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AP Calculus AB - Quiz Review 5.1-5.3 (WS #1)

1. Find the critical numbers of the function $f(x)=2x^3-8x^2+10x$

2. Find the absolute extrema of $f(x)=2x^3-3x^2-12x+1$ on the interval $[-2, 3]$.

3. If $f(x)=\dfrac{-x^2+4}{4x}$ on $[-2, 2]$, determine if Rolle's Theorem applies. If yes, find the value(s) of c defined by Rolle's Theorem. If no, explain why not.

4. If $f(x)=-(25-5x)^{\frac{1}{2}}$ on $[3, 5]$, determine if the Mean Value Theorem applies. If yes, find the value(s) of c defined by the Mean Value Theorem. If no, explain why not.

5. If $f(x)=x^3-4x^2+5x$, find the interval(s) where $f(x)$ is Increasing and Decreasing

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6. If $f(x) = x^3 - 12x^2 - 27x$, use the First Derivative Test to find all relative extrema (x-values).

7. If $f(x) = x^3 + x^2 - 21x$, find the intervals where $f(x)$ is concave up and concave down, and find all points of inflection.

8. If $f(x) = 2x^3 - x^2 + 20x - 10$, use the Second Derivative Test to find all relative extrema.

9. Sketch a labeled graph of a function, f, with the following characteristics:

$f(0) = f(2) = 0$
$f'(x) \geq 0$ if $x < 1$
$f'(1)$ does not exist
$f'(x) < 0$ if $x > 1$
$f''(x) > 0, x \neq 1$

Review

AP Calculus AB - Quiz #. 5.1-5.3 Review KEY

1. Find the critical numbers of the function $f(x) = 2x^3 - 8x^2 + 10x$

$f'(x) = 6x^2 - 16x + 10$

$= 2(3x^2 - 8x + 5)$

$0 = 2(3x - 5)(x - 1)$

$\boxed{x = 5/3, 1}$

$f(x)$ is continuous on $[-2, 3]$

2. Find the absolute extrema of $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval $[-2, 3]$.

$f'(x) = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

$x = 2, -1$

$f(-2) = -3$

$f(2) = -19$

$f(3) = -8$

$f(-1) = 8$

Absolute minimum is -19 at $x = 2$
Absolute maximum is 8 at $x = -1$

3. If $f(x) = \frac{-x^2 + 4}{4x}$ on $[-2, 2]$, determine if Rolle's Theorem applies. If yes, find the value(s) of c defined by Rolle's Theorem. If no, explain why not.

Check conditions: not continuous on $[-2, 2]$ since $f(x)$ is not continuous at $x = 0$. Rolle's theorem does not apply.

4. If $f(x) = -(25 - 5x)^{\frac{1}{2}}$ on $[3, 5]$, determine if the Mean Value Theorem applies. If yes, find the value(s) of c defined by the Mean Value Theorem. If no, explain why not.

$f'(x) = -\frac{1}{2}(25 - 5x)^{-1/2}(-5)$

$= \frac{+5}{2\sqrt{25 - 5x}}$

$25 - 5x = 0$

$5x = 25 \quad x = 5$

$f(x)$ is continuous on $[3, 5]$ and differentiable on $(3, 5)$ so MVT applies

$f(3) = -\sqrt{10}$

$f(5) = 0$

$m_{Avg} = \frac{-\sqrt{10} - 0}{3 - 5} = \frac{-\sqrt{10}}{-2} = \frac{\sqrt{10}}{2}$

* set $f'(x) = m_{Avg}$

$\frac{+5}{2\sqrt{25 - 5x}} = \frac{\sqrt{10}}{2}$

$+10 = 2\sqrt{10(25 - 5x)}$

$+5 = \sqrt{10(25 - 5x)} \rightarrow (-5)^2 = \left(\sqrt{10(25 - 5x)}\right)^2$

$25 = 10(25 - 5x)$

$5 = 2(25 - 5x)$

$5 = 50 - 10x$

$-45 = -10x$

$\frac{9}{2} = x$

$\boxed{c = 4.5}$

5. If $f(x) = x^3 - 4x^2 + 5x$, find the interval(s) where $f(x)$ is Increasing and Decreasing

$f'(x) = 3x^2 - 8x + 5$

$0 = (3x - 5)(x - 1)$

$x = 5/3, 1$

$+ \quad - \quad +$

$-1 \quad 5/3$

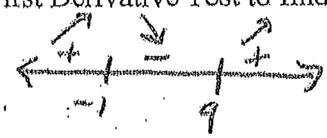
$f(x)$ increasing $(-\infty, 1) \cup (5/3, \infty)$ b/c $f'(x) > 0$

