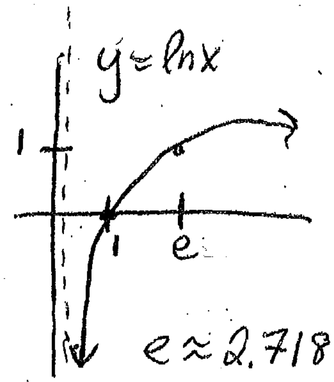
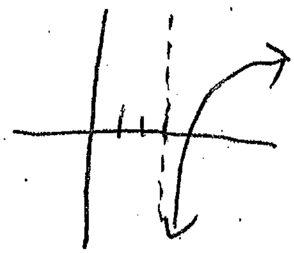
Calculus Ch. 5.1 Natural Log Function



Natural Log graph: Domain: $(0,\infty)$ Range: $(-\infty,\infty)$

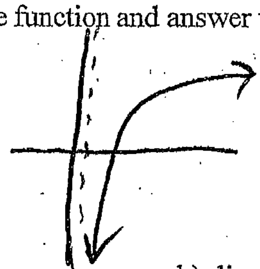
Graph characteristics: always continuous, always increasing, always concave down

Ex. 1: Sketch graph of ln(x – 3) and state domain:



D: $(3,\infty)$

Ex. 2 Draw the function and answer the examples.



a) $\lim_{x\to 0^+} \ln(x) = \boxed{-\infty}$ b) $\lim_{x\to 0^-} \ln(x) = \boxed{\text{DNE}}$ c) $\lim_{x\to 0} \ln(x) =$ DNE d) $\lim_{x\to \infty} \ln(x) = +\infty$

Properties: $\ln(1) = 0$ $\ln(a^n) = n\ln(a)$ $\ln(e) = 1$

$\ln(ab) = \ln(a) + \ln(b)$ $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

Ex. 3 Expand $\ln\left(3e^2\right)$

$\ln 3 + \ln e^2$

$\ln 3 + 2\ln e$

$\ln 3 + 2(1)$ $\boxed{\ln 3 + 2}$

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Properties: $\ln(1) = 0$ $\ln(a^n) = n\ln(a)$ $\ln(e) = 1$

$\ln(ab) = \ln(a) + \ln(b)$ $\ln(a/b) = \ln(a) - \ln(b)$

Ex. 4 condense $2[\ln(x) - \ln(x+1) - \ln(x-1)]$

$$2\left[\ln x - \left(\ln(x+1) + \ln(x-1)\right)\right]$$

$$2\left[\ln x - \ln(x^2-1)\right]$$

$$2\left[\ln\left(\frac{x}{x^2-1}\right)\right]$$

$$\ln\left(\frac{x}{x^2-1}\right)^2$$

Derivative of the Natural Logarithmic Function:

$$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$

Ex. 5: If $y = \ln(x)$, find y'

$$y' = \frac{1}{x}$$

Ex. 6: if $y = \ln(x^2 - 5)$, find y'

$$y' = \frac{2x}{x^2-5}$$

Ex. 8 Find $\frac{dy}{dx}$ $4xy + \ln(x^2y) = 7$

$$4xy + \ln x^2 + \ln y = 7$$

$$4xy + 2\ln x + \ln y = 7$$

$$4y + 4x\left(\frac{dy}{dx}\right) + 2\left(\frac{1}{x}\right) + \frac{1}{y}\left(\frac{dy}{dx}\right) = 0$$

$$4x\left(\frac{dy}{dx}\right) + \frac{1}{y}\left(\frac{dy}{dx}\right) = -\frac{2}{x} - 4y$$

$$\frac{dy}{dx}\left(4x + \frac{1}{y}\right) = -\frac{2}{x} - 4y$$

$$\frac{dy}{dx} = \frac{\left(-\frac{2}{x} - 4y\right)}{\left(4x + \frac{1}{y}\right)} \cdot \frac{(xy)}{(xy)}$$

$$\boxed{\frac{dy}{dx} = \frac{-2y - 4xy^2}{4xy^2 + x}}$$

$$y' = \frac{-4xy^2 - 2y}{4x^2y + x}$$

Ex. 7: if $y = \ln\left(\frac{x^2}{\sqrt{2x^3}}\right)$, find y' (always simplify logs before taking the derivative)

$$y = \ln x^2 - \ln(2x^3)^{1/2}$$

$$y = 2\ln x - \frac{1}{2}\ln(2x^3)$$

$$y' = 2\left(\frac{1}{x}\right) - \frac{1}{2}\left(\frac{6x^2}{2x^3}\right)$$

$$\boxed{y' = \frac{2}{x} - \frac{3}{2x}}$$

$$\boxed{y' = \frac{4-3}{2x}} = \boxed{\frac{1}{2x}}$$

Calculus Ch. 5.1b, 5.3 Notes

Recall $\frac{d}{dx}\ln u = \frac{u'}{u}$

Warm-up Problem: If g(2) = 3 and g ' (2) = -4, find f ' (2) if $f(x) = x^2\ln(g(x))$.

$$f'(x) = 2x\cdot\ln(g(x)) + 2^2\cdot\frac{g'(x)}{g(x)}$$

$$f'(2) = 2(2)\cdot\ln[g(2)] + 2^2\cdot\frac{g'(2)}{g(2)}$$

$$f'(2) = 4\cdot\ln(3) + 2^2\left(-\tfrac{4}{3}\right)$$

$$\boxed{f'(2) = 4\ln 3 - 16/3}$$

Logarithmic Differentiation : Simplifying non-log functions using log properties to expand before differentiating.

Log differentiation steps:

1. Take the **ln** (natural log) of both sides
2. Simplify and expand using log properties
3. Use implicit differentiation
4. Substitute for y

Example 1: Find the derivative of $y = \frac{(x-2)^2}{\sqrt{x^2+1}}$

$$\ln y = \ln\left[\frac{(x-2)^2}{(x^2+1)^{1/2}}\right]$$

$$\ln y = \ln(x-2)^2 - \ln(x^2+1)^{1/2}$$

$$\ln y = 2\ln(x-2) - \tfrac{1}{2}\ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = 2\left(\frac{1}{x-2}\right) - \frac{1}{2}\left(\frac{2x}{x^2+1}\right)$$

$$\frac{dy}{dx} = y\left[\frac{2}{x-2} - \frac{x}{x^2+1}\right]$$

$$\frac{dy}{dx} = \frac{(x-2)^2}{\sqrt{x^2+1}}\left[\frac{2}{x-2} - \frac{x}{x^2+1}\right]$$

Example 2: Find the derivative of $y = x^{2x+3}$

$$\ln y = \ln x^{2x+3}$$

$$\ln y = (2x+3)(\ln x)$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = (2)[\ln x] + (2x+3)\left(\frac{1}{x}\right)$$

$$\frac{dy}{dx} = y\left[2\ln x + \frac{2x+3}{x}\right]$$

$$\frac{dy}{dx} = x^{2x+3}\left[2\ln x + 2 + \frac{3}{x}\right]$$

Absolute Value Rule: $\frac{d}{dx}\ln|u| = \frac{u'}{u}$

Example 3: Find $\frac{d}{dx}\ln|x^2-5| = \frac{2x}{x^2-5}$

Example 4: Locate any relative extrema and inflection points for $y = \frac{x}{\ln x}$

$$y'(x) = \frac{1\ln x - x\left(\frac{1}{x}\right)}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2} \rightarrow$$

set $\ln x - 1 = 0$

$\ln x = 1$

$e^1 = x$

$-\quad -\quad +$

$0 \quad 1 \quad e$

Rel. min at $\left(e, \frac{e}{\ln e}\right) = (e, e)$

$$y''(x) = \frac{\left(\frac{1}{x}\right)(\ln x)^2 - (\ln x - 1)\,2(\ln x)\left(\frac{1}{x}\right)}{(\ln x)^4}$$

$$= \frac{\frac{1}{x}\ln x\,[\ln x - 2\ln x + 2]}{(\ln x)^4}$$

$$y''(x) = \frac{-\ln x + 2}{x(\ln x)^3}$$

$2 - \ln x = 0$

$2 = \ln x$

$e^2 = x$

$\cup \quad \cup \quad \cap$

$+ \quad + \quad -$

$0 \quad 1 \quad e^2$

POI at $\left(e^2, \frac{e^2}{2}\right)$ b/c $y''(x)$ change signs.

