

1)

$$\int (5x + 4)^5 dx$$

2)

$$\int 3t^2(t^3 + 4)^5 dt$$

3)

$$\int \sqrt{4x - 5} dx$$

4)

$$\int \frac{5x^2}{\sqrt[5]{x^3 - 2}} dx$$

5)

$$\int \cos(2x + 1) dx$$

6)

$$\int \sin^{10}(x) \cos(x) dx$$

$$7) \int \frac{\sin(x)}{(\cos(x))^5} dx$$

8)

$$\int \frac{2}{\sqrt{3x-1}} dx$$

9)

$$\int \frac{4}{x^2} \sec\left(\frac{5}{x}\right) \tan\left(\frac{5}{x}\right) dx$$

10)

$$\int \frac{3x^4}{(7-x^5)^6} dx$$

11)

$$\int \frac{x^3(2x-1)}{\sqrt{x}} dx$$

12)

$$\int 7x^2 \sqrt{3-2x^3} dx$$

Key

1)

$$\int (5x+4)^5 dx$$

$$u = 5x+4$$

$$\frac{du}{dx} = 5 \quad \left| \quad \frac{du}{5} = dx \right.$$

$$du = 5dx$$

$$\int u^5 \cdot \frac{du}{5}$$

$$\frac{1}{5} \int u^5 du = \frac{1}{5} \left( \frac{u^6}{6} \right) + C$$

$$= \frac{1}{30} u^6 + C$$

$$= \boxed{\frac{1}{30} (5x+4)^6 + C}$$

2)

$$\int 3t^2 (t^3+4)^5 dt$$

$$u = t^3+4$$

$$\frac{du}{dt} = 3t^2$$

$$du = 3t^2 dt$$

$$\frac{du}{3t^2} = dt$$

$$\int 3t^2 \cdot u^5 \cdot \frac{du}{3t^2}$$

$$\int u^5 du = \frac{u^6}{6} + C$$

$$= \boxed{\frac{1}{6} (t^3+4)^6 + C}$$

3)

$$\int \sqrt{4x-5} dx$$

$$\int (4x-5)^{1/2} dx$$

$$u = 4x-5$$

$$\frac{du}{dx} = 4$$

$$du = 4dx$$

$$\frac{du}{4} = dx$$

$$\int u^{1/2} \cdot \frac{du}{4}$$

$$\frac{1}{4} \int u^{1/2} du$$

$$\frac{1}{4} \cdot \frac{u^{3/2}}{3/2} + C$$

$$\frac{1}{4} \cdot \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{12} u^{3/2} + C$$

$$= \boxed{\frac{1}{6} (4x-5)^{3/2} + C}$$

4)

$$\int \frac{5x^2}{\sqrt[5]{x^3-2}} dx$$

$$u = x^3-2$$

$$\frac{du}{dx} = 3x^2$$

$$du = 3x^2 dx$$

$$\frac{du}{3x^2} = dx$$

$$\int \frac{5x^2}{(x^3-2)^{1/5}} dx$$

$$\int 5x^2 \cdot u^{-1/5} \cdot \frac{du}{3x^2}$$

$$\frac{5}{3} \int u^{-1/5} du$$

$$\frac{5}{3} \cdot \frac{u^{4/5}}{4/5} + C$$

$$= \frac{5 \cdot 5}{3 \cdot 4} u^{4/5} + C$$

$$= \boxed{\frac{25}{12} (x^3-2)^{4/5} + C}$$

5)

$$\int \cos(2x+1) dx$$

$$u = 2x+1$$

$$\frac{du}{dx} = 2$$

$$du = 2dx$$

$$\frac{du}{2} = dx$$

$$\int \cos u \cdot \frac{du}{2}$$

$$= \frac{1}{2} \int \cos u du$$

$$= \frac{1}{2} \sin u + C$$

$$= \boxed{\frac{1}{2} \sin(2x+1) + C}$$

6)

$$\int \sin^{10}(x) \cos(x) dx$$

$$\int (\sin x)^{10} \cos x dx$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$du = \cos x dx$$

