}\right) \approx 102.352$ thousand units

(b) $\frac{1}{3}\left[74.50t - \frac{262.5}{\pi}\cos\frac{\pi t}{6}\right]_3^6 = \frac{1}{3}\left(447 + \frac{262.5}{\pi} - 223.5\right) \approx 102.352$ thousand units

(c) $\frac{1}{12}\left[74.50t - \frac{262.5}{\pi}\cos\frac{\pi t}{6}\right]_0^{12} = \frac{1}{12}\left(894 - \frac{262.5}{\pi} + \frac{262.5}{\pi}\right) = 74.5$ thousand units

84. $\dfrac{1}{b-a}\int_a^b \left[2\sin(60\pi t) + \cos(120\pi t)\right] dt = \dfrac{1}{b-a}\left[-\dfrac{1}{30\pi}\cos(60\pi t) + \dfrac{1}{120\pi}\sin(120\pi t)\right]_a^b$

(a) $\dfrac{1}{(1/60) - 0}\left[-\dfrac{1}{30\pi}\cos(60\pi t) + \dfrac{1}{120\pi}\sin(120\pi t)\right]_0^{1/60} = 60\left[\left(\dfrac{1}{30\pi} + 0\right) - \left(-\dfrac{1}{30\pi}\right)\right] = \dfrac{4}{\pi} \approx 1.273$ amps

(b) $\dfrac{1}{(1/240) - 0}\left[-\dfrac{1}{30\pi}\cos(60\pi t) + \dfrac{1}{120\pi}\sin(120\pi t)\right]_0^{1/240} = 240\left[\left(-\dfrac{1}{30\sqrt{2}\pi} + \dfrac{1}{120\pi}\right) - \left(-\dfrac{1}{30\pi}\right)\right]$

$= \dfrac{2}{\pi}\left(5 - 2\sqrt{2}\right) \approx 1.382$ amps

(c) $\dfrac{1}{(1/30) - 0}\left[-\dfrac{1}{30\pi}\cos(60\pi t) + \dfrac{1}{120\pi}\sin(120\pi t)\right]_0^{1/30} = 30\left[\left(-\dfrac{1}{30\pi}\right) - \left(-\dfrac{1}{30\pi}\right)\right] = 0$ amp

85. $u = 1 - x$, $x = 1 - u$, $dx = -du$

When $x = a$, $u = 1 - a$. When $x = b$, $u = 1 - b$.

$P_{a,b} = \int_a^b \dfrac{15}{4}x\sqrt{1-x}\,dx = \dfrac{15}{4}\int_{1-a}^{1-b} -(1-u)\sqrt{u}\,du$

$= \dfrac{15}{4}\int_{1-a}^{1-b}\left(u^{3/2} - u^{1/2}\right)du = \dfrac{15}{4}\left[\dfrac{2}{5}u^{5/2} - \dfrac{2}{3}u^{3/2}\right]_{1-a}^{1-b} = \dfrac{15}{4}\left[\dfrac{2u^{3/2}}{15}(3u - 5)\right]_{1-a}^{1-b} = \left[-\dfrac{(1-x)^{3/2}}{2}(3x + 2)\right]_a^b$

(a) $P_{0.50,\,0.75} = \left[-\dfrac{(1-x)^{3/2}}{2}(3x + 2)\right]_{0.50}^{0.75} = 0.353 = 35.3\%$

(b) $P_{0,b} = \left[-\dfrac{(1-x)^{3/2}}{2}(3x + 2)\right]_0^b = -\dfrac{(1-b)^{3/2}}{2}(3b + 2) + 1 = 0.5$

$(1-b)^{3/2}(3b + 2) = 1$

$b \approx 0.586 = 58.6\%$

86. $u = 1 - x$, $x = 1 - u$, $dx = -du$

When $x = a$, $u = 1 - a$. When $x = b$, $u = 1 - b$.