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Ch.5.3 Notes **Derivatives of Inverse Function**

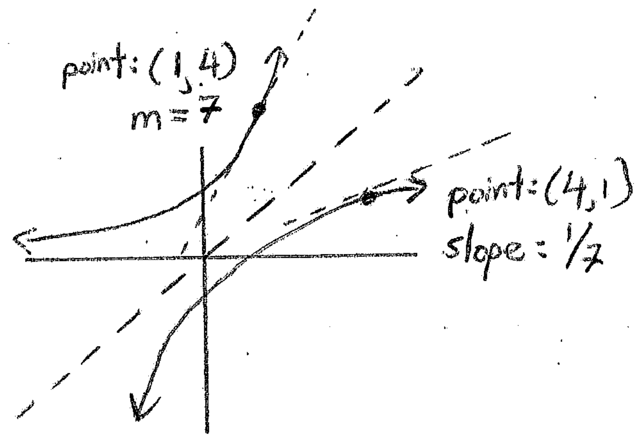
A. Inverse Functions
1) x's and y's are swapped
2) Domains and ranges are swapped
3) Graphs are symmetric over the line y = x
4) F(x) must be one-to-one (pass the horizontal line test) for its inverse to be a function
5) A function that is **monotonic** (always increasing or always decreasing) will always have an inverse that is a function
6) If f and g are inverses, then f(g(x)) = x and g(f(x)) = x

Example 4: find the inverse of f(x) = 6x + 2

*ALWAYS restrict the domain of the inverse function to the range of the original function

Ex. 5 f(x) = $\sqrt{x-5}$. Find the domain of the inverse function

*At their corresponding points, the slopes of tangent line will be reciprocals of each other



Evaluate derivative of inverse at a point: (find $(f^{-1})'(a)$)

$f(b) = a$	$(f^{-1})(a) = b$
$f'(b) = n$	$(f^{-1})'(a) = \frac{1}{n}$

Example 6: f(x) = $x^3 + 4x + 2$ find $(f^{-1})'(-3)$ $g'(-3)$

$f(-1)=-3$	$g(-3)=\underline{-1}$
$f'(-1)=7$	$g'(3)=\frac{1}{7}$

$-3 = x^3+4x+2$
$x = -1$

$f'(x) = 3x^2+4$
$f'(-1) = 3(-1)^2+4$
$= 3+4 = 7$

$\boxed{g'(3) = 1/7}$

Example 7: f(x) = $\sqrt{x^3-7}$ find $(f^{-1})'(1)$ Find $g'(1)$

$f(2)=1$	$g(1)=\underline{2}$
$f'(2)=6$	$g'(1)=1/6$

$1 = \sqrt{x^3-7}$
$1 = x^3-7$ $x^3=8, x=2$

$f(x) = (x^3-7)^{1/2}$
$f'(x) = \frac{1}{2}(x^3-7)^{-1/2}(3x^2)$
$= \frac{3x^2}{2\sqrt{x^3-7}}$

$f'(2) = \frac{12}{2(1)}$

Example 8: If g(f(x)) = x , g(7) = 2, and g'(7) = 10, then f'(2) is

$g(7)=2$	$f(2)=7$
$g'(7)=10$	$f'(2)=\frac{1}{10}$

$\boxed{f'(2)=1/10}$

Example 9: If g(f(x)) = x , g(9) = 3, and g'(9) = -4, then f'(3) is

$g(9)=3$	$f(3)=9$
$g'(9)=-4$	$f'(3)=\frac{-1}{4}$

$\boxed{f'(3)=1/4}$

Key ⑤

Ch. 5.1b Natural Log Differentiation Classwork

Finding a Derivative In Exercises 41–64, find the derivative of the function.

$$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$

*cannot expand: $\ln(a-b)$ and $\ln(a+b)$

Reminder: Expand log expression fully before differentiating

Log Properties

1. $\ln(1) = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln(a^n) = n \ln a$
4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

41. $f(x) = \ln(3x)$

$f'(x) = \frac{3}{3x} = \boxed{\frac{1}{x}}$

or $y = \ln 3 + \ln x$

$\boxed{y' = 0 + \frac{1}{x}}$

42. $f(x) = \ln(x - 1)$

$f'(x) = \boxed{\frac{1}{x-1}}$

43. $g(x) = \ln x^2$

$g(x) = 2\ln x$

$g'(x) = 2\left(\frac{1}{x}\right) = \boxed{\frac{2}{x}}$

44. $h(x) = \ln(2x^2 + 1)$

$h'(x) = \boxed{\frac{4x}{2x^2+1}}$

45. $y = (\ln x)^4$

*chain Rule : $\frac{d}{dx} f[g(x)] = f'[g(x)] \cdot g'(x)$

$y' = 4[\ln x]^3 \cdot \left(\frac{1}{x}\right)$

$\boxed{y' = \frac{4(\ln x)^3}{x}}$

46. $y = x^2 \ln x$

*product rule : $\frac{d}{dx} f(x) \cdot g(x) = f'(x)g(x) + f(x)g'(x)$

$y' = \underbrace{2x}_{f'} \cdot \underbrace{\ln x}_{g} + \underbrace{x^2}_{f} \cdot \underbrace{\frac{1}{x}}_{g'}$

$y' = 2x\ln x + \frac{x^2}{x} = \boxed{2x\ln x + x}$

47. $y = \ln(t + 1)^2$

$y = 2\ln(t+1)$

$y' = 2 \cdot \frac{1}{t+1} = \boxed{\frac{2}{t+1}}$

48. $y = \ln\sqrt{x^2 - 4}$

$y = \ln(x^2-4)^{1/2}$

$y = \frac{1}{2}\ln(x^2-4)$

$y' = \frac{1}{2} \cdot \frac{2x}{x^2-4}$

$\boxed{y' = \frac{x}{x^2-4}}$

49. $y = \ln\left(x\sqrt{x^2 - 1}\right)$

$y = \ln\left(x \cdot (x^2-1)^{1/2}\right)$

$y = \ln x + \ln(x^2-1)^{1/2}$

$y = \ln x + \frac{1}{2}\ln(x^2-1)$

$y' = \frac{1}{x} + \frac{1}{2} \cdot \frac{2x}{x^2-1}$

$\boxed{y' = \frac{1}{x} + \frac{x}{x^2-1}}$

50. $y = \ln[t(t^2 + 3)^3]$

$y = \ln t + \ln(t^2+3)^3$

$y = \ln t + 3\ln(t^2+3)$

$y' = \frac{1}{t} + 3 \cdot \frac{2t}{t^2+3}$

$\boxed{y' = \frac{1}{t} + \frac{6t}{t^2+3}}$

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51. $f(x) = \ln\left(\frac{x}{x^2+1}\right)$

$f(x) = \ln x - \ln(x^2+1)$

$\boxed{f'(x) = \frac{1}{x} - \frac{2x}{x^2+1}}$

52. $f(x) = \ln\left(\frac{2x}{x+3}\right)$

$f(x) = \ln(2x) - \ln(x+3)$

$f'(x) = \frac{2}{2x} - \frac{1}{x+3} = \boxed{\frac{1}{x} - \frac{1}{x+3}}$

53. $g(t) = \frac{\ln t}{t^2}$

* quotient rule: $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

$g'(t) = \frac{(\frac{1}{t})(t^2) - (\ln t)(2t)}{[t^2]^2}$

$= \frac{t - 2t\ln t}{t^4}$

$= \frac{\cancel{t}(1-2\ln t)}{t^{\cancel{4}3}}$

$= \boxed{\frac{1-2\ln t}{t^3}}$

54. $h(t) = \frac{\ln t}{t}$

* quotient rule

$h'(t) = \frac{(\frac{1}{t})(t) - (\ln t)(1)}{t^2}$

$\boxed{h'(t) = \frac{1-\ln t}{t^2}}$

55. $y = \ln(\ln x^2)$

* chain rule

$y = \ln[2\ln x]$

$y' = \frac{\cancel{2}(\frac{1}{x})}{\cancel{2}\ln x} = \frac{\frac{1}{x}}{\ln x} \cdot \frac{\frac{1}{\ln x}}{\frac{1}{\ln x}}$

$\boxed{y' = \frac{1}{x\ln x}}$

56. $y = \ln(\ln x)$

* chain rule

$y' = \frac{\frac{1}{x} \cdot \frac{1}{\ln x}}{\ln x \cdot \frac{1}{\ln x}}$

$\boxed{y' = \frac{1}{x\ln x}}$

57. $y = \ln\sqrt{\frac{x+1}{x-1}}$

$y = \ln\left(\frac{x+1}{x-1}\right)^{1/2}$

$y = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$

$y = \frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x-1)$

$y' = \frac{1}{2}\cdot\frac{1}{x+1} - \frac{1}{2}\cdot\frac{1}{x-1}$

$\boxed{y' = \frac{1}{2(x+1)} - \frac{1}{2(x-1)}}$

58. $y = \ln\sqrt[3]{\frac{x-1}{x+1}}$

$y = \ln\left(\frac{x-1}{x+1}\right)^{1/3}$

$y = \frac{1}{3}\ln\left(\frac{x-1}{x+1}\right)$

$y = \frac{1}{3}\ln(x-1) - \frac{1}{3}\ln(x+1)$

$y' = \frac{1}{3}\cdot\frac{1}{x-1} - \frac{1}{3}\cdot\frac{1}{x+1}$

$\boxed{y' = \frac{1}{3(x-1)} - \frac{1}{3(x+1)}}$

59. $f(x) = \ln\left(\frac{\sqrt{4+x^2}}{x}\right)$

$f(x) = \ln\sqrt{4+x^2} - \ln x$

$f(x) = \ln(4+x^2)^{1/2} - \ln x$

$f(x) = \frac{1}{2}\ln(4+x^2) - \ln x$

$f'(x) = \frac{1}{2}\cdot\frac{2x}{4+x^2} - \frac{1}{x}$

$\boxed{f'(x) = \frac{x}{4+x^2} - \frac{1}{x}}$

60. $f(x) = \ln(x + \sqrt{4+x^2})$

* cannot expand!! $f(x) = \ln[x + (4+x^2)^{1/2}]$

$f'(x) = \frac{1 + \frac{1}{\cancel{2}}(4+x^2)^{-1/2}(\cancel{2}x)}{x + \sqrt{4+x^2}}$

$f'(x) = \frac{1 + \frac{x}{\sqrt{4+x^2}}}{x + \sqrt{4+x^2}}$

$f'(x) = 1 + \frac{x}{\sqrt{4+x^2}} \cdot \frac{1}{x+\sqrt{4+x^2}}$

$\boxed{f'(x) = \left(1 + \frac{x}{\sqrt{4+x^2}}\right)\left(\frac{1}{x+\sqrt{4+x^2}}\right)}$

Finding a Derivative Implicitly In Exercises 73–76, use implicit differentiation to find dy/dx.

