2017

4.5b u-sub (Definite Integrals) p.301-303

#55-65 odd, 69-73 odd

55) $\int_{-1}^{1} x(x^2+1)^3\,dx$

Method 1: convert bounds:

If $x=-1$, $u=x^2+1=(-1)^2+1=2$

If $x=1$, $u=x^2+1=(1)^2+1=2$

$u=x^2+1$

$\frac{du}{dx}=2x$

$dx=\frac{du}{2x}$

$\int \cancel{x}\cdot u^3\cdot\frac{du}{2\cancel{x}}$

$\frac{1}{2}\int u^3\,du$

$\frac{1}{2}\frac{u^4}{4}\Big]_2^2 = \frac{2^4}{8}-\frac{2^4}{8}=\boxed{0}$

Method 2: solve in terms of x:

$\frac{1}{2}\left(\frac{u^4}{4}\right)=\frac{1}{8}(x^2+1)^4\Big]_{-1}^{1} = \frac{1}{8}(1^2+1)^4-\frac{1}{8}((-1)^2+1)^4 = \frac{16}{8}-\frac{16}{8}=\boxed{0}$

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57) $\int_1^2 2x^2\sqrt{x^3+1}\,dx = \int 2x^2(x^3+1)^{1/2}dx$

convert bounds:

If $x=1$, $u=x^3+1=1^3+1=2$

If $x=2$, $u=2^3+1=9$

$u=x^3+1$

$\frac{du}{dx}=3x^2$

$dx=\frac{du}{3x^2}$

$\int 2\cancel{x^2}\cdot u^{1/2}\cdot\frac{du}{3\cancel{x^2}}$

$\frac{2}{3}\int u^{1/2}\,du$

$\frac{2}{3}\cdot\frac{u^{3/2}}{3/2}\Big]_2^9 = \frac{2}{3}\cdot\frac{2}{3}u^{3/2}\Big]_2^9 = \frac{4}{9}(9)^{3/2}-\frac{4}{9}(2)^{3/2}$

$=\frac{4}{9}(27)-\frac{4}{9}(2)^{3/2}$

$=\boxed{12-\frac{8}{9}\sqrt{2}}$

Method 2:

$\frac{4}{9}u^{3/2} \to \frac{4}{9}(x^3+1)^{3/2}\Big]_1^2 = \frac{4}{9}\left[2^3+1\right]^{3/2}-\frac{4}{9}\left[1^3+1\right]^{3/2}$

$=\frac{4}{9}(9)^{3/2}-\frac{4}{9}(2)^{3/2}=\boxed{12-\frac{8}{9}\sqrt{2}}$

61) $\displaystyle\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$

$u = 1+\sqrt{x} = 1+x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$

$\displaystyle\int_1^9 \frac{1}{\cancel{\sqrt{x}}\cdot u^2}\cdot 2\cancel{\sqrt{x}}\,du$

convert bounds

If $x=1$, $u = 1+\sqrt{x} = 1+\sqrt{1} = 2$

If $x=9$, $u = 1+\sqrt{x} = 1+\sqrt{9} = 4$

$$2\int \frac{1}{u^2}du = 2\int u^{-2}du = \frac{2u^{-1}}{-1}\Bigg]_2^4 = -\frac{2}{4}+\frac{2}{2} = \boxed{\frac{1}{2}}$$

Use Even/odd Function properties:

69) $\displaystyle\int_{-2}^{2} x^2(x^2+1)\,dx$ (even function: $f(-x)=f(x)$)

$$= 2\int_0^2 x^2(x^2+1)\,dx = 2\int x^4+x^2\,dx = 2\cdot \frac{x^5}{5}+\frac{x^3}{3}\Bigg]_0^2 = 2\cdot\left[\frac{2^5}{5}+\frac{2^3}{3}\right]-0$$

$$= \boxed{\frac{275}{15}}$$

71) $\displaystyle\int_{-\pi/2}^{\pi/2} \sin^2 x\cos x\,dx$ ← (even function)

$u=\sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$$2\int_0^{\pi/2} (\sin x)^2\cos x\,dx$$

$$2\int u^2\cdot\cancel{\cos x}\cdot\frac{du}{\cancel{\cos x}}$$

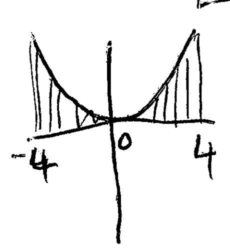
$$2\,\frac{u^3}{3} = \frac{2}{3}(\sin x)^3\Bigg]_0^{\pi/2} = \frac{2}{3}\left(\sin\tfrac{\pi}{2}\right)^3 - \frac{2}{3}(\sin 0)^3$$

$$= \frac{2}{3}(1)^3 - 0 = \boxed{\frac{2}{3}}$$

73) Given: even function and $\displaystyle\int_0^4 x^2\,dx = 64/3$

a) $\displaystyle\int_{-4}^{0} x^2\,dx = \int_0^4 x^2\,dx = \boxed{\frac{64}{3}}$

b) $\displaystyle\int_{-4}^{4} x^2\,dx = 2\int_0^4 x^2\,dx = 2\left(\frac{64}{3}\right) = \boxed{\frac{128}{3}}$



c) $\displaystyle\int_0^4 -x^2\,dx = -\int_0^4 x^2\,dx = -\left(\frac{64}{3}\right) = \boxed{-\frac{64}{3}}$

d) $\displaystyle\int_{-4}^{0} 3x^2\,dx = 3\int_0^4 x^2\,dx = 3\left(\frac{64}{3}\right) = \boxed{64}$