Calculus Ch. 5.1 Natural Log Function

Natural Log graph: Domain: ______________ Range: ______________

Graph characteristics: ____________________________________

Ex. 1: Sketch graph of ln(x – 3) and state domain:

Ex. 2 Draw the function and answer the examples.

a) $\lim_{x\to 0^+} \ln(x) =$ b) $\lim_{x\to 0^-} \ln(x) =$ c) $\lim_{x\to 0} \ln(x) =$ d) $\lim_{x\to \infty} \ln(x) =$

Properties: $\ln(1) = 0$ $\ln(a^n) = n\ln(a)$ $\ln(e) = 1$

$\ln(ab) = \ln(a) + \ln(b)$ $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

Ex. 3 Expand $\ln 3e^2$

②

Properties: $\ln(1) = 0$ $\ln(a^n) = n\ln(a)$ $\ln(e) = 1$

$\ln(ab) = \ln(a) + \ln(b)$ $\ln(a/b) = \ln(a) - \ln(b)$

Ex. 4 condense $2(\ln(x) - \ln(x + 1) - \ln(x - 1))$

Derivative of the Natural Logarithmic Function:

$$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$

Ex. 5: If $y = \ln(x)$, find y'

Ex. 6: if $y = \ln(x^2 - 5)$, find y'

Ex. 7: if $y = \ln\left(\frac{x^2}{\sqrt{2x^3}}\right)$, find y' (always simplify logs before taking the derivative)

Calculus Ch. 5.1b, 5.3 Notes

Warm-up Problem: If $g(2) = 3$ and $g'(2) = -4$, find $f'(2)$ if $f(x) = x^2\ln(g(x))$.

Logarithmic Differentiation : Simplifying non-log functions using log properties to expand before differentiating.

Log differentiation steps:

1. Take the **ln** (natural log) of both sides
2. Simplify and expand using log properties
3. Use implicit differentiation
4. Substitute for y

Example 1: Find the derivative of $y = \dfrac{(x-2)^2}{\sqrt{x^2+1}}$

Example 2: Find the derivative of $y = x^{2x+3}$

Absolute Value Rule: $\dfrac{d}{dx}\ln|u| = \dfrac{u'}{u}$

Example 3: Find $\dfrac{d}{dx}\ln|x^2-5|$

Example 4: Locate any relative extrema and inflection points for $y = \dfrac{x}{\ln x}$

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Ch.5.3 Notes — Derivatives of Inverse Function

A. Inverse Functions

1) x's and y's are swapped
2) Domains and ranges are swapped
3) Graphs are symmetric over the line y = x
4) F(x) must be one-to-one (pass the horizontal line test) for its inverse to be a function
5) A function that is **monotonic** (always increasing or always decreasing) will always have an inverse that is a function
6) If f and g are inverses, then f(g(x)) = x and g(f(x)) = x

Example 4: find the inverse of f(x) = 6x + 2

*ALWAYS restrict the domain of the inverse function to the range of the original function

Ex. 5 $f(x) = \sqrt{x-5}$. Find the domain of the inverse function

Evaluate derivative of inverse at a point: (find $(f^{-1})'(a)$

$f(b) = a$	$(f^{-1})(a) = b$
$f'(b) = n$	$(f^{-1})'(a) = \frac{1}{n}$

Example 6: $f(x) = x^3 + 4x + 2$ find $(f^{-1})'(-3)$

Example 7: $f(x) = \sqrt{x^3 - 7}$ find $(f^{-1})'(1)$

Example 8: If g(f(x)) = x , g(7) = 2, and g'(7) = 10, then f'(2) is

Example 9: If g(f(x)) = x , g(9) = 3, and g'(9) = -4, then f'(3) is

Ch. 5.1b Natural Log Differentiation Classwork

Finding a Derivative In Exercises 41–64, find the derivative of the function.