73. $x^2 - 3\ln y + y^2 = 10$

$$2x - 3\left(\frac{1}{y}\right)\frac{dy}{dx} + 2y\left(\frac{dy}{dx}\right) = 0$$

$$2x - \frac{3}{y}\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$$

$$2y\left(\frac{dy}{dx}\right) - \frac{3}{y}\left(\frac{dy}{dx}\right) = -2x$$

$$\frac{dy}{dx}\left(2y - \frac{3}{y}\right) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y - \frac{3}{y}} \cdot \frac{y}{y} = \boxed{\frac{-2xy}{2y^2 - 3}}$$

74. $\ln xy + 5x = 30$

*expand first

$$\ln(xy) + 5x = 30$$

$$\ln x + \ln y + 5x = 30$$

$$\frac{1}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right) + 5 = 0$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = -5 - \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{-5 - \frac{1}{x}}{\frac{1}{y}} \cdot \frac{xy}{xy} = \boxed{\frac{-5xy - y}{x}}$$

75. $4x^3 + \ln y^2 + 2y = 2x$

*expand first:

$$4x^3 + 2\ln y + 2y = 2x$$

$$12x^2 + 2\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) + 2\left(\frac{dy}{dx}\right) = 2$$

$$\frac{2}{y}\left(\frac{dy}{dx}\right) + 2\left(\frac{dy}{dx}\right) = 2 - 12x^2$$

$$\frac{dy}{dx}\left(\frac{2}{y} + 2\right) = 2 - 12x^2$$

$$\frac{dy}{dx} = \frac{2 - 12x^2}{\frac{2}{y} + 2} \cdot \frac{y}{y} = \frac{2y - 12x^2y}{2 + 2y}$$

$$= \frac{2(y - 6x^2y)}{2(1+y)}$$

$$= \boxed{\frac{y - 6x^2y}{1+y}}$$

76. $4xy + \ln x^2y = 7$

*expand first:

$$4xy + \ln(x^2y) = 7$$

$$4xy + \ln x^2 + \ln y = 7$$

$$\underbrace{4xy}_{\text{*product rule:}} + 2\ln x + \ln y = 7$$

$$(4)(y) + (4x)\left(\frac{dy}{dx}\right) + 2\left(\frac{1}{x}\right) + \frac{1}{y}\left(\frac{dy}{dx}\right) = 0$$

$$4x\left(\frac{dy}{dx}\right) + \frac{1}{y}\left(\frac{dy}{dx}\right) = -4y - \frac{2}{x}$$

$$\frac{dy}{dx}\left(4x + \frac{1}{y}\right) = -4y - \frac{2}{x}$$

$$\frac{dy}{dx} = \frac{-4y - \frac{2}{x}}{4x + \frac{1}{y}} \cdot \frac{xy}{xy} = \boxed{\frac{-4xy^2 - 2y}{4x^2y + x}}$$

(8)

Log Differentiation:
Use log differentiation to find dy/dx

1) Take "ln" of both sides
2) Expand using log properties
3) Take derivative (implicit diff. on left side)

Log Properties

1. $\ln(1) = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln(a^n) = n \ln a$
4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

89. $y = x\sqrt{x^2+1}, \quad x > 0$

$\ln y = \ln\left[x\sqrt{x^2+1}\right]$

$\ln y = \ln x + \ln\sqrt{x^2+1}$

$\ln y = \ln x + \ln(x^2+1)^{1/2}$

$\ln y = \ln x + \frac{1}{2}\ln(x^2+1)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{x} + \frac{1}{2}\cdot\frac{2x}{x^2+1}$

$\frac{dy}{dx} = y\left[\frac{1}{x} + \frac{x}{x^2+1}\right]$

$\boxed{\frac{dy}{dx} = \left[x\sqrt{x^2+1}\right]\left[\frac{1}{x} + \frac{x}{x^2+1}\right]}$

90. $y = \sqrt{x^2(x+1)(x+2)}, \quad x > 0$

$\ln y = \ln\left(x^2(x+1)(x+2)\right)^{1/2}$

$\ln y = \frac{1}{2}\ln\left[x^2(x+1)(x+2)\right]$

$\ln y = \frac{1}{2}\ln x^2 + \frac{1}{2}\ln(x+1) + \frac{1}{2}(x+2)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{2}\cdot\frac{2x}{x^2} + \frac{1}{2}\cdot\frac{1}{x+1} + \frac{1}{2}\cdot\frac{1}{x+2}$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{x} + \frac{1}{2(x+1)} + \frac{1}{2(x+2)}$

$\frac{dy}{dx} = y\left[\frac{1}{x} + \frac{1}{2(x+1)} + \frac{1}{2(x+2)}\right] = \boxed{\sqrt{x^2(x+1)(x+2)}\left[\frac{1}{x} + \frac{1}{2(x+1)} + \frac{1}{2(x+2)}\right]}$

91. $y = \frac{x^2\sqrt{3x-2}}{(x+1)^2}, \quad x > \frac{2}{3}$

$\ln y = \ln\left[\frac{x^2(3x-2)^{1/2}}{(x+1)^2}\right]$

$\ln y = \ln x^2 + \ln(3x-2)^{1/2} - \ln(x+1)^2$

$\ln y = 2\ln x + \frac{1}{2}\ln(3x-2) - 2\ln(x+1)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = 2\cdot\frac{1}{x} + \frac{1}{2}\cdot\frac{3}{3x-2} - 2\cdot\frac{1}{x+1}$

$\frac{dy}{dx} = y\left[\frac{2}{x} + \frac{3}{2(3x-2)} - \frac{2}{x+1}\right] = \boxed{\frac{x^2\sqrt{3x-2}}{(x+1)^2}\left[\frac{2}{x} + \frac{3}{2(3x-2)} - \frac{2}{x+1}\right]}$

92. $y = \sqrt{\frac{x^2-1}{x^2+1}}, \quad x > 1$

$\ln y = \ln\left(\frac{x^2-1}{x^2+1}\right)^{1/2}$

$\ln y = \frac{1}{2}\ln\left(\frac{x^2-1}{x^2+1}\right)$

$\ln y = \frac{1}{2}\ln(x^2-1) - \frac{1}{2}\ln(x^2+1)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{2}\cdot\frac{2x}{x^2-1} - \frac{1}{2}\cdot\frac{2x}{x^2+1}$

$\frac{dy}{dx} = y\cdot\left[\frac{x}{x^2-1} - \frac{x}{x^2+1}\right]$

$\boxed{\frac{dy}{dx} = \sqrt{\frac{x^2-1}{x^2+1}}\left[\frac{x}{x^2-1} - \frac{x}{x^2+1}\right]}$

93. $y = \frac{x(x-1)^{3/2}}{\sqrt{x+1}}, \quad x > 1$

$\ln y = \ln\left[\frac{x(x-1)^{3/2}}{(x+1)^{1/2}}\right]$

$\ln y = \ln x + \ln(x-1)^{3/2} - \ln(x+1)^{1/2}$

$\ln y = \ln x + \frac{3}{2}\ln(x-1) - \frac{1}{2}\ln(x+1)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{x} + \frac{3}{2}\cdot\frac{1}{x-1} - \frac{1}{2}\cdot\frac{1}{x+1}$

$\frac{dy}{dx} = y\left[\frac{1}{x} + \frac{3}{2(x-1)} - \frac{1}{2(x+1)}\right]$

$\boxed{\frac{dy}{dx} = \frac{x(x-1)^{3/2}}{\sqrt{x+1}}\left[\frac{1}{x} + \frac{3}{2(x-1)} - \frac{1}{2(x+1)}\right]}$

Key 9

Non-AP Calculus 5.1/5.3 Natural Logs Properties and Derivatives Quiz Review WS #1

Log Properties

1. $\ln(1) = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln(a^n) = n \ln a$
4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
5. $\ln(e) = 1$

$$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$

Expanding a Logarithmic Expression: Use properties of logarithms to expand the logarithmic expression