$P_{a,b} = \int_a^b \dfrac{1155}{32}x^3(1-x)^{3/2}\,dx = \dfrac{1155}{32}\int_{1-a}^{1-b} -(1-u)^3 u^{3/2}\,du$

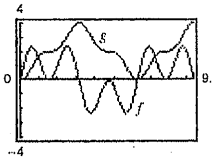
$= \dfrac{1155}{32}\int_{1-a}^{1-b}\left(u^{9/2} - 3u^{7/2} + 3u^{5/2} - u^{3/2}\right)du = \dfrac{1155}{32}\left[\dfrac{2}{11}u^{11/2} - \dfrac{2}{3}u^{9/2} + \dfrac{6}{7}u^{7/2} - \dfrac{2}{5}u^{5/2}\right]_{1-a}^{1-b}$

$= \dfrac{1155}{32}\left[\dfrac{2u^{5/2}}{1155}\left(105u^3 - 385u^2 + 495u - 231\right)\right]_{1-a}^{1-b} = \left[\dfrac{u^{5/2}}{16}\left(105u^3 - 385u^2 + 495u - 231\right)\right]_{1-a}^{1-b}$

(a) $P_{0,\,0.25} = \left[\dfrac{u^{5/2}}{16}\left(105u^3 - 385u^2 + 495u - 231\right)\right]_1^{0.75} \approx 0.025 = 2.5\%$

(b) $P_{0.5,\,1} = \left[\dfrac{u^{5/2}}{16}\left(105u^3 - 385u^2 + 495u - 231\right)\right]_{0.5}^{0} \approx 0.736 = 73.6\%$

87. (a)

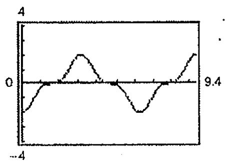


(b) g is nonnegative because the graph of f is positive at the beginning, and generally has more positive sections than negative ones.

(c) The points on g that correspond to the extrema of f are points of inflection of g.

(d) No, some zeros of f, like $x = \pi/2$, do not correspond to an extrema of g. The graph of g continues to increase after $x = \pi/2$ because f remains above the x-axis.

(e) The graph of h is that of g shifted 2 units downward.



$$g(t) = \int_0^t f(x)\,dx = \int_0^{\pi/2} f(x)\,dx + \int_{\pi/2}^t f(x)\,dx = 2 + h(t).$$

88. Let $f(x) = \sin \pi x,\ 0 \le x \le 1.$

Let $\Delta x = \dfrac{1}{n}$ and use righthand endpoints

$c_i = \dfrac{i}{n},\ i = 1, 2, \ldots, n.$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\sin(i\pi/n)}{n} = \lim_{\|\Delta x\|\to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x$$
$$= \int_0^1 \sin \pi x\,dx$$
$$= -\frac{1}{\pi}\cos \pi x\Big]_0^1$$
$$= -\frac{1}{\pi}(-1 - 1) = \frac{2}{\pi}$$

89. (a) Let $u = 1 - x,\ du = -dx,\ x = 1 - u$

$x = 0 \Rightarrow u = 1,\ x = 1 \Rightarrow u = 0$

$$\int_0^1 x^2(1 - x)^5\,dx = \int_1^0 (1 - u)^2 u^5(-du)$$
$$= \int_0^1 u^5(1 - u)^2\,du$$
$$= \int_0^1 x^5(1 - x)^2\,dx$$

(b) Let $u = 1 - x,\ du = -dx,\ x = 1 - u$

$x = 0 \Rightarrow u = 1,\ x = 1 \Rightarrow u = 0$

$$\int_0^1 x^a(1 - x)^b\,dx = \int_1^0 (1 - u)^a u^b(-du)$$
$$= \int_0^1 u^b(1 - u)^a\,du$$
$$= \int_0^1 x^b(1 - x)^a\,dx$$

90. (a) $\sin x = \cos\left(\dfrac{\pi}{2} - x\right)$ and $\cos x = \sin\left(\dfrac{\pi}{2} - x\right)$

Let $u = \dfrac{\pi}{2} - x,\ du = -dx,\ x = \dfrac{\pi}{2} - u$:

$$\int_0^{\pi/2} \sin^2 x\,dx = \int_0^{\pi/2} \cos^2\left(\frac{\pi}{2} - x\right)dx$$
$$= \int_{\pi/2}^0 \cos^2 u(-du)$$
$$= \int_0^{\pi/2} \cos^2 u\,du = \int_0^{\pi/2} \cos^2 x\,dx$$