$\frac{d}{dx}[\ln u] = \frac{u'}{u}$

Reminder: Expand log expression fully before differentiating

Log Properties

1. $\ln(1) = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln(a^n) = n \ln a$
4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

41. $f(x) = \ln(3x)$	**42.** $f(x) = \ln(x - 1)$
43. $g(x) = \ln x^2$	**44.** $h(x) = \ln(2x^2 + 1)$
45. $y = (\ln x)^4$	**46.** $y = x^2 \ln x$
47. $y = \ln(t + 1)^2$	**48.** $y = \ln\sqrt{x^2 - 4}$
49. $y = \ln\left(x\sqrt{x^2 - 1}\right)$	**50.** $y = \ln[t(t^2 + 3)^3]$

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51. $f(x) = \ln\left(\frac{x}{x^2 + 1}\right)$

52. $f(x) = \ln\left(\frac{2x}{x + 3}\right)$

53. $g(t) = \frac{\ln t}{t^2}$

54. $h(t) = \frac{\ln t}{t}$

55. $y = \ln(\ln x^2)$

56. $y = \ln(\ln x)$

57. $y = \ln\sqrt{\frac{x + 1}{x - 1}}$

58. $y = \ln\sqrt[3]{\frac{x - 1}{x + 1}}$

59. $f(x) = \ln\left(\frac{\sqrt{4 + x^2}}{x}\right)$

60. $f(x) = \ln\left(x + \sqrt{4 + x^2}\right)$

Finding a Derivative Implicitly **In Exercises 73–76, use implicit differentiation to find dy/dx.**

73. $x^2 - 3 \ln y + y^2 = 10$

74. $\ln xy + 5x = 30$

75. $4x^3 + \ln y^2 + 2y = 2x$

76. $4xy + \ln x^2y = 7$

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Log Differentiation:
Use log differentiation to find dy/dx

Log Properties

1. $\ln(1) = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln(a^n) = n \ln a$
4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

89. $y = x\sqrt{x^2 + 1}, \quad x > 0$

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90. $y = \sqrt{x^2(x + 1)(x + 2)}, \quad x > 0$

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91. $y = \frac{x^2\sqrt{3x - 2}}{(x + 1)^2}, \quad x > \frac{2}{3}$

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92. $y = \sqrt{\frac{x^2 - 1}{x^2 + 1}}, \quad x > 1$

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93. $y = \frac{x(x - 1)^{3/2}}{\sqrt{x + 1}}, \quad x > 1$

Non-AP Calculus 5.1/5.3 Natural Logs Properties and Derivatives Quiz Review WS #1

Log Properties

1. $\ln(1) = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln(a^n) = n \ln a$
4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
5. $\ln(e) = 1$

$$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$

Expanding a Logarithmic Expression: Use properties of logarithms to expand the logarithmic expression

1. $y = \ln\left(x\sqrt[3]{6 - x^4}\right)$

2. $f(x) = ln\sqrt{\frac{3-2x}{4x}}$

3. $y = ln\left(\frac{2x^4}{x\sqrt{3x^5-1}}\right)$

Condensing a Logarithmic Expression: Use properties of logarithms to write expression as a logarithm of a single quantity

4. $5\ln x + 3\ln y - 5\ln w - 6\ln z$

5. $\frac{1}{5}[\ln(x-4) - 3\ln(7-x^2) - \ln(8-x)]$

Find the derivative of the functions below:

6) $y = ln\left(x\sqrt{6 - x^3}\right)$

(10) Find the derivative of the functions below:

7) $y = ln\sqrt{\left(\frac{3-2x}{x^3}\right)^5}$

8) $y = x^3 \ln(x^2)$

Use Log differentiation to find the derivative of the function:

9) $y = \frac{x^3\left(\sqrt{5-4x^5}\right)}{(x-1)^2}$

Use Implicit Differentiation to find $\frac{dy}{dx}$:

10) $x + ln(xy) - 2y = x^3$

11) Find an inverse function for f(x): $f(x) = 2x^3 - 1$

Use function f(x) and the given real number a to find $(f^{-1})'(a)$

12) $f(x) = 2x^3 - 3x + 1$ a = 11

Calculus AB **Ch. 5.1-5.3 Natural Logs Quiz Review**

1. Find the domain for $y = \ln(2 + 3x) - 1$

2. Find $\frac{dy}{dx}$ $y = ln\sqrt{\frac{3-2x}{4x}}$

3. Find $\frac{dy}{dx}$ $y = x^{\sqrt{x+3}}$

4. $f(x) = \sqrt{5x - 1} - 4$
   a. Find $(f^{-1})(x)$
   b. Find the domain for $(f^{-1})(x)$

5. $f(x) = x^3 + 2x^2 - 3$ Find $(f^{-1})'(13)$

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Identify the domain and range of each.