1. $y = \ln(x\sqrt[3]{6 - x^4})$

$y = \ln x + \ln \sqrt[3]{6-x^4}$

$y = \ln x + \ln(6-x^4)^{1/3}$

$y = \ln x + \frac{1}{3}\ln(6-x^4)$

$\boxed{y = \ln x + \frac{1}{3}\ln(6-x^4)}$

2. $f(x) = \ln\sqrt{\frac{3-2x}{4x}}$

$f(x) = \ln\left(\frac{3-2x}{4x}\right)^{1/2}$

$f(x) = \frac{1}{2}\ln\left(\frac{3-2x}{4x}\right)$

$\boxed{f(x) = \frac{1}{2}\ln(3-2x) - \frac{1}{2}\ln(4x)}$

3. $y = \ln\left(\frac{2x^4}{x\sqrt{3x^5-1}}\right)$

$y = \ln(2x^4) - \ln x - \ln\sqrt{3x^5-1}$

$y = \ln(2x^4) - \ln x - \ln(3x^5-1)^{1/2}$

$\boxed{y = \ln(2x^4) - \ln x - \frac{1}{2}\ln(3x^5-1)}$

Condensing a Logarithmic Expression: Use properties of logarithms to write expression as a logarithm of a single quantity

4. $5\ln x + 3\ln y - 5\ln w - 6\ln z$

$\ln x^5 + \ln y^3 - \ln w^5 - \ln z^6$

$\ln\left(\frac{x^5 y^3}{w^5 z^6}\right)$

5. $\frac{1}{5}[\ln(x-4) - 3\ln(7-x^2) - \ln(8-x)]$

$\frac{1}{5}\left[\ln(x-4) - \ln(7-x^2)^3 - \ln(8-x)\right]$

$\frac{1}{5}\ln\left(\frac{x-4}{(7-x^2)^3(8-x)}\right) = \boxed{\ln\left(\frac{x-4}{(7-x^2)^3(8-x)}\right)^{1/5}}$

Find the derivative of the functions below:

6) $y = \ln(x\sqrt{6-x^3})$

*expand equation first:

$y = \ln x + \ln\sqrt{6-x^3}$

$y = \ln x + \ln(6-x^3)^{1/2}$

$y = \ln x + \frac{1}{2}\ln(6-x^3)$

$y' = \frac{1}{x} + \frac{1}{2}\cdot\frac{-3x^2}{6-x^3}$

$\boxed{y' = \frac{1}{x} - \frac{3x^2}{2(6-x^3)}}$

10

Find the derivative of the functions below:

7) $y = \ln\sqrt{\left(\frac{3-2x}{x^3}\right)^5}$

$y = \ln\left(\frac{3-2x}{x^3}\right)^{5/2}$

$y = \frac{5}{2}\ln\left(\frac{3-2x}{x^3}\right)$

$y = \frac{5}{2}\ln(3-2x) - \frac{5}{2}\ln(x^3)$

$y' = \frac{5}{2}\cdot\frac{-2}{3-2x} - \frac{5}{2}\cdot\frac{3x^2}{x^3}$

$\boxed{y' = \frac{-5}{3-2x} - \frac{15}{2x}}$

8) $y = x^3\ln(x^2)$ *product rule

$y' = \underbrace{3x^2}_{f'}\cdot\underbrace{\ln x^2}_{g} + \underbrace{x^3}_{f}\cdot\underbrace{\frac{2x}{x^2}}_{g'}$

$\boxed{y' = 3x^2\ln x^2 + 2x^2}$

Use Log differentiation to find the derivative of the function:

9) $y = \frac{x^3\left(\sqrt{5-4x^5}\right)}{(x-1)^2}$

$\ln y = \ln\left[\frac{x^3(5-4x^5)^{1/2}}{(x-1)^2}\right]$

$\ln y = \ln x^3 + \ln(5-4x^5)^{1/2} - \ln(x-1)^2$

$\ln y = 3\ln x + \frac{1}{2}\ln(5-4x^5) - 2\ln(x-1)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = 3\cdot\frac{1}{x} + \frac{1}{2}\cdot\frac{-20x^4}{5-4x^5} - 2\cdot\frac{1}{x-1}$

$\frac{dy}{dx} = \left[\frac{x^3\sqrt{5-4x^5}}{(x-1)^2}\right]\left[\frac{3}{x} - \frac{10x^4}{5-4x^5} - \frac{2}{x-1}\right]$

Use Implicit Differentiation to find $\frac{dy}{dx}$:

10) $x + \ln(xy) - 2y = x^3$

$x + \ln x + \ln y - 2y = x^3$

$1 + \ln x + \frac{1}{y}\left(\frac{dy}{dx}\right) - 2\left(\frac{dy}{dx}\right) = 3x^2$

$\frac{1}{y}\left(\frac{dy}{dx}\right) - 2\left(\frac{dy}{dx}\right) = -3x^2 - \ln x - 1$

$\frac{dy}{dx}\left(\frac{1}{y} - 2\right) = -3x^2 - \ln x - 1$

$\boxed{\frac{dy}{dx} = \frac{-3x^2 - \ln x - 1}{\frac{1}{y} - 2}}$

11) Find an inverse function for f(x): $f(x) = 2x^3 - 1$

$y = 2x^3 - 1$ | $x = 2y^3 - 1$ | $x + 1 = 2y^3$ | $\frac{x+1}{2} = y^3$

$y = \sqrt[3]{\frac{x+1}{2}}$

$\boxed{f^{-1}(x) = \sqrt[3]{\frac{x+1}{2}}}$

Use function f(x) and the given real number a to find $(f^{-1})'(a)$

12) $f(x) = 2x^3 - 3x + 1$ a = 11

$f(b) = a$	$f^{-1}(a) = b$
$f'(b) = n$	$f^{-1\prime}(a) = \frac{1}{n}$

$f(2) = 11$	$f^{-1}(11) = 2$
$f'(2) = \_$	$(f^{-1})'(11) = \_\_$

$11 = 2x^3 - 3x + 1$

$0 = 2x^3 - 3x - 10$

$x = 2$

$f'(x) = 6x^2 - 3$

$f'(2) = 6(2)^2 - 3$

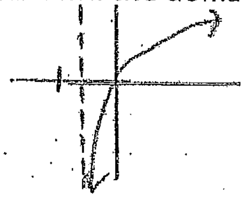
$f'(2) = 24 - 3$

$f'(2) = 21$

$\boxed{(f^{-1})'(11) = \frac{1}{21}}$

Calculus AB **Ch. 5.1-5.3 Natural Logs Quiz Review** Solution Key

1. Find the domain for y = ln(2 + 3x) – 1



VA: $x = -\frac{2}{3}$

$2 + 3x = 0$

$x = -\frac{2}{3}$

Domain: $\left(-\frac{2}{3}, \infty\right)$

2. Find $\frac{dy}{dx}$ $y = \ln\sqrt{\frac{3-2x}{4x}}$ *expand first using log properties

$y = \ln\left(\frac{3-2x}{4x}\right)^{1/2}$

$y = \frac{1}{2}\ln\left(\frac{3-2x}{4x}\right)$

$y = \frac{1}{2}\left[\ln(3-2x) - \ln(4x)\right]$

$y' = \frac{1}{2}\left(\frac{-2}{3-2x}\right) - \frac{1}{2}\left(\frac{4}{4x}\right)$

$\boxed{y' = \frac{-1}{3-2x} - \frac{1}{2x}}$

3. Find $\frac{dy}{dx}$ $y = x^{\sqrt{x+3}}$ *use log differentiation

$\ln y = \ln x^{(x+3)^{1/2}}$

$\ln y = (x+3)^{1/2} \cdot \ln x$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{2}(x+3)^{-1/2}(1)\ln x + (x+3)^{1/2}\left(\frac{1}{x}\right)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{\ln x}{2\sqrt{x+3}} + \frac{\sqrt{x+3}}{x}$

$\frac{dy}{dx} = y\left[\frac{\ln x}{2\sqrt{x+3}} + \frac{\sqrt{x+3}}{x}\right]$

$\boxed{\frac{dy}{dx} = x^{\sqrt{x+3}}\left[\frac{\ln x}{2\sqrt{x+3}} + \frac{\sqrt{x+3}}{x}\right]}$

4. f(x) = $\sqrt{5x-1} - 4$ $y = \sqrt{5x-1} - 4$
   a. Find $(f^{-1})(x)$
   b. Find the domain for $(f^{-1})(x)$

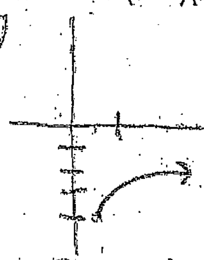
$x = \sqrt{5y-1} - 4$

$x + 4 = \sqrt{5y-1}$

$(x+4)^2 = 5y - 1$

$\frac{(x+4)^2 + 1}{5} = y$

$\boxed{f^{-1}(x) = \frac{(x+4)^2+1}{5}}$



D: $[1/5, \infty)$

R: $[-4, \infty)$

Domain for $(f^{-1})(x)$

D: $[-4, \infty)$

5. f(x) = $x^3 + 2x^2 - 3$ Find $(f^{-1})'(13)$

$f(\_) = 13$	$(f^{-1})(13) =$
$f'(2) = 20$	$(f^{-1})'(13) = \boxed{\frac{1}{20}}$

$x^3 + 2x^2 - 3 = 13$

$x^3 + 2x^2 = 16$

$x = 2$ ✓

$(2)^3 + 2(2)^2 = 16$ ✓

$f'(x) = 3x^2 + 4x$

$f'(2) = 3(2)^2 + 4(2)$

$= 12 + 8$

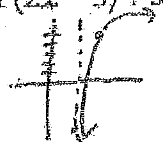
$= \underline{20}$

(12)

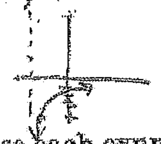
$x = {-17}/{3}$

Identify the domain and range of each.