$f(x)$ decreasing $(1, 5/3)$ b/c $f'(x) < 0$

4

6. If $f(x) = \cancel{x^4 - 12x^2 + 16x}$ $x^3 - 12x^2 - 27x$, use the First Derivative Test to find all relative extrema. (x-values)

$f'(x) = 3x^2 - 24x - 27$



$0 = 3(x^2 - 8x - 9)$

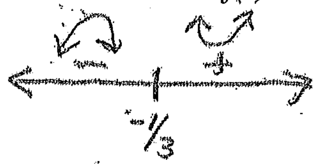
$= 3(x-9)(x+1)$

$x = 9, -1$

Relative max at $x = -1$ b/c $f'(x)$ change signs from + to −

Relative min at $x = 9$ b/c $f'(x)$ change signs from − to +

7. If $f(x) = x^3 + x^2 - 21x$, find the intervals where $f(x)$ is concave up and concave down, and find all points of inflection.



$f'(x) = 3x^2 + 2x - 21$

$f''(x) = 6x + 2$

$0 = 6x + 2$

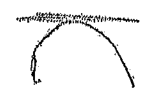
$x = -1/3$

$f(x)$ concave up $(-1/3, \infty)$ b/c $f''(x) > 0$

$f(x)$ concave down $(-\infty, -1/3)$ b/c $f''(x) < 0$

POI at $(-1/3, \underline{7.074})$ b/c $f''(x)$ change signs

8. If $f(x) = 2x^3 - x^2 - 20x - 10$, use the Second Derivative Test to find all relative extrema. Find Rel max/min



$f'(x) = 6x^2 - 2x - 20$

$= 2(3x^2 - x - 10)$

$2(3x+5)(x-2)$

$x = -5/3, 2$

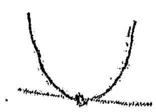
$f''(x) = 12x - 2$

$f''(-5/3) = 12(-5/3) - 2 = -22 < 0$, concave down

Relative max at $x = -5/3$

$f''(2) = 12(2) - 2 = 22 > 0$, concave up

Relative min at $x = 2$



9. Sketch a labeled graph of a function, f, with the following characteristics:

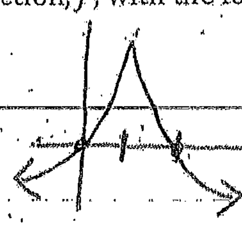
$f(0) = f(2) = 0$

~~$f'(x) > 0$ if $x < 1$~~

$f'(1)$ does not exist

$f'(x) < 0$ if $x > 1$

$f''(x) > 0, x \neq 1$



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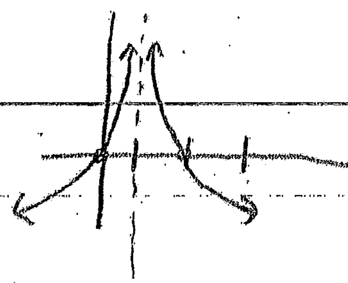
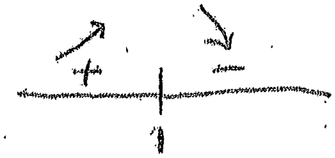
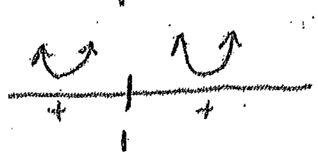


$f'(x)$



$f''(x)$



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AP Calculus AB Quiz Review: 5.1-5.3 Worksheet #2

1) Find all critical numbers for $g(x) = 16\sqrt{x} - x^2$

2) Find the value(s) of the absolute extrema of the function $f(x) = x^3 + 6x^2$ on the interval $[-3, 1]$. State theorem and conditions

3) If $f(x) = x^2 - 8x + 5$ on [2,6], determine if Rolle's Theorem can be applied. If yes, find the value(s) of c defined on Rolle's Theorem. State conditions and show steps.