(b) Let $u = \dfrac{\pi}{2} - x$ as in part (a):

$$\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n\left(\frac{\pi}{2} - x\right)dx$$
$$= \int_{\pi/2}^0 \cos^n u(-du)$$
$$= \int_0^{\pi/2} \cos^n u\,du = \int_0^{\pi/2} \cos^n x\,dx$$

91. False

$$\int (2x+1)^2\,dx = \frac{1}{2}\int (2x+1)^2 2\,dx = \frac{1}{6}(2x+1)^3 + C$$

92. False

$$\int x(x^2+1)\,dx = \frac{1}{2}\int (x^2+1)(2x)\,dx = \frac{1}{4}(x^2+1)^2 + C$$

93. True

$$\int_{-10}^{10} (ax^3 + bx^2 + cx + d)\,dx = \underbrace{\int_{-10}^{10} (ax^3 + cx)\,dx}_{\text{Odd}} + \underbrace{\int_{-10}^{10} (bx^2 + d)\,dx}_{\text{Even}} = 0 + 2\int_0^{10} (bx^2 + d)\,dx$$

94. True

$$\int_a^b \sin x\,dx = \left[-\cos x\right]_a^b = -\cos b + \cos a = -\cos(b + 2\pi) + \cos a = \int_a^{b+2\pi} \sin x\,dx$$

95. True

$$4\int \sin x\cos x\,dx = 2\int \sin 2x\,dx = -\cos 2x + C$$

96. False

$$\int \sin^2 2x\cos 2x\,dx = \frac{1}{2}\int (\sin 2x)^2(2\cos 2x)\,dx$$
$$= \frac{1}{2}\frac{(\sin 2x)^3}{3} + C$$
$$= \frac{1}{6}\sin^3 2x + C$$

97. Let $u = cx$, $du = c\,dx$:

$$c\int_a^b f(cx)\,dx = c\int_{ca}^{cb} f(u)\frac{du}{c}$$
$$= \int_{ca}^{cb} f(u)\,du$$
$$= \int_{ca}^{cb} f(x)\,dx$$

98. (a) $\frac{d}{du}[\sin u - u\cos u + C] = \cos u - \cos u + u\sin u$

$$= u\sin u$$

So, $\int u\sin u\,du = \sin u - u\cos u + C.$

(b) Let $u = \sqrt{x}$, $u^2 = x$, $2u\,du = dx$.

$$\int_0^{\pi^2} \sin\sqrt{x}\,dx = \int_0^{\pi} \sin u(2u\,du)$$
$$= 2\int_0^{\pi} u\sin u\,du$$
$$= 2[\sin u - u\cos u]_0^{\pi} \text{ (part (a))}$$
$$= 2[-\pi\cos(\pi)]$$
$$= 2\pi$$

99. Because f is odd, $f(-x) = -f(x)$. Then

$$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_0^a f(x)\,dx$$
$$= -\int_0^{-a} f(x)\,dx + \int_0^a f(x)\,dx.$$

Let $x = -u$, $dx = -du$ in the first integral.

When $x = 0$, $u = 0$. When $x = -a$, $u = a$.

$$\int_{-a}^{1} f(x)\,dx = -\int_0^a f(-u)(-du) + \int_0^a f(x)\,dx$$
$$= -\int_0^a f(u)\,du + \int_0^a f(x)\,dx = 0$$

100. Let $u = x + h$, then $du = dx$.

When $x = a$, $u = a + h$.

When $x = b$, $u = b + h$. So,

$$\int_a^b f(x+h)\,dx = \int_{a+h}^{b+h} f(u)\,du = \int_{a+h}^{b+h} f(x)\,dx.$$

101. Let $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$.

$$\int_0^1 f(x)\,dx = \left[a_0x + a_1\frac{x^2}{2} + a_2\frac{x^3}{3} + \cdots + a_n\frac{x^{n+1}}{n+1}\right]_0^1$$
$$= a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \cdots + \frac{a_n}{n+1} = 0 \text{ (Given)}$$

By the Mean Value Theorem for Integrals, there exists c in $[0, 1]$ such that

$$\int_0^1 f(x)\,dx = f(c)(1 - 0)$$
$$0 = f(c).$$

So the equation has at least one real zero.