6) $y = \ln(2x-3)+5$ D: $(3/2, \infty)$
VA: $x = 3/2$ R: $(-\infty, \infty)$



7) $y = \ln(3x+17)-5$
D: $(-17/3, \infty)$
R: $(-\infty, \infty)$



Expand each logarithm.

8) $\ln(a \cdot b \cdot c^3)$

$\boxed{\ln a + \ln b + 3\ln c}$

Condense each expression to a single logarithm.

9) $\frac{\ln u}{2} + \frac{\ln v}{2} + \frac{\ln w}{2}$

$\frac{1}{2}\ln u + \frac{1}{2}\ln v + \frac{1}{2}\ln w$

$\ln u^{1/2} + \ln v^{1/2} + \ln w^{1/2}$

$\ln u^{1/2} v^{1/2} w^{1/2}$

$= \boxed{\ln (uvw)^{1/2}}$

Differentiate each function with respect to x.

10) $f(x) = \ln \sqrt[4]{\frac{2x^3}{3x^2-4}} = \ln\left(\frac{2x^3}{3x^2-4}\right)^{1/4}$

$y = \frac{1}{4}\ln\left(\frac{2x^3}{3x^2-4}\right)$

$y = \frac{1}{4}\ln(2x^3) - \frac{1}{4}\ln(3x^2-4)$

$y' = \frac{1}{4}\left(\frac{6x^2}{2x^3}\right) - \frac{1}{4}\left(\frac{6x}{3x^2-4}\right)$

$\boxed{y' = \frac{3}{4x} - \frac{3x}{2(3x^2-4)}}$

11) $f(x) = \ln\left(\frac{4x^2}{5x^3-3}\right)^5$

$y = 5\ln\left(\frac{4x^2}{5x^3-3}\right)$

$y = 5\ln(4x^2) - 5\ln(5x^3-3)$

$y' = 5\left(\frac{8x}{4x^2}\right) - 5\left(\frac{15x^2}{5x^3-3}\right)$

$\boxed{y' = \frac{10}{x} - \frac{75x^2}{5x^3-3}}$

Use logarithmic differentiation to differentiate each function with respect to x.

12) $y = \sqrt[3]{x^2+1}$

$\ln y = \ln(x^2+1)^{1/3} = \frac{1}{3}\ln(x^2+1)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{3}\left(\frac{2x}{x^2+1}\right)$

$\frac{dy}{dx} = y\left(\frac{2x}{3(x^2+1)}\right) = \boxed{\sqrt[3]{x^2+1}\left(\frac{2x}{3(x^2+1)}\right)}$

13) $y = x^{2x}$

$\ln y = \ln x^{2x}$

$\ln y = 2x \ln x$

$\frac{1}{y}\frac{dy}{dx} = 2\ln x + 2x\left(\frac{1}{x}\right)$

$\boxed{\frac{dy}{dx} = x^{2x}\left[2\ln x + 2\right]}$

For each problem, find $(f^{-1})'(a)$

14) $f(x) = 3x^5 + 2x + 5, a = 5$

$f(0) = 5$ | $(f^{-1})(5) = 0$

$f'(0) = 2$ | $(f^{-1})'(5) = \boxed{\frac{1}{2}}$

$3x^5 + 2x + 5 = 5$

$3x^5 + 2x = 0$

$x = 0$

$f'(x) = 15x^4 + 2$

$f'(0) = 0 + 2 = 2$

15) $f(x) = 2x^3 + 4x + 5, a = 5$

$f(0) = 5$ | $(f^{-1})(5) = 0$

$f'(0) = 4$ | $(f^{-1})'(5) = \boxed{\frac{1}{4}}$

$5 = 2x^3 + 4x + 5$

$0 = 2x^3 + 4x$

$x = 0$

$f'(x) = 6x^2 + 4$

$f'(0) = 0 + 4 = 4$

Ch.5.4 Notes **Derivative of Exponential Function e^x**

$y = \ln x$ and $y = e^x$ are inverse functions (meaning $f(g(x)) = x$ and $g(f(x)) = x$)

Example 1: Solve $7 = e^{x+1}$

$\ln 7 = \ln e^{x+1}$

$\ln 7 = (x+1)\cancel{\ln e}$

$\boxed{\ln 7 - 1 = x}$

Example 2: solve $\ln(2x - 3) = 5$

$\log_e(2x-3) = 5$

$e^5 = 2x - 3$

$\boxed{\frac{e^5+3}{2} = x}$

Reminder: exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ Reminder: e is a NUMBER: if $y = e^2$, then $y' = 0$

Exponential Function e^x Derivative rule $\frac{d}{dx}e^u = e^u * u'$

$\frac{d}{dx}e^x = e^x \cdot (1) = e^x$

Example 3: find y' for $y = \ln(2x - e^{-2x})$

$y' = \frac{2 - e^{-2x}(-2)}{2x - e^{-2x}} = \frac{2 + \frac{2}{e^{2x}}}{2x - \frac{1}{e^{2x}}}$

$\boxed{y' = \frac{2e^{2x} + 2}{2xe^{2x} - 1}}$

Example 4: Find y' for $y = xe^{(x^2+2x+3)^3}$

$y' = 1e^{(x^2+2x+3)^3} + x \cdot e^{(x^2+2x+3)^3} \cdot 3(x^2+2x+3)^2 \cdot (2x+2)$

$y' = e^{(x^2+2x+3)^3} + (6x^2+6x)(x^2+2x+3)^2 e^{(x^2+2x+3)^3}$

$y' = e^{(x^2+2x+3)^3}\left[1 + (6x^2+6x)(x^2+2x+3)^2\right]$

Example 5: Find the equation of the tangent line to the graph at the given point:
$y = e^{-x}\ln x$ (1, 0)

$y' = e^{-x}(-1)\ln x + e^{-x}\left(\frac{1}{x}\right)$

$= \frac{-\ln x}{e^x} + \frac{1}{xe^x}$

$y' = \frac{-x\ln x + 1}{xe^x}$

$y'(1) = \frac{-\ln(1) + 1}{(1)e^1} = \frac{1}{e}$

$y - 0 = \frac{1}{e}(x-1)$

$\boxed{y = \frac{1}{e}(x-1)}$

Example 6: Find dy/dx

$xe^y - 10x + 3y = 0$

$1e^y + xe^y\left(\frac{dy}{dx}\right) - 10 + 3\frac{dy}{dx} = 0$

$\frac{dy}{dx}(xe^y + 3) = 10 - e^y$

$\boxed{\frac{dy}{dx} = \frac{10 - e^y}{xe^y + 3}}$

Example 7: Find the equation of the tangent line to the graph of the function at the given point.

$xe^y + ye^x = 1$ at (0, 1)

$1e^y + xe^y\left(\frac{dy}{dx}\right) + \frac{dy}{dx}e^x + ye^x = 0$

$1e^1 + \cancel{0e^1\left(\frac{dy}{dx}\right)} + \frac{dy}{dx}e^0 + 1e^0 = 0$

$e + \frac{dy}{dx} + 1 = 0$

$\frac{dy}{dx}\Big|_{(0,1)} = -e - 1$

$\boxed{y - 1 = (-e-1)[x - 0]}$

Example 8: Find the 2nd derivative of the function

$f(x) = (3 + 2x)e^{-3x}$

$f' = (2)e^{-3x} + (3+2x)e^{-3x}(-3) = \frac{2 - 9 - 6x}{e^{3x}}$

$= \frac{-7 - 6x}{e^{3x}}$

$f'' = \frac{(-6)e^{3x} - (-7-6x)e^{3x}(3)}{(e^{3x})^2}$

$\frac{-6e^{3x} + 21e^{3x} + 18xe^{3x}}{(e^{3x})^2} = \boxed{\frac{15 + 18x}{e^{3x}}}$

(14)

$$g(t) = 1 + \frac{2+t}{e^t}$$

Ex. 9: find the extrema and points of inflection for $g(t) = 1 + (2+t)e^{-t}$

$$g' = 0 + 1(e^{-t}) + (2+t)e^{-t}(-1) = \frac{1-2-t}{e^t} = \frac{-1-t}{e^t} \quad \boxed{t=-1}$$

$+ \quad | \quad -$; -1

Rel. max at $(-1, 1+e)$ b/c $f'(x)$ changes from + to −.

