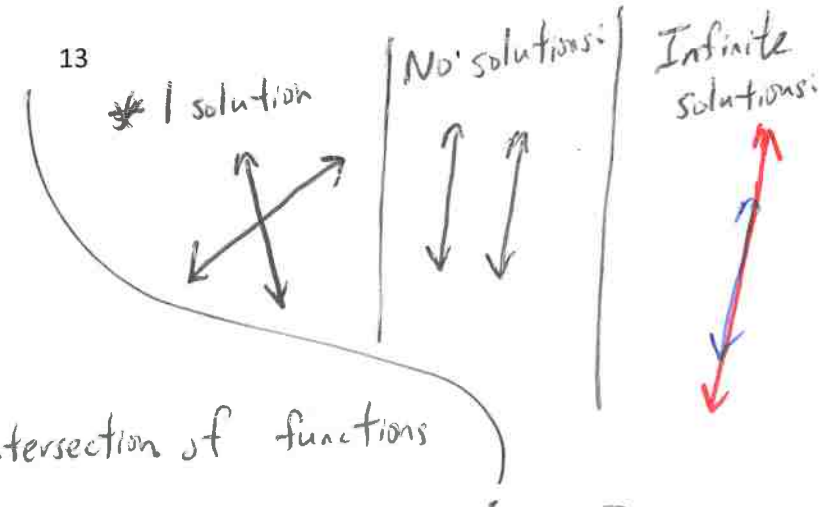
5.05 Solving a 2x2 System of Equations Notes



Ways to solve a 2-variable system of equations:

* 1) elimination
2) substitution
3) graphing

What does your solution represent?

point of intersection of functions

Review: Solve the system using the elimination method.

$\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$ → $-4x + 9y = 9$, $4(x - 3y = -6)$ → $-4x + 9y = 9$, $4x - 12y = -24$

$-3y = -15$

$\frac{-3y}{-3} = \frac{-15}{-3}$

$y = 5$

$x - 3y = -6$

$x - 3(5) = -6$

$x - 15 = -6$

$x = 9$

$(9, 5)$

This year, a NEW way to solve!

How can matrices help to solve a system of equations?

Matrix equations should take the form AX = B; where A = the coefficient matrix, X = the variable matrix, and B = the constant matrix

$[A]\ [X] = [B]$

Example:

As a system: $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$

As a matrix equation: $\begin{bmatrix} -4 & 9 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -6 \end{bmatrix}$

If A·X = B, how do we solve for X?

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 & 9 \\ 1 & -3 \end{bmatrix}^{-1} \begin{bmatrix} 9 \\ -6 \end{bmatrix}$

Solve $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$ using a matrix equation. $\det(A) = -4(-3) - (1)(9) = 3$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3} \begin{bmatrix} -3 & -9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 9 \\ -6 \end{bmatrix}$

$2 \times 2 \quad 2 \times 1$

$\begin{bmatrix} -3 & -9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 9 \\ -6 \end{bmatrix} \begin{matrix} -3(9) + (-9)(-6) \\ -1(9) + (-4)(-6) \end{matrix} \rightarrow \begin{bmatrix} 27 \\ 15 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 27 \\ 15 \end{bmatrix} \rightarrow \begin{bmatrix} 9 \\ 5 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$

$x = 9$

$y = 5 \rightarrow (9, 5)$

Using a Matrix Equation to Solve a System

1. Write the equation, AX = B, made of the coefficient matrix A times the variable matrix X equal to the constant matrix B.

2. Solve for the variable matrix by multiplying the inverse of the coeffiecient matrix times the answer matrix. $X = A^{-1}B$

3. If A^{-1} does not exist, then there are either:
   - NO SOLUTIONS
   - or INFINITELY MANY SOLUTIONS

Examples Cont'd: Solve the following systems using matrix equations.

2. $\begin{cases} x - 4y = 12 \\ 3x + 2y = 8 \end{cases}$

$$\begin{bmatrix} 1 & -4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 12 \\ 8 \end{bmatrix}$$

$\det(A) = 2 - (-12) = 14$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 8 \end{bmatrix} \rightarrow \begin{bmatrix} 2(12)+4(8) \\ 12(-3)+1(8) \end{bmatrix} \rightarrow \begin{bmatrix} 56 \\ -28 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 56 \\ -28 \end{bmatrix} \rightarrow \boxed{\begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}}$$

3. $\begin{cases} 2x + 12y = 20 \\ 5x + 30y = -7 \end{cases}$

$$\begin{bmatrix} 2 & 12 \\ 5 & 30 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 12 \\ 5 & 30 \end{bmatrix}^{-1}\begin{bmatrix} 20 \\ -7 \end{bmatrix}$$

$\det(A) = 2(30) - 5(12) = 0$

matrix A is singular. Determinant = 0, No inverse exists. No solution.

