

pg. 21. #2 (elimination method)

p. 26 #3, 4, 5 (matrix inverse)

5.08 Notes: Solving a 3x3 System of Equations

Solving a 3-variable system of equations: you have 3 equations and can still use the previous methods from 2-variable systems. Mainly, we use the **elimination method** when solving by hand. For this unit, we can use a **matrix equation** and use **technology**!

Solve each system using the elimination method.

1. $-4x + 2y + 4z = 4$
 $-6x + 2y - 2z = -30$
 $x - 3y - 6z = -16$

$-4x + 2y + 4z = 4$
 $4(x - 3y - 6z = -16)$

$-4x + 2y + 4z = 4$
 $4x - 12y - 24z = -64$

$-10y - 20z = -60$
 $-10y - 20(5) = -60$

2. $3x + 2y - 6z = 6$
 $3x + y + 4z = -20$
 $6x + 2y - z = -22$

$3x + 2y - 6z = 6$
 $-1(6x + 2y - z = -22)$

$3x + 2y - 6z = 6$
 $-6x - 2y + z = 22$

$-3x - 5z = 28$
 $-3x - 5(-2) = 28$
 $-3x + 10 = 28$

$-3x = 18$ $x = -6$

$-6x + 2y - 2z = -30$
 $6(x - 3y - 6z = -16)$

$-6x + 2y - 2z = -30$
 $6x - 18y - 36z = -96$

$-16y - 38z = -126$

$-10y - 100 = -60$ $y = -4$
 $-10y = 40$

$3x + 2y - 6z = 6$
 $-2(3x + y + 4z = -20)$

$3x + 2y - 6z = 6$
 $-6x - 2y - 8z = 40$

$-3x - 14z = 46$

$3x + 2y - 6z = 6$
 $3(-6) + 2y - 6(-2) = 6$

$-8(-10y - 20z = -60)$
 $5(-16y - 38z = -126)$

$80y + 160z = 480$
 $-80y - 190z = -630$

$-30z = -150$
 $z = 5$

$x - 3y - 6z = -16$
 $x - 3(-4) - 6(5) = -16$
 $x + 12 - 30 = -16$
 $x - 18 = -16$

$x = 2$
 $(2, -4, 5)$

$-3x - 5z = 28$
 $-1(-3x - 14z = 46)$

$-3x - 5z = 28$
 $3x + 14z = -46$

$9z = -18$
 $z = -2$

$3x + 2y - 6z = 6$
 $(-6, 6, -2)$

$-18 + 2y + 12 = 6$
 $2y - 6 = 6$
 $2y = 12$ $y = 6$

Write each system as a matrix equation and then solve with technology.

$$\begin{aligned}
 & -x - 9y + 2z = -13 \\
 3. \quad & 6y - 5z = -6 \\
 & 6z + 5 = 2y - 3x
 \end{aligned}$$

$$-x - 9y + 2z = -13$$

$$0x + 6y - 5z = -6$$

$$3x - 2y + 6z = -5$$

$$\begin{bmatrix} -1 & -9 & 2 \\ 0 & 6 & -5 \\ 3 & -2 & 6 \end{bmatrix}
 \begin{bmatrix} x \\ y \\ z \end{bmatrix}
 =
 \begin{bmatrix} -13 \\ -6 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}
 =
 \begin{bmatrix} -1 & -9 & 2 \\ 0 & 6 & -5 \\ 3 & -2 & 6 \end{bmatrix}^{-1}
 \begin{bmatrix} -13 \\ -6 \\ -5 \end{bmatrix}$$

$$(-11, 4, 6)$$

$$\begin{aligned}
 & 2x - 3y + 2z = 16 \\
 4. \quad & -4x + 6y - 4z = 9 \\
 & 8x + z = -11
 \end{aligned}$$

Write each system of equations as a matrix equations. Solving using an inverse matrix and technology, if possible.

$$\begin{aligned} 2x + y - z &= -13 \\ 3. \quad 3x + 2y - 4z &= -36 \\ x + 6y - 3z &= 12 \end{aligned}$$

$$\begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & -4 \\ 1 & 6 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -13 \\ -36 \\ 12 \end{bmatrix}$$

$$\begin{aligned} 2x + 1y - z &= -13 \\ 3x + 2y - 4z &= -36 \\ x + 6y - 3z &= 12 \end{aligned}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & -4 \\ 1 & 6 & -3 \end{bmatrix}^{-1} \begin{bmatrix} -13 \\ -36 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 7 \\ 8 \end{bmatrix}$$

$(-6, 7, 8)$

$$\begin{aligned} 3x - 2y + 6z &= 38 - 2z \\ 4. \quad 6x + 3y - 9z &= -12 \\ 4y + 20z &= -4x \end{aligned}$$

$$\begin{aligned} 3x - 2y + 8z &= 38 \\ 6x + 3y - 9z &= -12 \\ 4x + 4y + 20z &= 0 \end{aligned}$$

$$\begin{bmatrix} 3 & -2 & 8 \\ 6 & 3 & -9 \\ 4 & 4 & 20 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 38 \\ -12 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & -2 & 8 \\ 6 & 3 & -9 \\ 4 & 4 & 20 \end{bmatrix}^{-1} \begin{bmatrix} 38 \\ -12 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -9 \\ 1 \end{bmatrix}$$

$(4, -9, 1)$

$$\begin{aligned} -x + 2y - z &= 2 - 2x \\ 5. \quad 2x + 3z &= y + 4 \\ 3x + y + 2z &= 6 \end{aligned}$$

$$\begin{aligned} 1x + 2y - 1z &= 2 \\ 2x - 1y + 3z &= 4 \\ 3x + 1y + 2z &= 6 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \text{no solution}$$