

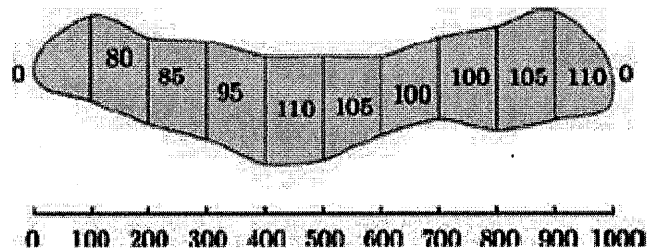
**AP Calculus AB 4-2, 4-6 Quiz Review**  
Calculators permitted.

Name \_\_\_\_\_

1. Find the sum:  $\sum_{i=2}^4 [(i+1)^2 - (2-i)^3]$

2. Use Sigma notation to write the sum:  $\frac{2}{\sqrt[3]{5-2}} + \frac{4}{\sqrt[3]{5-4}} + \frac{6}{\sqrt[3]{5-6}} + \frac{8}{\sqrt[3]{5-8}}$

3. The width, in feet, at various points along the fairway of a hole on a golf course is given to the right. If one pound of fertilizer covers 200 square feet, estimate the amount of fertilizer needed to fertilize the fairway using trapezoids.



4. Use 3 midpoint rectangles to approximate the area of the region bounded by  $f(x) = x^2 + 3$ , the  $x$ -axis,  $x = 0$ , and  $x = 5$ .

5. Use the table of values on the right to estimate the below:

$x$	0	4	6	7	10
$f(x)$	5	3	2	3	5

a. Use 3 left-handed rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on  $[0, 7]$

b. Use 2 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on  $[0, 10]$

c. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on  $[4, 10]$

d. Use 3 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on  $[0, 7]$

6. Given the region bounded by  $g(x) = 6 - x^2$ , the x-axis,  $x = -1$ , and  $x = 2$ . Use the limit definition to find the exact area of the region.

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Name Solution Key

1. Find the sum:  $\sum_{i=2}^4 [(i+1)^2 - (2-i)^3]$

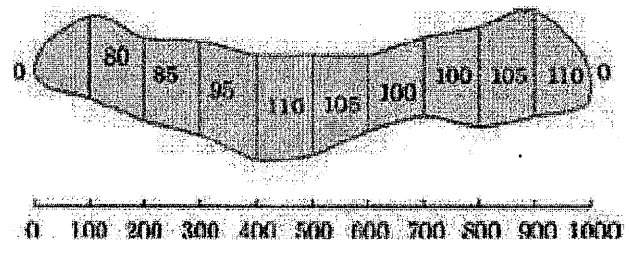
$$(2+1)^2 - (2-2)^3 + (3+1)^2 - (2-3)^3 + (4+1)^2 - (2-4)^3$$

$$9 - 0 + 16 - (-1) + 25 - (-8) = \boxed{59}$$

2. Use Sigma notation to write the sum:  $\frac{2}{\sqrt[3]{5-2}} + \frac{4}{\sqrt[3]{5-4}} + \frac{6}{\sqrt[3]{5-6}} + \frac{8}{\sqrt[3]{5-8}}$

$$\sum_{i=1}^4 \frac{2i}{\sqrt[3]{5-2i}}$$

3. The width, in feet, at various points along the fairway of a hole on a golf course is given to the right. If one pound of fertilizer covers 200 square feet, estimate the amount of fertilizer needed to fertilize the fairway using trapezoids.



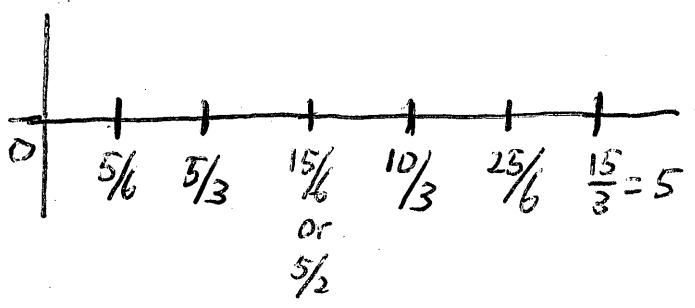
$$A = \frac{w}{2} [h_1 + 2h_2 + 2h_3 + 2h_4 + \dots + h_n]$$

$$A \approx \frac{100}{2} [0 + 2(80) + 2(85) + 2(95) + 2(110) + 2(105) + 2(100) + 2(100) + 2(105) + 2(110) + 0]$$

$$A \approx 50(1780) = 89000 \text{ ft}^2$$

$$\text{fertilizer needed} = 89000 \text{ ft}^2 \cdot \frac{1 \text{ lb fertilizer}}{200 \text{ ft}^2} = \boxed{445 \text{ pounds of fertilizers}}$$