6) $y = \ln(2x - 3) + 5$

7) $y = \ln(3x + 17) - 5$

Expand each logarithm.

8) $\ln(a \cdot b \cdot c^3)$

Condense each expression to a single logarithm.

9) $\frac{\ln u}{2} + \frac{\ln v}{2} + \frac{\ln w}{2}$

Differentiate each function with respect to x.

10) $f(x) = \ln \sqrt[4]{\frac{2x^3}{3x^2 - 4}}$

11) $f(x) = \ln\left(\frac{4x^2}{5x^3 - 3}\right)^5$

Use logarithmic differentiation to differentiate each function with respect to x.

12) $y = \sqrt[3]{x^2 + 1}$

13) $y = x^{2x}$

For each problem, find $(f^{-1})'(a)$

14) $f(x) = 3x^5 + 2x + 5,\ a = 5$

15) $f(x) = 2x^3 + 4x + 5,\ a = 5$

Ch.5.4 Notes — Derivative of Exponential Function e^x

$y = \ln x$ and $y = e^x$ are inverse functions (meaning $f(g(x)) = x$ and $g(f(x)) = x$)

Example 1: Solve $7 = e^{x+1}$

Example 2: solve $\ln(2x - 3) = 5$

Reminder: exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ Reminder: e is a NUMBER: if $y = e^2$, then $y' = 0$

Exponential Function e^x Derivative rule $\frac{d}{dx}e^u = e^u * u'$

Example 3: find y' for $y = \ln(2x - e^{-2x})$

Example 4: Find y' for $y = xe^{(x^2+2x+3)^3}$

Example 5: Find the equation of the tangent line to the graph at the given point:
$y = e^{-x}\ln x$ $(1, 0)$

Example 6: Find dy/dx

$xe^y - 10x + 3y = 0$

Example 7: Find the equation of the tangent line to the graph of the function at the given point.

$xe^y + ye^x = 1$ at (0, 1)

Example 8: Find the 2nd derivative of the function

$f(x) = (3 + 2x)e^{-3x}$

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Ex. 9: find the extrema and points of inflection for $g(t) = 1 + (2 + t)e^{-t}$

Ex. 10: find the extrema and points of inflection for $f(x) = \frac{e^{x} - e^{-x}}{2}$ (use common denominators)

5.4 Exponential Functions e^x Classwork Worksheet

exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ $\frac{d}{dx}e^u = e^u * u'$

Additional $y = \ln x$ and $y = e^x$ Properties: $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln 1 = 0$ | $\ln e = 1$

Solving an Exponential or Logarithmic Equation In Exercises 1–16, solve for x accurate to three decimal places.

1. $e^{\ln x} = 4$	**2.** $e^{\ln 3x} = 24$
3. $e^x = 12$	**4.** $5e^x = 36$
5. $9 - 2e^x = 7$	**8.** $100e^{-2x} = 35$
11. $\ln x = 2$	**12.** $\ln x^2 = 10$
13. $\ln(x - 3) = 2$	**14.** $\ln 4x = 1$
15. $\ln\sqrt{x + 2} = 1$	**16.** $\ln(x - 2)^2 = 12$

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exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ $\frac{d}{dx}e^u = e^u * u'$

Additional $y = \ln x$ and $y = e^x$ Properties: $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln 1 = 0$ | $\ln e = 1$

