4.6 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Using the Trapezoidal Rule and Simpson's Rule In Exercises 1–10, use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the given value of n. Round your answer to four decimal places and compare the results with the exact value of the definite integral.

1. $\int_0^2 x^2\,dx, \quad n = 4$
2. $\int_1^2 \left(\frac{x^2}{4} + 1\right) dx, \quad n = 4$
3. $\int_0^2 x^3\,dx, \quad n = 4$
4. $\int_2^3 \frac{2}{x^2}\,dx, \quad n = 4$
5. $\int_1^3 x^3\,dx, \quad n = 6$
6. $\int_0^8 \sqrt[3]{x}\,dx, \quad n = 8$
7. $\int_4^9 \sqrt{x}\,dx, \quad n = 8$
8. $\int_1^4 (4 - x^2)\,dx, \quad n = 6$
9. $\int_0^1 \frac{2}{(x+2)^2}\,dx, \quad n = 4$
10. $\int_0^2 x\sqrt{x^2+1}\,dx, \quad n = 4$

Using the Trapezoidal Rule and Simpson's Rule In Exercises 11–20, approximate the definite integral using the Trapezoidal Rule and Simpson's Rule with $n = 4$. Compare these results with the approximation of the integral using a graphing utility.

11. $\int_0^2 \sqrt{1 + x^3}\,dx$
12. $\int_0^2 \frac{1}{\sqrt{1 + x^3}}\,dx$
13. $\int_0^1 \sqrt{x}\,\sqrt{1 - x}\,dx$
14. $\int_{\pi/2}^{\pi} \sqrt{x}\sin x\,dx$
15. $\int_0^{\sqrt{\pi/2}} \sin x^2\,dx$
16. $\int_0^{\sqrt{\pi/4}} \tan x^2\,dx$
17. $\int_3^{3.1} \cos x^2\,dx$
18. $\int_0^{\pi/2} \sqrt{1 + \sin^2 x}\,dx$
19. $\int_0^{\pi/4} x \tan x\,dx$
20. $\int_0^{\pi} f(x)\,dx, \quad f(x) = \begin{cases} \frac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases}$

WRITING ABOUT CONCEPTS

21. **Polynomial Approximations** The Trapezoidal Rule and Simpson's Rule yield approximations of a definite integral $\int_a^b f(x)\,dx$ based on polynomial approximations of f. What is the degree of the polynomials used for each?
22. **Describing an Error** Describe the size of the error when the Trapezoidal Rule is used to approximate $\int_a^b f(x)\,dx$ when $f(x)$ is a linear function. Use a graph to explain your answer.

Estimating Errors In Exercises 23–26, use the error formulas in Theorem 4.20 to estimate the errors in approximating the integral, with $n = 4$, using (a) the Trapezoidal Rule and (b) Simpson's Rule.

23. $\int_1^3 2x^3\,dx$
24. $\int_3^5 (5x + 2)\,dx$
25. $\int_2^4 \frac{1}{(x-1)^2}\,dx$
26. $\int_0^{\pi} \cos x\,dx$

Estimating Errors In Exercises 27–30, use the error formulas in Theorem 4.20 to find n such that the error in the approximation of the definite integral is less than or equal to 0.00001 using (a) the Trapezoidal Rule and (b) Simpson's Rule.

27. $\int_1^3 \frac{1}{x}\,dx$
28. $\int_0^1 \frac{1}{1+x}\,dx$
29. $\int_0^2 \sqrt{x+2}\,dx$
30. $\int_0^{\pi/2} \sin x\,dx$

Estimating Errors Using Technology In Exercises 31–34, use a computer algebra system and the error formulas to find n such that the error in the approximation of the definite integral is less than or equal to 0.00001 using (a) the Trapezoidal Rule and (b) Simpson's Rule.

31. $\int_0^2 \sqrt{1 + x}\,dx$
32. $\int_0^2 (x + 1)^{2/3}\,dx$
33. $\int_0^1 \tan x^2\,dx$
34. $\int_0^1 \sin x^2\,dx$

35. **Finding the Area of a Region** Approximate the area of the shaded region using
    (a) the Trapezoidal Rule with $n = 4$.
    (b) Simpson's Rule with $n = 4$.