4) If $g(x) = x^3 - x$ on $[1, 2]$, determine if the Mean Value Theorem can be applied. If yes, find the value(s) of c defined in the Mean Value Theorem. State conditions and show steps.

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5) If $f(x) = \frac{x^2}{x^2-9}$ find the following (where-appropriate): Intervals where f(x) is increasing, decreasing, relative maximum points, and relative minimums points. Justify your answer(s)

6) If $f(x) = \frac{1}{2}x^4 + 2x^3$ Find the intervals where f(x) is concave up and concave down, and find all points of inflection. (Justify your answers)

7) If $f(x) = -x^3 + 7x^2 - 15x$ use the Second Derivative Test to find all relative extrema. Justify your answers.

8) Sketch a labeled graph of a function, f, with the following characteristics:

$f(0) = 4$, $f(6) = 0$, $f(2) = 0$, $f(4) = 2$

$f'(x) < 0$ for $x < 2$ or $x > 4$

$f'(2)$ does not exist

$f'(4) = 0$

$f'(x) > 0$ for $2 < x < 4$

$f''(x) < 0$ for $x \neq 2$

Solution Key

AP Calculus AB **Quiz Review 5.1-5.3 Worksheet #2**

1) Find all critical numbers for $g(x) = 16\sqrt{x} - x^2$

$g(x) = 16x^{1/2} - x^2$

$g'(x) = 16 \cdot \frac{1}{2}x^{-1/2} - 2x$

$g'(x) = \frac{8}{x^{1/2}} - 2x$

$g'(x) = \frac{8 - 2x^{3/2}}{x^{1/2}}$

$8 - 2x^{3/2} = 0$
$-2x^{3/2} = -8$
$x^{3/2} = 4 \quad x = (4)^{2/3}$

$x^{1/2} = 0$
$x = 0$

$\boxed{\text{critical points: } x = 4^{2/3}, x = 0}$

2) Find the value(s) of the absolute extrema of the function $f(x) = x^3 + 6x^2$ on the interval [-3, 1]. State theorem and conditions

*Apply EVT: $f(x)$ continuous $[-3, 1]$

$f'(x) = 3x^2 + 12x$

$0 = 3x(x+4)$

$x = 0, \cancel{x = -4}$ outside interval

$f(-3) = (-3)^3 + 6(-3)^2 = 27$

$f(0) = 0$

$f(1) = 1 + 6 = 7$

$\boxed{\text{Abs maximum is } 27 \text{ at } x = -3 \quad \text{Abs minimum is } 0 \text{ at } x = 0}$

3) If $f(x) = x^2 - 8x + 5$ on [2,6], determine if Rolle's Theorem can be applied. If yes, find the value(s) of c defined on Rolle's Theorem. State conditions and show steps. $f(x)$ continuous on $[2,6]$, $f(x)$ differentiable $(2,6)$

$f(2) = -7$
$f(6) = -7$

$f(2) = f(6)$ ✓

$f'(x) = 2x - 8$

set $f'(x) = 0$

$2x - 8 = 0$

$x = 4$

$\boxed{\text{By Rolle's theorem } c = 4}$

4) If $g(x) = x^3 - x$ on [1, 2], determine if the Mean Value Theorem can be applied. If yes, find the value(s) of c defined in the Mean Value Theorem. State conditions and show steps. $g(x)$ continuous $[1,2]$ $g(x)$ differentiable $(1,2)$

*MVT: $g'(c) = \frac{g(b) - g(a)}{b - a}$

$g(1) = 1 - 1 = 0$

$g(2) = 2^3 - 2 = 6$

$\frac{g(2) - g(1)}{2 - 1} = \frac{6 - 0}{1} = 6$

$g'(x) = 3x^2 - 1$

$6 = 3x^2 - 1$

$7 = 3x^2$

$\frac{7}{3} = x^2$

$x^2 = 7/3$

$x = \pm\sqrt{7/3}$

$c = \sqrt{7/3}, \cancel{c = -\sqrt{7/3}}$

$\boxed{\text{By MVT, } c = \sqrt{7/3}}$

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VA: $x=3, x=-3$

5) If $f(x) = \frac{x^2}{x^2-9}$ find the following (where appropriate): Intervals where f(x) is increasing, decreasing, relative maximum points, and relative minimums points. Justify your answer(s)

