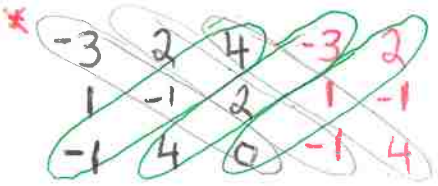
5.07 Matrix Inverses (3x3) Notes

Determinant of a 3x3: If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ then $\det[A] = (aei + bfg + cdh) - (bdi + afh + ceg)$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \begin{matrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{matrix}$$

Examples: Find the determinant of the following matrices, then use a calculator to find the inverse, if it exists.

1. $E = \begin{bmatrix} -3 & 2 & 4 \\ 1 & -1 & 2 \\ -1 & 4 & 0 \end{bmatrix}$

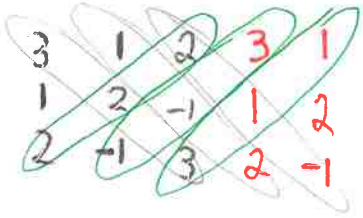


$0 - 4 + 16 - (4 - 24 + 0)$

$12 - (-20)$

$\det(E) = 32$

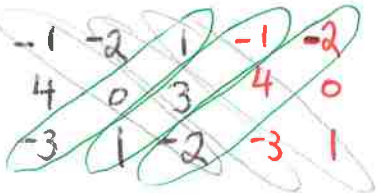
2. $F = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix}$



$18 - 2 - 2 - (8 + 3 + 3)$

$\det(F) = 0$

3. $G = \begin{bmatrix} -1 & -2 & 1 \\ 4 & 0 & 3 \\ -3 & 1 & -2 \end{bmatrix}$



$0 + 18 + 4 - (0 - 3 + 16)$

$\det(G) = 9$

4. Given A and AB, find B.

$A = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 0 & 2 \\ 4 & 1 & -2 \end{bmatrix}$ and $AB = \begin{bmatrix} 17 & -2 \\ 3 & 3 \\ 12 & -4 \end{bmatrix}$

$[A][B] = [AB]$

$[B] = [A^{-1}][AB]$

$$[B] = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 0 & 2 \\ 4 & 1 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 17 & -2 \\ 3 & 3 \\ 12 & -4 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 5 & -1 \\ 0 & 2 \\ 4 & 1 \end{bmatrix}$$

5.07 Practice

Find the determinant of each matrix. Then, find the inverse of the matrix, if it exists.

1. $\begin{bmatrix} 3 & 1 & -2 \\ 8 & -5 & 2 \\ -4 & 3 & -1 \end{bmatrix}$ $\det(A) = -11$

$A^{-1} = \begin{bmatrix} 1/11 & 5/11 & 8/11 \\ 0 & 1 & 2 \\ -4/11 & 13/11 & 23/11 \end{bmatrix}$

2. $\begin{bmatrix} 1 & -1 & -2 \\ 5 & 9 & 3 \\ 2 & 7 & 4 \end{bmatrix}$ $\det(A) = -5$

3. $\begin{bmatrix} 9 & 3 & 7 \\ -6 & -2 & -5 \\ 3 & 1 & 4 \end{bmatrix}$ $\det(A) = 0$

4. $\begin{bmatrix} 2 & 3 & -1 \\ -4 & -5 & 2 \\ 6 & 1 & 3 \end{bmatrix}$ $\det(A) = 12$

5. $\begin{bmatrix} -1 & 3 & 2 \\ 3 & -5 & -3 \\ 4 & 2 & 6 \end{bmatrix}$ $\det(A) = -14$

6. $\begin{bmatrix} 6 & -1 & 2 \\ 1 & -2 & -4 \\ -3 & 1 & -5 \end{bmatrix}$ $\det(A) = 57$

7. Given A and AB, find B. $A = \begin{bmatrix} 5 & 0 & 1 \\ 2 & -3 & 2 \\ 1 & -1 & 4 \end{bmatrix}, AB = \begin{bmatrix} 1 & 4 \\ -16 & -6 \\ -2 & -5 \end{bmatrix}$

$B = \begin{bmatrix} 0 & 1 \\ 6 & 2 \\ 1 & -1 \end{bmatrix}$

Find the determinant of each matrix.

8. $\begin{bmatrix} r & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & t \end{bmatrix}$ $\begin{matrix} r & 0 \\ 0 & s \\ 0 & 0 \end{matrix}$

$rst + 0 + 0 - (0 - 0 - 0)$

$\det(A) = rst$

9. $\begin{bmatrix} c & c & c \\ 0 & c & c \\ 0 & 0 & c \end{bmatrix}$ $\begin{matrix} c & c \\ 0 & c \\ 0 & 0 \end{matrix}$

$c^3 + 0 - 0 - (0 + 0 + 0)$

c^3

Matrices on the Calculator

Entering matrices into your TI calculator:

1. Access the "Matrix" window by pressing [2nd] and the button showing "Matrix" in blue.
2. The first menu, "NAMES", is how you select matrices to work with. The second menu, "MATH", is where you will find operations to perform on matrices (like *determinant* and *transpose*!). The third menu, "EDIT", is where you will enter matrices.
3. Select a matrix under the "EDIT" menu. Enter the dimensions of the matrix (number of rows first and then number of columns). Enter all elements of the matrix. When you reach the end of the matrix, double check all elements are entered correctly.
4. If you need to enter another matrix, start the process all over, [2nd], "Matrix", go to the "EDIT" menu, select a matrix that is NOT the one you just entered, set its dimensions, and enter its elements.
5. Press [2nd] followed by [mode] to quit to the home screen.

Using matrices on your TI calculator:

- Find the <u>determinant</u> of A: det [A] or |A|
  1. Store the matrix in the "EDIT" menu (described above). Be sure to "quit" at the end.
  2. Press [2nd], "Matrix", go to the "MATH" menu. Select "determinant" from the "MATH" list of the matrix window. You should see "det(" on the home screen.
  3. Press [2nd], "Matrix". Select the matrix from the "NAMES" menu that was entered.
  4. Press [ENTER]. The calculator returns the determinant.

- Find the <u>inverse</u> of A: A^{-1}
  1. Store the matrix in the "EDIT" menu (described above). Be sure to "quit" at the end.
  2. Press [2nd], "Matrix". Select the matrix from the "NAMES" menu that was entered.
  3. Enter the 'inverse' exponent of -1. You should see "$[A]^{-1}$" on the home screen
     a. Graphing Calc use the button [x^{-1}]
     b. TI36xPro: inverse is in the matrix "MATH" menu OR use [$x^{\square}$] and enter -1.
  4. [ENTER] = WHOA! Ugly decimals. Convert that to fractions!
     a. Graphing Calc: [MATH], "1:▶Frac", [ENTER]
     b. TI-36xPro: [◀▶≈] is found right above [enter].

- <u>Solve a system</u> using a matrix equation:
  1. Write the matrix equation on paper: CV = A.
     - Not here, but if an equation in the system is missing variables, fill in 0 for the coefficient in the <u>correct spot</u> in the row (x is first, then y, and z). Also, if an equation is not in standard form (x first, then y, followed by z), then rearrange the equation so that it is.
  2. Store the *coefficient matrix* and the *answers matrix* in the matrix "EDIT" menu. Be sure to "quit" at the end of the process.
  3. Multiply the *inverse of the coefficient matrix times the answers matrix.* You should see something similar to this "$[A]^{-1}[B]$" on the home screen. The calculator will return a 3x1 matrix with the solution for (x, y, z).