$$g(-1) = 1 + (2-1)e^1$$
$$= 1 + e$$

$$g''(t) = \frac{(-1)e^t - (-1-t)e^t}{e^{2t}}$$

$$g''(t) = \frac{-e^t + e^t + te^t}{e^{2t}} = \frac{t}{e^t}$$

$$0 = \frac{t}{e^t}$$
$$t = 0$$

$- \quad | \quad +$; 0

$$f(0) = 1 + (2+0)e^0$$
$$= 1 + 2 = 3$$

POI at $(0, 3)$ b/c $f''(x)$ changes signs

Ex. 10: find the extrema and points of inflection for $f(x) = \frac{e^x - e^{-x}}{2}$ (use common denominators)

$$f(x) = \frac{e^x - \frac{1}{e^x}}{2} = \frac{\frac{e^{2x}-1}{e^x}}{2} = \frac{e^{2x}-1}{2e^x}$$

$$f'(x) = \frac{e^{2x}(2)[2e^x] - (e^{2x}-1)(2e^x)}{4e^{2x}}$$

$$= \frac{4e^{3x} - 2e^{3x} + 2e^x}{4e^{2x}} = \frac{2e^{3x} + 2e^x}{4e^{2x}} = \frac{2e^x(e^{2x}+1)}{4e^{2x}} = \frac{e^{2x}+1}{2e^x} > 0$$

$$e^{2x} + 1 = 0$$
$$e^{2x} = -1$$

$\boxed{\text{f(x) always increasing, no relative extrema}}$

$$f''(x) = \frac{e^{2x}(2)[2e^x] - [e^{2x}+1]2e^x}{(2e^x)^2}$$

$$= \frac{4e^{3x} - 2e^{3x} - 2e^x}{4e^{2x}}$$

$$= \frac{2e^{3x} - 2e^x}{4e^{2x}}$$

$$= \frac{2e^x(e^{2x}-1)}{4e^{2x}} = \frac{e^{2x}-1}{2e^x} = 0$$

$$e^{2x} - 1 = 0$$
$$e^{2x} = 1$$
$$\ln e^{2x} = \ln 1$$
$$2x = 0$$
$$x = 0$$

$$f(0) = \frac{e^0 - e^{-0}}{2} = \frac{1-1}{2} = 0$$

$- \quad | \quad +$; 0

$\boxed{\text{POI at (0,0) b/c f''(x) changes sign}}$

5.4 Exponential Functions e^x Classwork Worksheet

Key 15

exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ $\frac{d}{dx}e^u = e^u * u'$ $\frac{d}{dx}\ln u = \frac{u'}{u}$

Additional $y = \ln x$ and $y = e^x$ Properties: $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln 1 = 0$ | $\ln e = 1$

Solving an Exponential or Logarithmic Equation In Exercises 1–16, solve for x accurate to three decimal places.

$* \ln a^n = n \cdot \ln a$

1. $e^{\ln x} = 4$

$\boxed{x=4}$

2. $e^{\ln 3x} = 24$

$3x = 24$

$\boxed{x=8}$

3. $e^x = 12$

$\ln e^x = \ln 12$

$x \cancel{\ln e} = \ln 12$

$\boxed{x = \ln 12}$

4. $5e^x = 36$

$\frac{5e^x}{5} = \frac{36}{5}$

$e^x = \frac{36}{5}$

$\ln e^x = \ln\left(\frac{36}{5}\right)$

$x \cancel{\ln e} = \ln\left(\frac{36}{5}\right)$

$\boxed{x = \ln\left(\frac{36}{5}\right)}$

5. $9 - 2e^x = 7$

$-9 \quad -9$

$-2e^x = -2$

$\frac{-2e^x}{-2} = \frac{-2}{-2}$

$e^x = 1$

$\ln e^x = \ln 1$

$x \cancel{\ln e} = 0$

$\boxed{x=0}$

8. $100e^{-2x} = 35$

$\frac{100e^{-2x}}{100} = \frac{35}{100}$

$e^{-2x} = \frac{7}{20}$

$\ln e^{-2x} = \ln\left(\frac{7}{20}\right)$

$-2x \cdot \cancel{\ln e} = \ln\left(\frac{7}{20}\right)$

$-2x = \ln\left(\frac{7}{20}\right)$

$\boxed{x = -\frac{1}{2}\ln\left(\frac{7}{20}\right)}$

11. $\ln(x) = 2$

$e^{\ln x} = e^2$

$\boxed{x = e^2}$

12. $\ln x^2 = 10$

$2\ln x = 10$

$\ln x = \frac{10}{2}$

$\ln x = 5$

$e^{\ln x} = e^5$

$\boxed{x = e^5}$

13. $\ln(x-3) = 2$

$e^{\ln(x-3)} = e^2$

$x - 3 = e^2$

$\boxed{x = e^2 + 3}$

14. $\ln(4x) = 1$

$e^{\ln 4x} = e^1$

$4x = e$

$\boxed{x = \frac{e}{4} = \frac{1}{4}e}$

15. $\ln\sqrt{x+2} = 1$

$\ln(x+2)^{1/2} = 1$

$2\left(\frac{1}{2}\ln(x+2) = 1\right)$

$\ln(x+2) = 2$

$e^{\ln(x+2)} = e^2$

$x + 2 = e^2$

$\boxed{x = e^2 - 2}$

16. $\ln(x-2)^2 = 12$

$\frac{2\ln(x-2)}{2} = \frac{12}{2}$

$\ln(x-2) = 6$

$e^{\ln(x-2)} = e^6$

$x - 2 = e^6$

$\boxed{x = e^6 + 2}$

16

exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ $\frac{d}{dx} e^u = e^u * u'$

Additional $y = \ln x$ and $y = e^x$ Properties: $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln 1 = 0$ | $\ln e = 1$

Finding a Derivative **In Exercises 33–54, find the derivative**

33. $f(x) = e^{2x}$

$f'(x) = e^{2x} \cdot 2 = \boxed{2e^{2x}}$

34. $y = e^{-8x}$

$y' = e^{-8x} \cdot -8$ $\boxed{y' = -8e^{-8x}}$

35. $y = e^{\sqrt{x}}$

$y = e^{x^{1/2}}$

$y' = e^{x^{1/2}} \cdot \frac{1}{2}x^{-1/2}$

$\boxed{y' = \frac{e^{\sqrt{x}}}{2\sqrt{x}}}$

36. $y = e^{-2x^3}$

$y' = e^{-2x^3} \cdot -6x^2$

$\boxed{y' = -6x^2 e^{-2x^3}}$

39. $y = e^x \ln x$

$y' = \underbrace{e^x}_{f'} \cdot \underbrace{\ln x}_{g} + \underbrace{e^x}_{f} \cdot \underbrace{\frac{1}{x}}_{g'}$

*product rule

$\boxed{y' = e^x \ln x + \frac{e^x}{x}}$

40. $y = xe^{4x}$

*product rule

$y' = \underbrace{1}_{f'} \cdot \underbrace{e^{4x}}_{g} + \underbrace{x}_{f} \cdot \underbrace{e^{4x} \cdot 4}_{g'}$

$\boxed{y' = e^{4x} + 4xe^{4x}}$

41. $y = x^3 e^x$

*product rule

$y' = \underbrace{3x^2}_{f'} \cdot \underbrace{e^x}_{g} + \underbrace{x^3}_{f} \cdot \underbrace{e^x}_{g'}$

$\boxed{y' = 3x^2 e^x + x^3 e^x}$

42. $y = x^2 e^{-x}$

*product rule

$y' = \underbrace{2x}_{f'} \cdot \underbrace{e^{-x}}_{g} + \underbrace{x^2}_{f} \cdot \underbrace{e^{-x}(-1)}_{g'}$

$\boxed{y' = 2xe^{-x} - x^2 e^{-x}}$

43. $g(t) = (e^{-t} + e^t)^3$

*chain rule:

outside: $(\ )^3$

inside: $e^{-t} + e^t$

$g'(t) = 3(\ )^2 \cdot (e^{-t}(-1) + e^t)$

$\boxed{g'(t) = 3(e^{-t} + e^t)^2(-e^{-t} + e^t)}$

44. $g(t) = e^{-3/t^2}$

$g'(t) = e^{-3t^{-2}} \cdot 6t^{-3}$

$\boxed{g'(t) = \frac{6e^{-3/t^2}}{t^3}}$

45. $y = \ln(1 + e^{2x})$

$y' = \frac{u'}{u}$

$y' = \frac{e^{2x} \cdot 2}{1 + e^{2x}}$

$\boxed{y' = \frac{2e^{2x}}{1 + e^{2x}}}$

46. $y = \ln\left(\frac{1 + e^x}{1 - e^x}\right)$

*expand first:

$y = \ln(1 + e^x) - \ln(1 - e^x)$

$y' = \frac{e^x}{1 + e^x} - \frac{-e^x}{1 - e^x}$

$\boxed{y' = \frac{e^x}{1 + e^x} + \frac{e^x}{1 - e^x}}$

Finding a Derivative In Exercises 33–54, find the derivative.