$f(x)=\frac{x^2}{(x+3)(x-3)}$

$$f'(x) = \frac{2x(x^2-9)-x^2(2x)}{(x^2-9)^2}$$

$$f'(x)= \frac{2x^3-18x-2x^3}{(x^2-9)^2}$$

$$f'(x)= \frac{-18x}{(x^2-9)^2}$$

$-18x=0$, $x=0$ | $x^2-9=0$, $x=3,-3$

Critical pts: $x=0, 3, -3$

$+ \;|\; + \;|\; - \;|\; -$ at -3, 0, 3

f(x) increasing $(-\infty,-3),(-3,0)$ b/c $f'(x)>0$

f(x) decreasing $(0,3),(3,\infty)$ b/c $f'(x)<0$

Rel. max at $(0,0)$ b/c $f'(x)$ changes from + to −

6) If $f(x) = \frac{1}{2}x^4 + 2x^3$ Find the intervals where f(x) is concave up and concave down, and find all points of inflection. (Justify your answers)

$$f'(x)= \tfrac{1}{2}\cdot 4x^3+6x^2 = 2x^3+6x^2$$

$$f''(x)= 6x^2+12x$$

$$0=6x(x+2)$$

$$x=0,-2$$

$\cup \;\cap\; \cup$

$+ \;|\; - \;|\; +$ at -2, 0

f(x) concave up $(-\infty,-2),(0,\infty)$ b/c $f''(x)>0$

f(x) concave down $(-2,0)$ b/c $f''(x)<0$

POI at $(-2, f(-2))$ and $(0, f(0))$ b/c $f''(x)$ change signs.

7) If $f(x) = -x^3 + 7x^2 - 15x$ use the Second Derivative Test to find all relative extrema. Justify your answers.

$$f'(x)=-3x^2+14x-15$$

$$0=-(3x^2-14x+15)$$

$$0=-(3x-5)(x-3)$$

$$\boxed{x=5/3,\ x=3}$$

$$f''(x)=-6x+14$$

$f''(5/3)=-6(5/3)+14>0$, concave up

$\boxed{\text{Rel. minimum at } (5/3, f(5/3))}$

$f''(3)=-6(3)+14=-4<0$ concave down

$\boxed{\text{Rel. maximum at } (3, f(3))}$

8) Sketch a labeled graph of a function, f, with the following characteristics:

f(0) = 4, f(6) = 0, $f(2)=0$, $f(4)=2$

f '(x) < 0 for x < 2 or x > 4

f '(2) does not exist

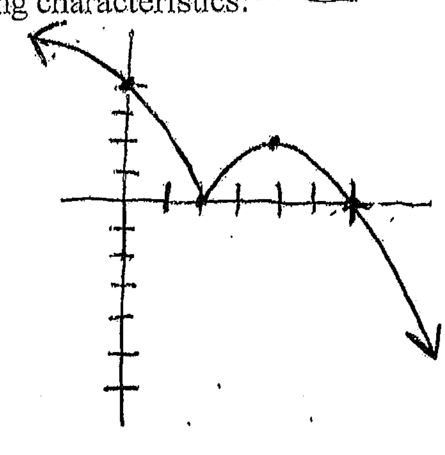
f '(4) = 0

f '(x) > 0 for 2 < x < 4

f ''(x) < 0 for x $\neq$ 2

$f'(x)$: $- \;|\; + \;|\; -$ at 2, 4

$f''(x)$: $\cap\; - \;|\; \cap\; -$ at 2



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AP Calculus AB **Quiz Review 5.1-5.3 Worksheet 3**

1) Find the critical points of $f(x) = x^{8/5} + x^{3/5}$

2) Find the value(s) of the absolute extrema of the function $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval $[-2, 3]$. State theorem and conditions

3) If $f(x) = \frac{x^2-2x-3}{x+2}$ on $[-1,3]$, determine if Rolle's Theorem can be applied. If yes, find the value(s) of c defined on Rolle's Theorem. State conditions and show steps.