4. Use 3 midpoint rectangles to approximate the area of the region bounded by  $f(x) = x^2 + 3$ , the x-axis,  $x = 0$ , and  $x = 5$ .  $w = \frac{b-a}{n} = \frac{5-0}{3} = \frac{5}{3}$



$$A \approx \frac{5}{3} f\left(\frac{5}{6}\right) + \frac{5}{3} f\left(\frac{5}{2}\right) + \frac{5}{3} f\left(\frac{25}{6}\right)$$

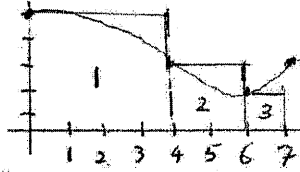
$$\approx \frac{5}{3} (33.3) = \boxed{55.5}$$

5. Use the table of values on the right to estimate the below:

x	0	4	6	7	10
f(x)	5	3	2	3	5

a. Use 3 left-handed rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [0, 7]

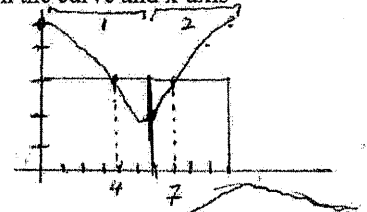
x	0	4	6	7
f(x)	5	3	2	3



$$\begin{aligned}
 A &= 4 \cdot f(0) + 2 \cdot f(4) + 1 \cdot f(6) \\
 &= 4(5) + 2(3) + 1(2) \\
 &= \boxed{28}
 \end{aligned}$$

b. Use 2 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [0, 10]

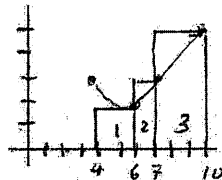
x	0	4	6	7	10
f(x)	5	3	2	3	5



$$\begin{aligned}
 A &= 6 \cdot f(3) + 4 \cdot f(7) \\
 &= 6(3) + 4(3) \\
 &= \boxed{30}
 \end{aligned}$$

c. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on [4, 10]

x	4	6	7	10
f(x)	3	2	3	5



$$\begin{aligned}
 A &= 2 \cdot f(4) + 1 \cdot f(7) + 3 \cdot f(10) \\
 &= 2(3) + 1(3) + 3(5) \\
 &= 4 + 3 + 15 = \boxed{22}
 \end{aligned}$$

d. Use 3 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on [0, 7]

x	0	4	6	7
f(x)	5	3	2	3

$$A = \frac{w}{2} [h_1 + h_2]$$

$$\begin{aligned}
 A &= \frac{4}{2} [f(0) + f(4)] + \frac{2}{2} [f(4) + f(6)] + \frac{1}{2} [f(6) + f(7)] \\
 &= 2(5+3) + 1(3+2) + \frac{1}{2}(2+3) \\
 &= 2(8) + 1(5) + \frac{1}{2}(5) \\
 &= 16 + 5 + \frac{5}{2} = \boxed{23.5 \text{ or } 47/2}
 \end{aligned}$$

6. Given the region bounded by  $g(x) = 6 - x^2$ , the x-axis,  $x = -1$ , and  $x = 2$ . Use the limit definition to find the exact area of the region.

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n \text{width} \cdot f(\text{left} + \text{width} \cdot i) \quad \text{width} = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$$

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \cdot f\left(-1 + \frac{3}{n}i\right) \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \cdot \left[6 - \left(-1 + \frac{3i}{n}\right)^2\right] \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \left[6 - \left(-1 + \frac{3i}{n}\right)\left(-1 + \frac{3i}{n}\right)\right] \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \left[6 - \left(1 - \frac{6i}{n} + \frac{9i^2}{n^2}\right)\right] \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \left[5 + \frac{6i}{n} - \frac{9i^2}{n^2}\right]
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ \frac{15}{n} + \frac{18}{n^2}i - \frac{27i^2}{n^3} \right] \\
 &= \lim_{n \rightarrow \infty} \left[ \frac{15}{n} \sum_{i=1}^n 1 + \frac{18}{n^2} \sum_{i=1}^n i - \frac{27}{n^3} \sum_{i=1}^n i^2 \right] \\
 &= \lim_{n \rightarrow \infty} \left[ \frac{15}{n}(n) + \frac{18}{n^2} \left(\frac{n^2}{2} + \frac{n}{2}\right) - \frac{27}{n^3} \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) \right] \\
 &= \lim_{n \rightarrow \infty} \left[ \frac{15n}{n} + \frac{18n^2}{2n^2} + \frac{18n}{2n^2} - \frac{27n^3}{3n^3} - \frac{27n^2}{2n^3} - \frac{27n}{6n^3} \right] \\
 &= 15 + \frac{18}{2} - \frac{27}{3} = 15 + 9 - 9 = \boxed{15}
 \end{aligned}$$