$$\frac{du}{\cos x} = dx$$

$$\int u^{10} \cdot \cos x \cdot \frac{du}{\cos x}$$

$$= \int u^{10} du$$

$$= \frac{u^{11}}{11} + C$$

$$= \boxed{\frac{1}{11} (\sin x)^{11} + C}$$

$$7. \int \frac{\sin(x)}{(\cos(x))^5} dx$$

$$\int \frac{\sin x}{(\cos x)^5} dx$$

$$\int \sin x \cdot (\cos x)^{-5} dx$$

$$u = \cos x \quad \left| \begin{array}{l} du = -\sin x dx \\ \frac{du}{- \sin x} = dx \end{array} \right.$$

$$\int \frac{1}{4u^4} + C$$

$$\boxed{\frac{1}{4(\cos x)^4} + C}$$

8)

$$\int \frac{2}{\sqrt{3x-7}} dx$$

$$\int \frac{2}{(3x-7)^{1/2}} dx$$

$$u = 3x-7 \quad \left| \begin{array}{l} du = 3 dx \\ \frac{du}{3} = dx \end{array} \right.$$

$$\int 2 \cdot u^{-1/2} \cdot \frac{du}{3} = \frac{2}{3} \int u^{-1/2} du$$

$$\frac{2}{3} \cdot \frac{u^{1/2}}{1/2} + C = \frac{2}{3} \cdot \frac{2}{1} u^{1/2} + C$$

$$\boxed{\frac{4}{3} (3x-7)^{1/2} + C}$$

9)

$$\int \frac{4}{x^2} \sec\left(\frac{5}{x}\right) \tan\left(\frac{5}{x}\right) dx$$

$$u = \frac{5}{x} = 5x^{-1} \quad \left| \begin{array}{l} \frac{du}{dx} = -5x^{-2} \\ \frac{du}{dx} = \frac{-5}{x^2} \\ x^2 du = -5 dx \end{array} \right.$$

$$\int \frac{4}{x^2} \sec \tan u \cdot \frac{x^2 du}{-5}$$

$$= \frac{-4}{5} \int \sec u \tan u du$$

$$= \frac{-4}{5} \sec u = \boxed{\frac{-4}{5} \sec\left(\frac{5}{x}\right) + C}$$

10)

$$\int \frac{3x^4}{(7-x^5)^6} dx$$

$$\int 3x^4 (7-x^5)^{-6} dx$$

$$u = 7-x^5 \quad \left| \begin{array}{l} \frac{du}{dx} = -5x^4 \\ du = -5x^4 dx \end{array} \right.$$

$$\int \frac{3x^4 \cdot u^{-6} \cdot \frac{du}{-5x^4}}{-5}$$

$$= \frac{-3}{5} \int u^{-6} du$$

$$= \frac{-3}{5} \cdot \frac{u^{-5}}{-5} + C$$

$$= \frac{3}{25} (7-x^5)^{-5} + C$$

$$\boxed{\frac{3}{25(7-x^5)^5} + C}$$

11)

$$\int \frac{x^3(2x-1)}{\sqrt{x}} dx$$

$$\int (2x^4 - x^3) x^{-1/2} dx$$

$$\int 2x^{7/2} - x^{5/2} dx$$

$$\frac{2x^{9/2}}{9/2} - \frac{x^{7/2}}{7/2} + C$$

$$\boxed{\frac{4}{9} x^{9/2} - \frac{2}{7} x^{7/2} + C}$$

12)

$$\int 7x^2 \sqrt{3-2x^3} dx$$

$$\int 7x^2 (3-2x^3)^{1/2} dx$$

$$u = 3-2x^3 \quad \left| \begin{array}{l} \frac{du}{dx} = -6x^2 \\ du = -6x^2 dx \\ \frac{du}{-6x^2} = dx \end{array} \right.$$

$$\int \frac{7x^2 \cdot u^{1/2} \cdot \frac{du}{-6x^2}}{-6x^2}$$

$$= \frac{-7}{6} \int u^{1/2} du$$

$$= \frac{-7}{6} \cdot \frac{2}{3} u^{3/2} + C$$

$$= \frac{-14}{18} u^{3/2} + C$$

$$\boxed{\frac{-7}{9} (3-2x^3)^{3/2} + C}$$