4) If $g(x) = x^3 - x - 1$ on $[-1, 2]$, determine if the Mean Value Theorem can be applied. If yes, find the value(s) of c defined in the Mean Value Theorem. State conditions and show steps.

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5) If $f(x) = 2x^3 + 3x^2 - 12x$ find the following (where appropriate): Intervals where f(x) is increasing, decreasing, relative maximum points, and relative minimums points. Justify your answer(s)

6) If $f(x) = x^5 - 5x^4 + 3x + 7$ Find the intervals where f(x) is concave up and concave down, and find all points of inflection. (Justify your answers)

7) Sketch a labeled graph of a function, f, with the following characteristics:

$f(-4) = 5$, $f(-1) = -2$, $f(0) = 0$, $f(2) = 4$

$f'(x) < 0$ for $x < -1$ and $x > 2$

$f'(-1) = 0$, $f'(2) = 0$

$f'(x) > 0$ for $-1 < x < 2$

$f''(x) < 0$ for $x < -4$, $x > 0$

$f''(x) > 0$ for $-4 < x < 0$

AP Calculus AB Quiz Review 5.1-5.3 Worksheet 3

Key (11)

1) Find the critical points of $f(x) = x^{8/5} + x^{3/5}$

$f'(x) = \frac{8}{5}x^{3/5} + \frac{3}{5}x^{-2/5}$

$f'(x) = \frac{8x^{3/5}}{5} + \frac{3}{5x^{2/5}}$

$f'(x) = \frac{8x^1 + 3}{5x^{2/5}}$

$8x+3=0$ $\boxed{x = \frac{-3}{8}}$

$5x^{2/5} = 0$ $\boxed{x = 0}$

2) Find the value(s) of the absolute extrema of the function $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval [-2, 3]. State theorem and conditions $f(x)$ continuous $[-2,3]$, EVT

$f'(x) = 6x^2 - 6x - 12$

$f'(x) = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

$x = 2,\ x = -1$

$f(-2) = -3$

$f(-1) = 8$

$f(2) = -19$

$f(3) = -8$

$\boxed{\text{Abs max is } 8 \text{ (at } x=-1) \quad \text{Abs min is } -19 \text{ (at } x = 2)}$

3) If $f(x) = \frac{x^2-2x-3}{x+2}$ on [-1,3], determine if Rolle's Theorem can be applied. If yes, find the value(s) of c defined on Rolle's Theorem. State conditions and show steps.

V.A. at $x = -2$ (outside interval)

i) $f(x)$ continuous $[-1, 3]$

ii) $f(x)$ differentiable $(-1, 3)$

iii) $f(-1) = 0$, $f(3) = 0$ $\rightarrow f(-1) = f(3)$

Rolle's theorem applies:

$\boxed{\text{Set } f'(x) = 0}$

* quotient rule

$f'(x) = \frac{\overbrace{(2x-2)}^{f'}\overbrace{(x+2)}^{g} - \overbrace{(x^2-2x-3)}^{f}\overbrace{(1)}^{g'}}{\underbrace{(x+2)^2}_{g^2}} = \frac{2x^2+2x-4-x^2+2x+3}{(x+2)^2}$

$f'(x) = \frac{x^2+4x-1}{(x+2)^2}$

$x^2 + 4x - 1 = 0$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\frac{-4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$

$\frac{-4 \pm \sqrt{20}}{2} = x$

$x = \frac{-4+\sqrt{20}}{2}$

$\cancel{x = \frac{-4-\sqrt{20}}{2}}$

$\boxed{c = -2 + \frac{\sqrt{20}}{2}}$

4) If $g(x) = x^3 - x - 1$ on [-1, 2], determine if the Mean Value Theorem can be applied. If yes, find the value(s) of c defined in the Mean Value Theorem. State conditions and show steps.