$\frac{d}{dx}[\sin u] = (\cos u)u'$ 14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$ 15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$

$\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$ 17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$ 18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$

47. $y = \dfrac{2}{e^x + e^{-x}}$

$y = 2(e^x + e^{-x})^{-1}$

*chain rule:
outside: $2(\ )^{-1}$
inside: $e^x + e^{-x}$

$y' = -2(\ )^{-2} \cdot (e^x + e^{-x}(-1))$

$y' = -2(e^x + e^{-x})^{-2}(e^x - e^{-x})$

$\boxed{y' = \dfrac{-2(e^x - e^{-x})}{(e^x + e^{-x})^2}}$

48. $y = \dfrac{e^x - e^{-x}}{2}$

$y = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

$y' = \frac{1}{2}e^x - \frac{1}{2}e^{-x}(-1)$

$\boxed{y' = \frac{1}{2}e^x + \frac{1}{2}e^{-x}}$

49. $y = \dfrac{e^x + 1}{e^x - 1}$

*quotient rule

$y' = \dfrac{\overbrace{e^x}^{f'} \cdot \overbrace{(e^x - 1)}^{g} - \overbrace{(e^x + 1)}^{f} \cdot \overbrace{e^x}^{g'}}{\underbrace{(e^x - 1)^2}_{g^2}}$

50. $y = \dfrac{e^{2x}}{e^{2x} + 1}$

*quotient rule

$y' = \dfrac{\overbrace{e^{2x} \cdot 2}^{f'} \overbrace{(e^{2x} + 1)}^{g} - \overbrace{e^{2x}}^{f} \cdot \overbrace{(e^{2x} \cdot 2)}^{g'}}{\underbrace{(e^{2x} + 1)^2}_{g^2}}$

51. $y = e^x(\sin x + \cos x)$

$y' = \overbrace{e^x}^{f'} \cdot \overbrace{(\sin x + \cos x)}^{g} + \overbrace{e^x}^{f} \cdot \overbrace{(\cos x - \sin x)}^{g'}$

*product rule

52. $y = e^{2x} \tan 2x$

*product rule

$y' = \overbrace{e^{2x} \cdot 2}^{f'} \overbrace{\tan 2x}^{g} + \overbrace{e^{2x}}^{f} \cdot \overbrace{\sec^2(2x) \cdot 2}^{g'}$

(18) **Finding an Equation of a Tangent Line** **In Exercises 55–62, find an equation of the tangent line to the graph of the function at the given point.**

Steps for finding Tangent Line Equation:

1) Identify Ordered Pair: Point: ___________
2) Find Slope: Find f '(x) and evaluate the slope at x-value: Slope: m = ___________
3) Put equation into point-slope form: $y - y_1 = m(x - x_1)$

55. $f(x) = e^{3x}, \quad (0, 1)$

$f'(x) = e^{3x} \cdot 3$

$f'(0) = e^{3(0)} \cdot 3 = 1(3) = 3$

point: (0,1) slope: m = 3

$\boxed{y - 1 = 3(x - 0)}$

56. $f(x) = e^{-2x}, \quad (0, 1)$

$f'(x) = e^{-2x}(-2)$

$f'(0) = -2e^{-2(0)} = -2(1) = -2$

point: (0,1)

slope: m = -2

$\boxed{y - 1 = -2(x - 0)}$

57. $f(x) = e^{1-x}, \quad (1, 1)$

$f'(x) = e^{1-x} \cdot (-1)$

$f'(1) = e^{1-1} \cdot (-1) = e^0(-1) = 1(-1) = -1$

point: (1,1)

slope: m = -1

$\boxed{y - 1 = -1(x - 1)}$

58. $y = e^{-2x + x^2}, \quad (2, 1)$

$y' = e^{-2x+x^2} \cdot (-2 + 2x)$

$y'(2) = e^{-4+4} \cdot (-2 + 4) = e^0(2) = 1(2) = 2$

point: (2,1)

slope: m = 2

$\boxed{y - 1 = 2(x - 2)}$

59. $f(x) = e^{-x} \ln x, \quad (1, 0)$

$f'(x) = \overbrace{e^{-x}(-1)}^{f'}\ \overbrace{\ln x}^{g} + \overbrace{e^{-x}}^{f} \cdot \overbrace{\frac{1}{x}}^{g'}$

$f'(1) = -e^{-1}(\ln 1) + e^{-1}\left(\frac{1}{1}\right)$

$= 0 - \frac{1}{e} = \frac{1}{e}$

point: (1,0)

slope: $m = \frac{1}{e}$

$\boxed{y - 0 = \frac{1}{e}(x - 1)}$

62. $y = xe^x - e^x, \quad (1, 0)$

$y' = \overbrace{1}^{f'}\ \overbrace{e^x}^{g} + \overbrace{x}^{f} \cdot \overbrace{e^x}^{g'} - e^x$

$y'(1) = e + 1e - e = e$

point: (1,0)

slope: m = e

$\boxed{y - 0 = e(x - 1)}$

Ch.5.5

~~Ch.5.4~~ Notes Derivative of Logs of other bases

Change of Base: $log_a x = \frac{lnx}{lna}$

Ex. 1 solve for x: $3^x = 1/81$

$$\log_3 3^x = \log_3(1/81)$$

$$x = \log_3(3)^{-4}$$

$$x = -4$$

Ex. 2 solve: $\log_2 x = -4$

$$2^{\log_2 x} = 2^{-4}$$

$$x = 2^{-4} = \boxed{\frac{1}{16}}$$

Derivative Rule for logs of other bases : $\frac{d}{dx}\log_a u = \frac{u'}{(\ln a)u'}$ $= \left(\frac{1}{\ln a}\right)\left(\frac{u'}{u}\right)$

$$\frac{d}{dx}\log_a x = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \left(\frac{1}{\ln a}\right)\cdot\left(\frac{1}{x}\right) = \frac{1}{(\ln a)x}$$

Recall: $\frac{d}{dx}\ln u = \frac{u'}{u}$

$$\frac{d}{dx}\log_a u = \frac{d}{dx}\left(\frac{\ln u}{\ln a}\right) = \left(\frac{1}{\ln a}\right)\cdot\frac{d}{dx}(\ln u) = \left(\frac{1}{\ln a}\right)\left(\frac{u'}{u}\right)$$

Ex. 3 Find f '(x) for f(x) = $log_5 \sqrt[3]{(2x^2+7)}$

$$f(x) = \log_5(2x^2+7)^{1/3}$$

$$= \frac{1}{3}\log_5(2x^2+7)$$

$$f'(x) = \frac{1}{3}\left(\frac{1}{\ln 5}\right)\left(\frac{4x}{2x^2+7}\right) = \boxed{\frac{4x}{3\ln 5(2x^2+7)}}$$

Ex. 4 Find f '(x) for f(x) = $log\,\frac{5x^3}{(x^2-3x)^3}$

$$f(x) = \log 5x^3 - \log(x^2-3x)^3$$

$$= \log 5x^3 - 3\log(x^2-3x)$$

$$f'(x) = \left(\frac{1}{\ln 10}\right)\left(\frac{15x^2}{5x^3}\right) - 3\left(\frac{1}{\ln 10}\right)\left(\frac{2x-3}{x^2-3x}\right)$$

$$= \frac{3}{x\ln 10} - \frac{3(2x-3)}{(\ln 10)(x^2-3x)}$$

20

$* e^{\ln x} = x$

Derivative Rule for Exponential functions of base a^x : $\frac{d}{dx}a^u = (\ln a)a^u * u'$

$\frac{d}{dx}a^x = e^{(\ln a)x} \rightarrow \frac{d}{dx}e^{(\ln a)x} = e^{(\ln a)(x)} \cdot \ln a = (\ln a)e^{(\ln a)x} = (\ln a)a^x$

Recall: $\frac{d}{dx}e^u = e^u \cdot u'$

$\frac{d}{dx}a^u = \ln a \cdot a^u \cdot u'$

Ex. 5 Find f '(x) for $f(x) = 5^{x^2-2x}$

$f'(x) = (\ln 5)(5^{x^2-2x})(2x-2)$

Ex. 6 Find f '(x) for $f(x) = x(4^{-x})$

$f'(x) = (1)(4^{-x}) + x \cdot (\ln 4)(4^{-x})(-1)$

$= \frac{1}{4^x} - \frac{x\ln 4}{4^x} = \boxed{\frac{1-x\ln 4}{4^x}}$

Ex 7

$v(t) = t\ln t - t$

a) $a(t) = v'(t) = \ln t + t(\frac{1}{t}) - 1 = \ln t + 1 - 1 = \ln t$

b) Find critical points : set $v(t) = 0$

$v(t) = t\ln t - t$

$0 = t(\ln t - 1)$

$\ln t - 1 = 0$

$\ln t = 1$

$e^1 = t$

$t = 0, \quad t = e$

$-$ $+$

0 e

particle moving right (e, ∞) b/c $v(t) > 0$

c) * minimum velocity occurs where $f''(x)$ changes from $-$ to $+$

$a(t) = \ln t$

$0 = \ln t$

$e^0 = t$

$t = 1$

$-$ $+$

0 1

$v(1) = 1\ln(1) - 1$

$\boxed{v(1) = -1}$

Minimum velocity is -1 b/c $a(t) < 0$ for t in $(0, 1)$ and $a(t) > 0$ for all $t > 1$

Ch. 5.5 Log and Exponential Derivatives for base a

11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$

12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$

Find the Derivative of the below functions: (Consider Expanding Log Expressions before Deriving if applicable)

39. $y = 5^{-4x}$

$y' = \ln 5 \cdot 5^{-4x} \cdot (-4)$ $\boxed{y' = -4\ln 5 \cdot 5^{-4x}}$

40. $y = 6^{3x-4}$

$y' = \ln 6 \cdot 6^{3x-4} \cdot 3$ $\boxed{y' = 3\ln 6 \cdot 6^{3x-4}}$

41. $f(x) = x\,9^x$

$f'(x) = \overbrace{1}^{f'} \cdot \overbrace{9^x}^{g} + \overbrace{x}^{f} \cdot \overbrace{\ln 9 \cdot 9^x}^{g'}$

