2017

4.5a U-substitution p.301-302

#7-29 odd, 43-53 odd

Find an indefinite integral

7) $\int \sqrt{25-x^2}(-2x)\,dx$ | $\int (25-x^2)^{1/2}(-2x)\,dx$ | $\frac{u^{3/2}}{3/2}+c$

$u=25-x^2$ | $dx=\frac{du}{-2x}$ | $\int u^{1/2}\cancel{(-2x)}\cdot\frac{du}{\cancel{(-2x)}}$ | $=\boxed{\frac{2}{3}(25-x^2)^{3/2}+c}$

$\frac{du}{dx}=-2x$

17) $\int \frac{x}{(1-x^2)^3}dx = \int \frac{\cancel{x}}{(u)^3}\cdot\frac{du}{-2\cancel{x}} = \frac{-1}{2}\int u^{-3}du = \frac{-1}{2}\frac{u^{-2}}{-2}+c$

$u=1-x^2$ | $dx=\frac{du}{-2x}$

$\frac{du}{dx}=-2x$

$= \frac{+1}{4}(1-x^2)^{-2}+c$

$= \boxed{\frac{1}{4(1-x^2)^2}+c}$

23) $\int \left(1+\frac{1}{t}\right)^3\left(\frac{1}{t^2}\right)dt$

$u=1+\frac{1}{t}=1+t^{-1}$ | $\frac{du}{dt}=\frac{-1}{t^2}$ | $\int u^3\cdot\left(\frac{1}{\cancel{t^2}}\right)\cdot\left(-\cancel{t^2}du\right)$ | $-\frac{u^4}{4}+c$

$\frac{du}{dt}=-t^{-2}$ | $-dt=t^2du$ | $-\int u^3du$ | $=\boxed{-\frac{1}{4}\left(1+\frac{1}{t}\right)^4+c}$

$dt=-t^2du$

29) Solve differential equation (take the integral)

$\frac{dy}{dx}=\frac{x+1}{(x^2+2x-3)^2}$ | $u=x^2+2x-3$ | $y=\int\frac{\cancel{x+1}}{u^2}\cdot\frac{du}{2\cancel{(x+1)}}$ | $y=\frac{-1}{2u}+c$

$\frac{du}{dx}=2x+2$

$\int dy=\int\frac{x+1}{(x^2+2x-3)^2}dx$ | $dx=\frac{du}{2(x+1)}$ | $y=\frac{1}{2}\int u^{-2}du$ | $\boxed{y=\frac{-1}{2(x^2+2x-3)}+c}$

$y=\frac{1}{2}\left(\frac{u^{-1}}{-1}\right)+c$

43) Find particular solution $f'(x) = -\sin\left(\frac{x}{2}\right)$ point: $(0,6)$

$f(x) = \int -\sin(x/2)\,dx$

$u = \frac{1}{2}x$ | $dx = 2du$

$\frac{du}{dx} = \frac{1}{2}$

$-\int \sin u \cdot 2du$

$-2\int \sin u\,du$

$= -2(-\cos u) + c$

$y = 2\cos x/2 + c$

$6 = 2\cos 0 + c$

$6 = 2 + c$

$4 = c$

$\boxed{y = 2\cos x/2 + 4}$

Change of variable u-sub Method

47) $\int x\sqrt{x+6}\,dx = \int x(x+6)^{1/2}dx$

$u = x+6$ | $dx = du$

$\frac{du}{dx} = 1$

$\int x \cdot u^{1/2} du$

$x = u - 6$

$\int (u-6)\cdot u^{1/2} du$

$\int u^{3/2} - 6u^{1/2} du$

$\frac{u^{5/2}}{5/2} - \frac{6u^{3/2}}{3/2} + c$

$\frac{2}{5}(x+6)^{5/2} - \frac{2}{3}(6)(x+6)^{3/2} + c$

$= \boxed{\frac{2}{5}(x+6)^{5/2} - 4(x+6)^{3/2} + c}$

49) $\int x^2\sqrt{1-x}\,dx = \int x^2(1-x)^{1/2}dx$

$u = 1-x$ | $dx = -du$

$\frac{du}{dx} = -1$

$x = 1-u$

$\int x^2 \cdot u^{1/2}(-du)$

$-\int (1-u)^2 u^{1/2} du$

$-\int (1-2u+u^2)u^{1/2} du$

$\int -u^{1/2} - 2u^{3/2} + u^{5/2} du$

$-\frac{u^{3/2}}{3/2} - \frac{2u^{5/2}}{5/2} + \frac{u^{7/2}}{7/2} + c$

$-\frac{2}{3}u^{3/2} - \frac{4}{5}u^{5/2} + \frac{2}{7}u^{7/2} + c$

$\boxed{-\frac{2}{3}(1-x)^{3/2} - \frac{4}{5}(1-x)^{5/2} + \frac{2}{7}(1-x)^{7/2} + c}$

53) $\int \frac{-x}{(x+1) - \sqrt{x+1}}\,dx$

$x = u-1$

$u = x+1$

$\frac{du}{dx} = 1$

$dx = du$

$\int \frac{-x}{u - u^{1/2}}\,du$

$\int \frac{-(u-1)}{\sqrt{u}(\sqrt{u}-1)}$

$\int \frac{-(\sqrt{u}+1)\cancel{(\sqrt{u}-1)}}{\sqrt{u}\cancel{(\sqrt{u}-1)}}$

$-\int (\sqrt{u}+1)u^{-1/2}$

$-\int 1 + u^{-1/2} du$

$-u - \frac{u^{1/2}}{1/2} + c$

$-(x+1) - 2\sqrt{x+1} + c$

$-x - 1 - 2\sqrt{x+1} + c$

$\boxed{-x - 2\sqrt{x+1} + c}$