$g(x)$ continuous $[-1, 2]$, $g(x)$ differentiable $(-1, 2)$

MVT: $f'(c) = \frac{f(b)-f(a)}{b-a}$

$g(-1) = -1 + 1 - 1 = -1$

$g(2) = 2^3 - 2 - 1 = 5$

slope: $\frac{5 - -1}{2 - -1} = \frac{6}{3} = \boxed{2}$

$g'(x) = \boxed{3x^2 - 1}$

$3x^2 - 1 = 2$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$\boxed{c = 1}$

$\cancel{c = -1}$

(12)

4) If $f(x) = 2x^3 + 3x^2 - 12x$ find the following (where appropriate): Intervals where f(x) is increasing, decreasing, relative maximum points, and relative minimums points. Justify your answer(s)

$f'(x) = 6x^2 + 6x - 12$

$0 = 6(x^2 + x - 2)$

$0 = 6(x+2)(x-1)$

$x = -2,\ x = 1$

$f'(x)$: $+$ (-3) | -2 | $-$ (0) | 1 | $+$ (2)

Rel. max $(-2, 20)$ b/c $f'(x)$ changes from + to −

Rel. min $(1, -7)$ b/c $f'(x)$ changes from − to +

$f(x)$ increasing $(-\infty, -2) \cup (1, \infty)$ b/c $f'(x) > 0$

$f(x)$ decreasing $(-2, 1)$ b/c $f'(x) < 0$

5) If $f(x) = x^5 - 5x^4 + 3x + 7$ Find the intervals where f(x) is concave up and concave down, and find all points of inflection. (Justify your answers)

$f'(x) = 5x^4 - 20x^3 + 3$

$f''(x) = 20x^3 - 60x^2$

$0 = 20x^2(x-3)$

$x = 0,\ x = 3$

$f''(x)$: $-$ (-1) | 0 | $-$ (1) | 3 | $+$ (4)

Point of Inflection at $(3, -146)$ b/c $f''(x)$ change signs.

$f(x)$ is concave up $(3, \infty)$ b/c $f''(x) > 0$

$f(x)$ is concave down $(-\infty, 0) \cup (0, 3)$ b/c $f''(x) < 0$

6) Sketch a labeled graph of a function, f, with the following characteristics:

f(-4) = 5, f(-1) = -2, f(0) = 0, f(2) = 4

f'(x) < 0 for x < -1 or x > 2

f'(-1) = 0 , f'(2) = 0

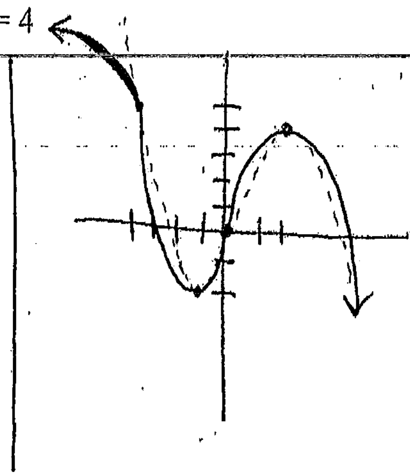
f'(x) > 0 for -1 < x < 2

f''(x) < 0 for x < −4 , x > 0

f''(x) > 0 for - 4 < x < 0

$f'(x)$: $-$ | -1 | $+$ | 2 | $-$

$f''(x)$: $-$ | -4 | $+$ | 0 | $-$



AP Calculus AB 5.1-5.3 Quiz Review (WS #4)

1. If $f(x) = \frac{x^2-6x}{x+2}$, does Rolle's theorem apply to the function on the interval [0,6]. If yes, find the value of c defined by Rolle's theorem.

2. Determine whether the Mean Value Theorem can be applied to $f(x) = \frac{x+1}{x}$ on $\left[\frac{1}{2}, 2\right]$.

   If so, then find all values of c on (a, b) defined by MVT.

3. Use the Second Derivative Test to classify the critical points of $y = \frac{1}{4}x^4 - 2x^3 + 6$.

AP Calculus AB Ch. 3.1-3.4 Quiz Review

4. Sketch a labeled graph with the following characteristics:
   a) $f(-1) = 4$ and $f(2) = -1$
   b) $f'(x) > 0$ if $x < -1$ and if $x > 2$
   c) $f'(x) < 0$ if $-1 < x < 2$
   d) POI at point (1, 2)
   e) $f''(x) < 0$ if $x < 1$
   f) $f''(x) > 0$ if $x > 1$