42. $y = x(6^{-2x})$

$y' = \overbrace{1}^{f'} \cdot \overbrace{6^{-2x}}^{g} + \overbrace{x}^{f} \cdot \overbrace{\ln 6 \cdot 6^{-2x} \cdot (-2)}^{g'}$

49. $h(t) = \log_5(4-t)^2$

$h(t) = 2\log_5(4-t)$

$h'(t) = 2 \cdot \frac{1}{\ln 5} \cdot \frac{-1}{4-t} = \boxed{\frac{-2}{\ln 5(4-t)}}$

48. $y = \log_3(x^2 - 3x)$

$y' = \frac{1}{\ln 3} \cdot \frac{2x-3}{x^2-3x} = \boxed{\frac{2x-3}{\ln 3(x^2-3x)}}$

51. $y = \log_5 \sqrt{x^2 - 1}$

$y = \log_5(x^2-1)^{1/2}$

$y = \frac{1}{2}\log_5(x^2-1)$

$y' = \frac{1}{2} \cdot \frac{1}{\ln 5} \cdot \frac{2x}{x^2-1}$

$\boxed{y' = \frac{x}{(\ln 5)(x^2-1)}}$

50. $g(t) = \log_2(t^2 + 7)^3$

$g(t) = 3\log_2(t^2+7)$

$g'(t) = 3 \cdot \frac{1}{\ln 2} \cdot \frac{2t}{t^2+7}$

$\boxed{g'(t) = \frac{6t}{(\ln 2)(t^2+7)}}$

53. $f(x) = \log_2 \frac{x^2}{x-1}$

$f(x) = \log_2(x^2) - \log_2(x-1)$

$f'(x) = \frac{1}{\ln 2} \cdot \frac{2x}{x^2} - \frac{1}{\ln 2} \cdot \frac{1}{x-1}$

52. $f(x) = \log_2 \sqrt[3]{2x+1}$

$f(x) = \log_2(2x+1)^{1/3}$

$f(x) = \frac{1}{3}\log_2(2x+1)$

$f'(x) = \frac{1}{3} \cdot \frac{1}{\ln 2} \cdot \frac{2}{2x+1}$

$f'(x) = \boxed{\frac{2}{3\ln 2(2x+1)}}$

55. $h(x) = \log_3 \frac{x\sqrt{x-1}}{2}$

$h(x) = \log_3 x + \log_3(x-1)^{1/2} - \log_3 2$

$h(x) = \log_3(x) + \frac{1}{2}\log_3(x-1) - \log_3 2$

$h'(x) = \frac{1}{\ln 3} \cdot \frac{1}{x} + \frac{1}{2} \cdot \frac{1}{\ln 3} \cdot \frac{1}{x-1} - 0$

56. $g(x) = \log_5 \frac{4}{x^2\sqrt{1-x}}$

$g(x) = \log_5 4 - \log_5 x^2 - \log_5(1-x)^{1/2}$

$g(x) = \log_5 4 - \log_5 x^2 - \frac{1}{2}\log_5(1-x)$

$g'(x) = 0 - \frac{1}{\ln 5} \cdot \frac{2x}{x^2} - \frac{1}{2} \cdot \frac{1}{\ln 5} \cdot \frac{-1}{1-x}$

(22) **Implicit Differentiation** In Exercises 63 and 64, use implicit differentiation to find dy/dx.

63. $xe^y - 10x + 3y = 0$

$$\underbrace{1}_{f'}\cdot\underbrace{e^y}_{g} + \underbrace{x}_{f}\cdot\underbrace{e^y\left(\frac{dy}{dx}\right)}_{g'} - 10 + 3\left(\frac{dy}{dx}\right) = 0$$

$$xe^y\left(\frac{dy}{dx}\right) + 3\left(\frac{dy}{dx}\right) = 10 - e^y$$

$$\frac{dy}{dx}(xe^y + 3) = 10 - e^y$$

$$\boxed{\frac{dy}{dx} = \frac{10 - e^y}{xe^y + 3}}$$

64. $e^{xy} + x^2 - y^2 = 10$

$$e^{xy}\left[1y + x\left(\frac{dy}{dx}\right)\right] + 2x - 2y\left(\frac{dy}{dx}\right) = 0$$

$$xe^{xy}\left(\frac{dy}{dx}\right) + ye^{xy} + 2x - 2y\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx}(xe^{xy} - 2y) = -2x - ye^{xy}$$

$$\boxed{\frac{dy}{dx} = \frac{-2x - ye^{xy}}{xe^{xy} - 2y}}$$

Finding the Equation of a Tangent Line In Exercises 65 and 66, find an equation of the tangent line to the graph of the function at the given point.

65. $xe^y + ye^x = 1, \quad (0, 1)$

$$1\cdot e^y + x\cdot e^y\left(\frac{dy}{dx}\right) + \frac{dy}{dx}e^x + ye^x = 0$$

$$1e^1 + 0(e^1)\left(\frac{dy}{dx}\right) + \frac{dy}{dx}e^0 + 1e^0 = 0$$

$$e + \frac{dy}{dx} + 1 = 0 \qquad \frac{dy}{dx} = -e - 1$$

$$\boxed{y - 1 = (-1 - e)(x - 0)}$$

66. $1 + \ln xy = e^{x-y}, \quad (1, 1)$

$$1 + \ln x + \ln y = e^{x-y}$$

$$0 + \frac{1}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right) = e^{x-y}\left[1 - \frac{dy}{dx}\right]$$

$$1 + 1\left(\frac{dy}{dx}\right) = e^0\left[1 - \frac{dy}{dx}\right]$$

$$1 + \frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 0$$

point $(1, 1)$

slope $= m = 0$

$$y - 1 = 0(x - 1)$$

$$\boxed{y = 1}$$

5.4-5.5 Quiz Review Derivatives of Exponential function e^x and a^x

Find $\frac{dy}{dx}$

1. f(x) $= ln\frac{(3+4x)^5}{\sqrt[4]{1-3x}}$

2. $y = \log_3\left(\frac{3x^5}{2x^4-3}\right)^3$

3. f(x) = xe^{2-x^2}

4. f(x) = $log_5\left(\frac{3-x}{\sqrt{1-x}}\right)$

5. f(x) = $8^{x-3x^2}(log(3-2x)^3)$

5.4-5.5 Quiz Review Derivatives of Exponential function e^x and a^x

Solution Key

Find $\frac{dy}{dx}$

1. $f(x) = \ln\frac{(3+4x)^5}{\sqrt[4]{1-3x}}$ $= \ln(3+4x)^5 - \ln(1-3x)^{1/4}$ $= 5\ln(3+4x) - \frac{1}{4}\ln(1-3x)$

$f'(x) = 5\left(\frac{4}{3+4x}\right) - \frac{1}{4}\left(\frac{-3}{1-3x}\right) = \boxed{\frac{20}{3+4x} + \frac{3}{4(1-3x)}}$

2. $y = \log_3\left(\frac{3x^5}{2x^4-3}\right)^3 = 3\log_3\left(\frac{3x^5}{2x^4-3}\right) = 3\log_3(3x^5) - 3\log_3(2x^4-3)$

$y' = 3\cdot\frac{1}{\ln 3}\left(\frac{15x^4}{3x^5}\right) - 3\left(\frac{1}{\ln 3}\right)\left(\frac{16x^3}{2x^4-3}\right)$

Reminder
$* \frac{d}{dx}\log_a u = \frac{1}{\ln a}\left[\frac{u'}{u}\right]$

$= \boxed{\frac{3}{\ln 3}\left(\frac{5}{x}\right) - \frac{3}{\ln x}\left(\frac{16x^3}{2x^4-3}\right)}$

3. $f(x) = xe^{2-x^2}$

$* $ product rule

$f'(x) = (1)e^{2-x^2} + x\cdot e^{2-x^2}(-2x)$

$\boxed{= e^{2-x^2}(1-2x^2)}$

4. $f(x) = \log_5\left(\frac{3-x}{\sqrt{1-x}}\right)$ $= \log_5(3-x) - \log_5(1-x)^{1/2}$ $= \log_5(3-x) - \frac{1}{2}\log_5(1-x)$

$f'(x) = \frac{1}{\ln 5}\left(\frac{-1}{3-x}\right) - \frac{1}{2}\left(\frac{1}{\ln 5}\right)\left(\frac{-1}{1-x}\right)$

$\boxed{= \frac{-1}{\ln 5(3-x)} + \frac{1}{2\ln 5(1-x)}}$

Reminder
$* \frac{d}{dx}a^u = \ln a\cdot a^u\cdot u'$

5. $f(x) = 8^{x-3x^2}(\log(3-2x)^3)$

$f(x) = 8^{x-3x^2}\cdot 3\log_{10}(3-2x)$

$f'(x) = (\ln 8)8^{x-3x^2}\cdot(1-6x)\cdot\log(3-2x)^3 + 8^{x-3x^2}\cdot\frac{3}{\ln 10}\left(\frac{-2}{3-2x}\right)$

$\boxed{8^{x-3x^2}\left[(\ln 8)(1-6x)\log(3-2x)^3 - \frac{6}{\ln 10(3-2x)}\right]}$

DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$
22. $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1 + u^2}$
23. $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$
24. $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$