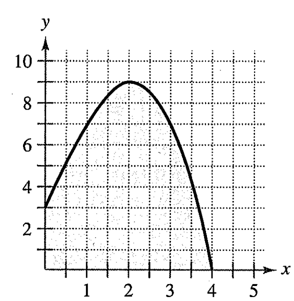


Figure for 35

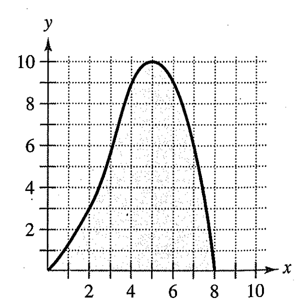


Figure for 36

36. **Finding the Area of a Region** Approximate the area of the shaded region using
    (a) the Trapezoidal Rule with $n = 8$.
    (b) Simpson's Rule with $n = 8$.
37. **Area** Use Simpson's Rule with $n = 14$ to approximate the area of the region bounded by the graphs of $y = \sqrt{x}\cos x$, $y = 0$, $x = 0$, and $x = \pi/2$.

38. Circumference The **elliptic integral**

$$8\sqrt{3}\int_0^{\pi/2}\sqrt{1-\tfrac{2}{3}\sin^2\theta}\,d\theta$$

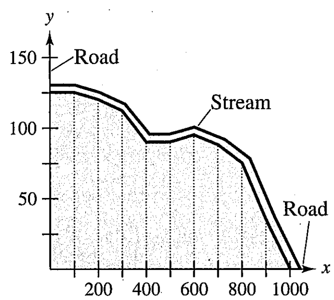
gives the circumference of an ellipse. Use Simpson's Rule with $n = 8$ to approximate the circumference.

39. Surveying

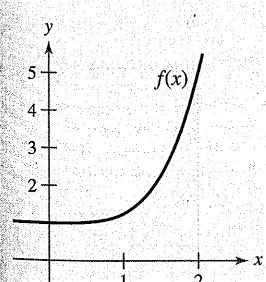
Use the Trapezoidal Rule to estimate the number of square meters of land, where x and y are measured in meters, as shown in the figure. The land is bounded by a stream and two straight roads that meet at right angles.

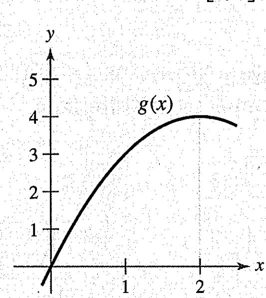
x	0	100	200	300	400	500
y	125	125	120	112	90	90

x	600	700	800	900	1000
y	95	88	75	35	0



40. HOW DO YOU SEE IT? The function $f(x)$ is concave upward on the interval $[0, 2]$ and the function $g(x)$ is concave downward on the interval $[0, 2]$.





(a) Using the Trapezoidal Rule with $n = 4$, which integral would be overestimated? Which integral would be underestimated? Explain your reasoning.

(b) Which rule would you use for more accurate approximations of $\int_0^2 f(x)\,dx$ and $\int_0^2 g(x)\,dx$, the Trapezoidal Rule or Simpson's Rule? Explain your reasoning.

41. Work To determine the size of the motor required to operate a press, a company must know the amount of work done when the press moves an object linearly 5 feet. The variable force to move the object is

$$F(x) = 100x\sqrt{125 - x^3}$$

where F is given in pounds and x gives the position of the unit in feet. Use Simpson's Rule with $n = 12$ to approximate the work W (in foot-pounds) done through one cycle when

$$W = \int_0^5 F(x)\,dx.$$

42. Approximating a Function The table lists several measurements gathered in an experiment to approximate an unknown continuous function $y = f(x)$.

x	0.00	0.25	0.50	0.75	1.00
y	4.32	4.36	4.58	5.79	6.14

x	1.25	1.50	1.75	2.00
y	7.25	7.64	8.08	8.14

(a) Approximate the integral

$$\int_0^2 f(x)\,dx$$

using the Trapezoidal Rule and Simpson's Rule.

(b) Use a graphing utility to find a model of the form $y = ax^3 + bx^2 + cx + d$ for the data. Integrate the resulting polynomial over $[0, 2]$ and compare the result with the integral from part (a).

Approximation of Pi In Exercises 43 and 44, use Simpson's Rule with $n = 6$ to approximate π using the given equation. (In Section 5.7, you will be able to evaluate the integral using inverse trigonometric functions.)

43. $\pi = \displaystyle\int_0^{1/2}\frac{6}{\sqrt{1-x^2}}\,dx$ **44.** $\pi = \displaystyle\int_0^1\frac{4}{1+x^2}\,dx$

45. Using Simpson's Rule Use Simpson's Rule with $n = 10$ and a computer algebra system to approximate t in the integral equation

$$\int_0^t \sin\sqrt{x}\,dx = 2.$$

46. Proof Prove that Simpson's Rule is exact when approximating the integral of a cubic polynomial function, and demonstrate the result with $n = 4$ for

$$\int_0^1 x^3\,dx.$$

47. Proof Prove that you can find a polynomial

$$p(x) = Ax^2 + Bx + C$$

that passes through any three points (x_1, y_1), (x_2, y_2), and (x_3, y_3), where the x_i's are distinct.