

AP Calculus AB 4-2, 4-6 Morning Review
Calculators permitted.

Name _____

1. Find the sum: $\sum_{i=1}^3 [(2i+1)^2 - (3i+1)^3]$

2. Use Sigma notation to write the sum: $\frac{7\sqrt{3}}{27} + \frac{7\sqrt{4}}{64} + \frac{7\sqrt{5}}{125} + \frac{7\sqrt{6}}{216} + \dots + \frac{7\sqrt{n}}{n^3}$

3. Use 4 middle rectangles to approximate the area of the region bounded by $f(x) = 3 + 2x^2$, the x -axis, $x = 1$, and $x = 7$.

4. Use the table of values on the right to estimate the below:

x	1	5	6	8	11	13	15
f(x)	4	2	7	3	1	6	5

a. Use 3 middle rectangles with intervals indicated by the table to estimate the area between the curve and x -axis on $[1, 15]$

b. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x -axis on $[5, 11]$

c. Use 4 trapezoids with interval indicated by the table to estimate area between the curve and x -axis on $[6, 15]$

5. Given the region bounded by $g(x) = 3 - 2x^2$, the x -axis, $x = -1$, and $x = 1$. Use the **limit definition** to find the exact area of the region.

AP Calculus AB 4-2, 4-6 Morning Review

Name Key

Calculators permitted.

1. Find the sum: $\sum_{i=1}^3 [(2i+1)^2 - (3i+1)^3] = (2+1)^2 - (3+1)^3 + (4+1)^2 - (6+1)^3 + (6+1)^2 - (9+1)^3$
 $= 3^2 - 4^3 + 5^2 - 7^3 + 7^2 - 10^3 = \boxed{-1324}$

2. Use Sigma notation to write the sum: $\frac{7\sqrt{3}}{27} + \frac{7\sqrt{4}}{64} + \frac{7\sqrt{5}}{125} + \frac{7\sqrt{6}}{216} + \dots + \frac{7\sqrt{n}}{n^3}$

$$\sum_{i=3}^n \frac{7\sqrt{i}}{i^3}$$

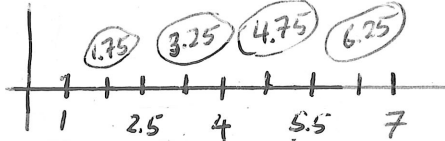
3. Use 4 middle rectangles to approximate the area of the region bounded by

$f(x) = 3+2x^2$, the x-axis, $x = 1$, and $x = 7$.

$$A \approx (1.5)f(1.75) + 1.5 \cdot f(3.25) + 1.5 \cdot f(4.75) + 1.5 \cdot f(6.25)$$

$$\approx 1.5(9.125 + 24.125 + 48.125 + 81.125)$$

$$w = \frac{b-a}{n} = \frac{7-1}{4} = \frac{3}{2}$$



4. Use the table of values on the right to estimate the below:

x	1	5	6	8	11	13	15
f(x)	4	2	7	3	1	6	5

$$\approx (1.5)(162.5)$$

$$\approx \boxed{243.75}$$

a. Use 3 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [1, 15]

$$5 \cdot f(5) + 5f(8) + 4f(13)$$

$$5(2) + 5(3) + 4(6)$$

$$\boxed{49}$$

b. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on [5, 11]

$$1f(6) + 2f(8) + 3f(11)$$

$$1(7) + 2(3) + 3(1)$$

$$\boxed{16}$$

c. Use 4 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on [6, 15]

$$A = \frac{w}{2}(h_1 + h_2)$$

$$= \frac{2}{2}[f(6) + f(8)] + \frac{3}{2}[f(8) + f(11)]$$

$$+ \frac{2}{2}[f(11) + f(13)] + \frac{2}{2}[f(13) + f(15)]$$

$$= \frac{2}{2}(10) + \frac{3}{2}(4) + \frac{2}{2}(7) + \frac{2}{2}(11)$$

$$\boxed{34}$$

5. Given the region bounded by $g(x) = 3 - 2x^2$, the x-axis, $x = -1$, and $x = 1$. Use the limit definition to find the exact area of the region.

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n \text{width} \cdot f[a + \text{width} \cdot i]$$

$$= \frac{2}{n} \cdot f\left[-1 + \frac{2}{n}i\right]$$

$$= \frac{2}{n} \cdot \left[3 - 2\left(-1 + \frac{2}{n}i\right)^2\right]$$

$$= \frac{2}{n} \left[3 - 2\left(1 - \frac{4}{n}i + \frac{4}{n^2}i^2\right)\right]$$

$$= \frac{2}{n} \left[3 - 2 + \frac{8}{n}i - \frac{8}{n^2}i^2\right]$$

$$\sum \frac{2}{n} + \frac{16}{n^2}i - \frac{16}{n^3}i^2$$

$$w = \frac{1 - (-1)}{n} = \frac{2}{n}$$

$$\lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n 1 + \frac{16}{n^2} \sum_{i=1}^n i - \frac{16}{n^3} \sum_{i=1}^n i^2$$

$$\lim_{n \rightarrow \infty} \frac{2}{n}(n) + \frac{16}{n^2} \left[\frac{n(n+1)}{2}\right] - \frac{16}{n^3} \left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$\lim_{n \rightarrow \infty} 2 + \frac{16n^2}{2n^2} - \frac{16(2n^3)}{6n^3}$$

$$= 2 + 8 - \frac{32}{6}$$

$$= \boxed{\frac{14}{3}}$$