Finding a Derivative **In Exercises 33–54, find the derivative.**

33. $f(x) = e^{2x}$	**34.** $y = e^{-8x}$
35. $y = e^{\sqrt{x}}$	**36.** $y = e^{-2x^3}$
39. $y = e^x \ln x$	**40.** $y = xe^{4x}$
41. $y = x^3 e^x$	**42.** $y = x^2 e^{-x}$
43. $g(t) = (e^{-t} + e^t)^3$	**44.** $g(t) = e^{-3/t^2}$
45. $y = \ln(1 + e^{2x})$	**46.** $y = \ln\left(\frac{1 + e^x}{1 - e^x}\right)$

Finding a Derivative **In Exercises 33–54, find the derivative**

$\frac{d}{dx}[\sin u] = (\cos u)u'$ 14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$ 15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$

$\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$ 17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$ 18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$

47. $y = \dfrac{2}{e^x + e^{-x}}$

48. $y = \dfrac{e^x - e^{-x}}{2}$

49. $y = \dfrac{e^x + 1}{e^x - 1}$

50. $y = \dfrac{e^{2x}}{e^{2x} + 1}$

51. $y = e^x(\sin x + \cos x)$

52. $y = e^{2x} \tan 2x$

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Finding an Equation of a Tangent Line In Exercises 55–62, find an equation of the tangent line to the graph of the function at the given point.

Steps for finding Tangent Line Equation:

1) Identify Ordered Pair: Point: ___________
2) Find Slope: Find f '(x) and evaluate the slope at x-value: Slope: m = ___________
3) Put equation into point-slope form: $y - y_1 = m(x - x_1)$

55. $f(x) = e^{3x}$, $(0, 1)$

56. $f(x) = e^{-2x}$, $(0, 1)$

57. $f(x) = e^{1-x}$, $(1, 1)$

58. $y = e^{-2x+x^2}$, $(2, 1)$

59. $f(x) = e^{-x}\ln x$, $(1, 0)$

62. $y = xe^x - e^x$, $(1, 0)$

Ch. 5.5 Notes **Derivative of Logs of other bases**

Change of Base: $log_a x = \frac{lnx}{lna}$

Ex. 1 solve for x: $3^x = 1/81$

Ex. 2 solve: $\log_2 x = -4$

Derivative Rule for logs of other bases : $\frac{d}{dx}\log_a \cdot u = \frac{u'}{(\ln a)\,u'}$

Ex. 3 Find f '(x) for $f(x) = log_5 \sqrt[3]{(2x^2+7)}$

Ex. 4 Find f '(x) for $f(x) = log\,\frac{5x^3}{(x^2-3x)^3}$

Derivative Rule for Exponential functions of base a^x : $\frac{d}{dx}a^u = (\ln a)a^u * u'$

Ex. 5 Find f '(x) for $f(x) = 5^{x^2-2x}$

Ex. 6 Find f '(x) for $f(x) = x(4^{-x})$

Ex. 7:

1994 #4: A particle moves along the *x*-axis so that at any time $t > 0$ its velocity is given by $v(t) = t\ln t - t$.

a) Write an expression for the acceleration of the particle.

b) For what values of *t* is the particle moving to the right?

c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

Ch. 5.5 Log and Exponential Derivatives for base a

11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$

12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$

Find the Derivative of the below functions: (Consider Expanding Log Expressions before Deriving if applicable)

39. $y = 5^{-4x}$	40. $y = 6^{3x-4}$
41. $f(x) = x\,9^x$	42. $y = x(6^{-2x})$
49. $h(t) = \log_5(4 - t)^2$	48. $y = \log_3(x^2 - 3x)$
51. $y = \log_5 \sqrt{x^2 - 1}$	50. $g(t) = \log_2(t^2 + 7)^3$
53. $f(x) = \log_2 \frac{x^2}{x - 1}$	52. $f(x) = \log_2 \sqrt[3]{2x + 1}$
55. $h(x) = \log_3 \frac{x\sqrt{x - 1}}{2}$	56. $g(x) = \log_5 \frac{4}{x^2\sqrt{1 - x}}$

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Implicit Differentiation In Exercises 63 and 64, use implicit differentiation to find dy/dx.