4. $\begin{cases} 5x + 4y = -30 \\ 3x - 9y = -18 \end{cases}$

$$\begin{bmatrix} 5 & 4 \\ 3 & -9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -30 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & -9 \end{bmatrix}^{-1}\begin{bmatrix} -30 \\ -18 \end{bmatrix}$$

$\det(A) = -45 - 12 = -57$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-57}\begin{bmatrix} -9 & -4 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} -30 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} -9 & -4 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} -30 \\ -18 \end{bmatrix} \rightarrow \begin{bmatrix} -9(-30)+4(18) \\ -3(-30)+5(-18) \end{bmatrix} \rightarrow \begin{bmatrix} 342 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{57}\begin{bmatrix} 342 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \end{bmatrix}$$

$x = -6$, $y = 0$ $\rightarrow (-6, 0)$

5.06 Practice

Solve each system of equations using the elimination method.

1. $5x = -3y - 31$
$2y = -4x - 22$

$2(5x + 3y = -31)$
$-3(4x + 2y = -22)$

$10x + 6y = -62$
$-12x - 6y = 66$

$-2x = 4$

$x = -2$

$4x + 2y = -22$
$4(-2) + 2y = -22$
$-8 + 2y = -22$
$2y = -14$
$y = -7$

$(-2, -7)$

2. $4y + 17 = -7x$
$8x + 5y = -19$

$5(7x + 4y = -17)$
$-4(8x + 5y = -19)$

$35x + 20y = -85$
$-32x - 20y = 76$

$3x = -9$

$x = -3$

$7x + 4y = -17$
$7(-3) + 4y = -17$
$-21 + 4y = -17$
$4y = 4$
$y = 1$

$(-3, 1)$

3. $12x = 21 - 3y$
$2y = -4x - 22$

$2(12x + 3y = 21)$
$3(4x + 2y = -22)$

$24x - 6y = -42$
$12x + 6y = -66$

$-12x = -108$

$x = 9$

$2y = -4x - 22$
$2y = -4(9) - 22$
$2y = -58$
$y = -29$

$(9, -29)$

4. $4y = 12x - 3$
$9x = 20y - 2$

$5(-12x + 4y = -3)$

$9x - 20y = -2$

$-60x + 20y = -15$
$9x - 20y = -2$

$-51x = -17$

$x = \frac{1}{3}$

$9\left(\frac{1}{3}\right) = 20y - 2$
$3 = 20y - 2$
$5 = 20y$
$\frac{5}{20} = y$
$y = \frac{1}{4}$

$\left(\frac{1}{3}, \frac{1}{4}\right)$

Write each system of equations as a matrix equation. Solving using an inverse matrix.

5. $\begin{matrix} 5x - 2y = 11 \\ -4x + 7y = 2 \end{matrix}$

$$\begin{bmatrix} 5 & -2 \\ -4 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -4 & 7 \end{bmatrix}^{-1}\begin{bmatrix} 11 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 7 & 2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 11 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 2 \\ 4 & 5 \end{bmatrix}\begin{matrix} \begin{bmatrix} 11 \\ 2 \end{bmatrix} \\ \hline 7(11)+2(2) \\ 11(4)+5(2) \end{matrix} = \begin{bmatrix} 81 \\ 54 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 81 \\ 54 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$(3, 2)$$

6. $\begin{matrix} 2x + 3y = 2 \\ x - 4y = -21 \end{matrix}$

$$\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -21 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}^{-1}\begin{bmatrix} 2 \\ -21 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ -21 \end{bmatrix}$$

$$\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}\begin{matrix} \begin{bmatrix} 2 \\ -21 \end{bmatrix} \\ \hline -4(2)-21(-3) \\ 2(-1)+2(-21) \end{matrix} = \begin{bmatrix} 55 \\ -44 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} 55 \\ -44 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$$

7. $\begin{matrix} -3x + 5y = 33 \\ 2x - 4y = -26 \end{matrix}$

$$\begin{bmatrix} -3 & 5 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 33 \\ -26 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ 2 & -4 \end{bmatrix}^{-1}\begin{bmatrix} 33 \\ -26 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -4 & -5 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 33 \\ -26 \end{bmatrix}$$

$$\begin{bmatrix} -4 & -5 \\ -2 & -3 \end{bmatrix}\begin{matrix} \begin{bmatrix} 33 \\ -26 \end{bmatrix} \\ \hline 33(-4)-5(-26) \\ 33(-2)-3(-26) \end{matrix} = \begin{bmatrix} -2 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -2 \\ 12 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

8. $\begin{matrix} -4x + y = 19 \\ 3x - 2y = -18 \end{matrix}$

$$\begin{bmatrix} -4 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 3 & -2 \end{bmatrix}^{-1}\begin{bmatrix} 19 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -2 & -1 \\ -3 & -4 \end{bmatrix}\begin{bmatrix} 19 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \cdot \begin{bmatrix} -2 & -1 \\ -3 & -4 \end{bmatrix}\begin{matrix} \begin{bmatrix} 19 \\ -18 \end{bmatrix} \\ \hline -2(19)+1(18) \\ -3(19)-18(4) \end{matrix} = \begin{bmatrix} -20 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -20 \\ 15 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$