5. Find the absolute maximum and absolute minimum of the function f on the given interval:

   $f(x) = \frac{x^2-4}{x^2+4}$ on [-4,4]

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AP Calculus AB   Ch. 3.1-3.4 Quiz Review   KEY

1. If $f(x)=\frac{x^2-6x}{x+2}$, does Rolle's theorem apply to the function on the interval [0,6]. If yes, find the value of c defined by Rolle's theorem. $f(x)$ continuous, differentiable on interval $[0,6]$

$f'(x)=\frac{(2x-6)(x+2)-(x^2-6x)(1)}{(x+2)^2}$

$=\frac{2x^2-2x-12-x^2+6x}{(x+2)^2}$

$=\frac{x^2+4x-12}{(x+2)^2}$

$=\frac{(x+6)(x-2)}{(x+2)^2}$

$f(0)=0$ $f(0)=f(6)$, Rolle's applies

$f(6)=0$

set $f'(x)=0$

$\frac{(x+6)(x-2)}{(x+2)^2}=0$

$x=2,-6$ $\boxed{c=2}$

2. Determine whether the Mean Value Theorem can be applied to $f(x)=\frac{x+1}{x}$ on $\left[\frac{1}{2},2\right]$. If so, then find all values of c on (a, b) defined by MVT. $x\neq 0$, $f(x)$ is continuous, differentiable on interval $[\frac{1}{2},2]$.

$f'(x)=\frac{(1)(x)-(x+1)(1)}{x^2}$

$f'(x)=\frac{-1}{x^2}$

Find endpoints

$f(\frac{1}{2})=3$

$f(2)=\frac{3}{2}$

$m_{Avg}=\frac{3-\frac{3}{2}}{\frac{1}{2}-2}=\frac{\frac{3}{2}}{-\frac{3}{2}}=-1$

set $f'(x)=m_{Avg}$

$\frac{-1}{x^2}=-1$

$x^2=1$

$x=\pm 1$ $\boxed{c=1}$

3. Use the Second Derivative Test to classify the critical points of $y=\frac{1}{4}x^4-2x^3+6$.

$y'(x)=\frac{1}{4}\cdot 4x^3-6x^2=x^3-6x^2=x^2(x-6)$ $\boxed{x=0,6}$

$y''(x)=3x^2-12x$

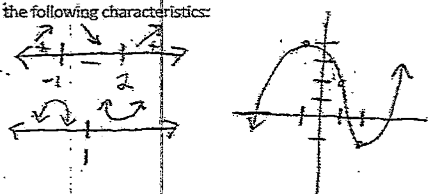
$y''(0)=0-0$ inconclusive test at $x=0$

$y''(6)=3(6)^2-12(6)=36>0$, concave up $\boxed{\text{Relative min at } x=6}$

$108-72=36$

4. Sketch a labeled graph with the following characteristics:

a) $f(-1)=4$ and $f(2)=-1$
b) $f'(x)>0$ if $x<-1$ and if $x>2$
c) $f'(x)<0$ if $-1<x<2$
d) POI at point (1, 2)
e) $f''(x)<0$ if $x<1$
f) $f''(x)>0$ if $x>1$



5) Find the absolute max and absolute min of function f on the given interval: $f(x)=\frac{x^2-4}{x^2+4}$; $[-4,4]$

$f'(x)=\frac{(2x)(x^2+4)-(x^2-4)(2x)}{(x^2+4)^2}=\frac{2x^3+8x-2x^3+8x}{(x^2+4)^2}=\frac{16x}{(x^2+4)^2}$

* $f(x)$ is continuous on $[-4,4]$

find critical pts: $\frac{16x}{(x^2+4)^2}$ $16x=0\rightarrow\boxed{x=0}$ $\cancel{(x^2+4)^2}=0\rightarrow$ no critical pts

$f(-4)=\frac{(-4)^2-4}{4^2+4}=\frac{12}{20}=\frac{3}{5}$

$f(4)=\frac{(4)^2-4}{4^2+4}=\frac{12}{20}=\frac{3}{5}$

$f(0)=\frac{0-4}{0+4}=\frac{-4}{4}=-1$

Abs. max is $\frac{3}{5}$ at $x=-4$, $x=4$

Abs. min is -1 at $x=0$