63. $xe^y - 10x + 3y = 0$

64. $e^{xy} + x^2 - y^2 = 10$

Finding the Equation of a Tangent Line In Exercises 65 and 66, find an equation of the tangent line to the graph of the function at the given point.

65. $xe^y + ye^x = 1, \quad (0, 1)$

66. $1 + \ln xy = e^{x-y}, \quad (1, 1)$

5.4-5.5 Quiz Review Derivatives of Exponential function e^x and a^x

Find $\frac{dy}{dx}$

1. f(x) $= ln\frac{(3+4x)^5}{\sqrt[4]{1-3x}}$

2. $y = \log_3\left(\frac{3x^5}{2x^4-3}\right)^3$

3. f(x) = xe^{2-x^2}

4. f(x) = $log_5\left(\frac{3-x}{\sqrt{1-x}}\right)$

5. f(x) = $8^{x-3x^2}(log(3-2x)^3)$

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5.4-5.5 Quiz Review Derivatives of Exponential function e^x and a^x

Solution Key

Find $\frac{dy}{dx}$.

1. $f(x) = ln\frac{(3+4x)^5}{\sqrt[4]{1-3x}}$

$= \ln(3+4x)^5 - \ln(1-3x)^{1/4}$

$= 5\ln(3+4x) - \frac{1}{4}\ln(1-3x)$

$f'(x) = 5\left(\frac{4}{3+4x}\right) - \frac{1}{4}\left(\frac{-3}{1-3x}\right) = \boxed{\frac{20}{3+4x} + \frac{3}{4(1-3x)}}$

2. $y = \log_3\left(\frac{3x^5}{2x^4-3}\right)^3 = 3\log_3\left(\frac{3x^5}{2x^4-3}\right) = 3\log_3(3x^5) - 3\log_3(2x^4-3)$

Reminder: $* \frac{d}{dx}\log_a u = \frac{1}{\ln a}\left[\frac{u'}{u}\right]$

$y' = 3\cdot\frac{1}{\ln 3}\left(\frac{15x^4}{3x^5}\right) - 3\left(\frac{1}{\ln 3}\right)\left(\frac{16x^3}{2x^4-3}\right)$

$= \boxed{\frac{3}{\ln 3}\left(\frac{5}{x}\right) - \frac{3}{\ln x}\left(\frac{16x^3}{2x^4-3}\right)}$

3. $f(x) = xe^{2-x^2}$

$*$ product rule

$f'(x) = (1)e^{2-x^2} + x\cdot e^{2-x^2}(-2x)$

$\boxed{= e^{2-x^2}(1-2x^2)}$

4. $f(x) = log_5\left(\frac{3-x}{\sqrt{1-x}}\right) = \log_5(3-x) - \log_5(1-x)^{1/2} = \log_5(3-x) - \frac{1}{2}\log_5(1-x)$

$f'(x) = \frac{1}{\ln 5}\left(\frac{-1}{3-x}\right) - \frac{1}{2}\left(\frac{1}{\ln 5}\right)\left(\frac{-1}{1-x}\right)$

$\boxed{= \frac{-1}{\ln 5(3-x)} + \frac{1}{2\ln 5(1-x)}}$

Reminder: $* \frac{d}{dx}a^u = \ln a\cdot a^u\cdot u'$

5. $f(x) = 8^{x-3x^2}(log(3-2x)^3)$

$f(x) = 8^{x-3x^2}\cdot 3\log_{10}(3-2x)$

$f'(x) = (\ln 8)8^{x-3x^2}\cdot(1-6x)\cdot\log(3-2x)^3 + 8^{x-3x^2}\cdot\frac{3}{\ln 10}\left(\frac{-2}{3-2x}\right)$

$\boxed{8^{x-3x^2}\left[(\ln 8)(1-6x)\log(3-2x)^3 - \frac{6}{\ln 10(3-2x)}\right]}$

DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$
22. $\frac{d}{dx}[\text{arccot}\, u] = \frac{-u'}{1 + u^2}$
23. $\frac{d}{dx}[\text{arcsec}\, u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$
24. $\frac{d}{dx}[\text{arccsc}\, u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$