

$$A^{-1} = \frac{1}{\det} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

5.10 More Practice with Matrix Applications

Identify the variables, create a system of equations, write the matrix equation, and solve. Show all work for solving 2x2 matrix equations. You may use technology to solve any 3x3 matrix equations.

1. One group of people purchased 10 hot dogs and 5 soft drinks at a cost of \$8.75. A second group bought 7 hot dogs and 4 soft drinks at a cost of \$6.25. What is the cost of a single hot dog and a single soft drink?

$x = \text{cost of hot dog}$
 $y = \text{cost of soft drink}$

$$\begin{cases} 10x + 5y = 8.75 \\ 7x + 4y = 6.25 \end{cases}$$

$$\begin{bmatrix} 10 & 5 \\ 7 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8.75 \\ 6.25 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 7 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 8.75 \\ 6.25 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 4 & -5 \\ -7 & 10 \end{bmatrix} \begin{bmatrix} 8.75 \\ 6.25 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.75 \\ 0.25 \end{bmatrix}$$

2. Four large cheeseburgers and two chocolate milkshakes cost a total of \$7.90. Two milkshakes cost \$0.15 more than one cheeseburger. What is the cost of a cheeseburger and what is the cost of a milkshake?

$x = \text{cost of cheeseburger}$
 $y = \text{cost of milkshake}$

$$\begin{cases} 4x + 2y = 7.90 \\ -1x + 2y = 0.15 \end{cases}$$

$$\begin{bmatrix} 4 & 2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7.90 \\ 0.15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 7.90 \\ 0.15 \end{bmatrix}$$

$$2y = 1x + 0.15$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \$1.55 \\ 0.85 \end{bmatrix}$$

3. Thompson's Furniture Store borrowed \$650,000 to expand its facilities and extend its product line. Some of the money was borrowed at 4%, some at 6.5%, and the rest at 9%. How much was borrowed at each rate if the annual interest was \$46,250 and the amount borrowed at 9% was twice the amount borrowed at 4%?

$x = \text{Amt borrowed at 4\%}$
 $y = \text{Amt at 6.5\%}$
 $z = \text{Amt at 9\%}$

$$\begin{cases} x + y + z = 650,000 \\ 0.04x + 0.065y + 0.09z = 46,250 \\ -2x + 0y + 1z = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.04 & 0.065 & 0.09 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 650,000 \\ 46,250 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160,000 \\ 170,000 \\ 320,000 \end{bmatrix}$$

$$z = 2x$$

4. At the Pittsburgh zoo, children ride a train for 25 cents, adults pay \$1.00, and senior citizens 75 cents. On a given day, 1400 passengers paid a total of \$740 for the rides. There were 250 more children riders than all other riders. Find the number of children, adult, and senior riders.

$x = \# \text{ of children}$
 $y = \# \text{ of adults}$
 $z = \# \text{ of seniors}$

$$\begin{cases} x + y + z = 1400 \\ 0.25x + 1y + 0.75z = 740 \\ 1x - 1y - 1z = 250 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.25 & 1 & 0.75 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1400 \\ 740 \\ 250 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 825 \\ 410 \\ 165 \end{bmatrix}$$

825 → children
 410 → adults
 165 → seniors

5. Grace receives an \$80,000 inheritance. She invests part of it in CDs (certificates of deposit) earning 6.7% APY (annual percentage yield), part in bonds earning 9.3% APY, and the remainder in a growth fund earning 15.6% APY. She invests three times as much in the growth fund as in the other two combined. How much does she have in each investment if she receives \$10,843 interest the first year?

$x = \text{CD's}$
 $y = \text{Amt in bonds}$
 $z = \text{Amt in Growth Fund}$

$$\begin{cases} x + y + z = 80000 \\ 0.067x + 0.093y + 0.156z = 10843 \\ -3x - 3y + z = 0 \end{cases}$$

$$\begin{cases} z = 3(x+y) \\ z = 3x + 3y \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.067 & 0.093 & 0.156 \\ -3 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 80,000 \\ 10,843 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \$14,500 \\ \$5,500 \\ \$10,000 \end{bmatrix}$$

6. Sophia has 74 coins consisting of nickels, dimes, and quarters in her coin box. The total value of the coins is \$8.85. If the number of nickels and quarters is four more than the number of dimes, find how many of each coin Sophia has.

7. A company manufactures tables, chairs, and stools. Last week it built a total of 275 items. The number of chairs built was four times the total number of tables and stools built. The total value of these items is \$42,125 with a chair selling for \$150, a table for \$200, and a stool for \$75. Determine the number of each item built last week.

8. A sports equipment company took out three different loans totaling \$350,000 from a bank to buy treadmills. The interest rates for each are: Loan 1 = 6.5%, Loan 2 = 7%, and Loan 3 = 9%. After one year the interest paid was \$24,950.00. The amount borrowed in Loan 1 was \$50,000 less than the amounts borrowed in the other two Loans combined. How much was borrowed in each loan?

5. Grace receives an \$80,000 inheritance. She invests part of it in CDs (certificates of deposit) earning 6.7% APY (annual percentage yield), part in bonds earning 9.3% APY, and the remainder in a growth fund earning 15.6% APY. She invests three times as much in the growth fund as in the other two combined. How much does she have in each investment if she receives \$10,843 interest the first year?

C = amt in CDs

B = amt in bonds

G = amt in growth fund

$$C + B + G = 80000$$

$$.067C + .093B + .156G = 10843$$

$$G = 3(C+B) \rightarrow -3C - 3B + G = 0$$

$$\begin{bmatrix} C \\ B \\ G \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ .067 & .093 & .156 \\ -3 & -3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 80000 \\ 10843 \\ 0 \end{bmatrix} = \begin{bmatrix} 14500 \\ 5500 \\ 60000 \end{bmatrix}$$

\$14,500 in CDs
\$5,500 in bonds
\$60,000 in growth fund

6. Sophia has 74 coins consisting of nickels, dimes, and quarters in her coin box. The total value of the coins is \$8.85. If the number of nickels and quarters is four more than the number of dimes, find how many of each coin Sophia has.

N = number of nickels

D = number of dimes

Q = number of quarters

$$N + D + Q = 74$$

$$.05N + .10D + .25Q = 8.85$$

$$(N+Q) = 4 + D \rightarrow N - D + Q = 4$$

$$\begin{bmatrix} N \\ D \\ Q \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ .05 & .10 & .25 \\ 1 & -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 74 \\ 8.85 \\ 4 \end{bmatrix} = \begin{bmatrix} 22 \\ 35 \\ 17 \end{bmatrix}$$

22 nickels
35 dimes
17 quarters

7. A company manufactures tables, chairs, and stools. Last week it built a total of 275 items. The number of chairs built was four times the total number of tables and stools built. The total value of these items is \$42,125 with a chair selling for \$150, a table for \$200, and a stool for \$75. Write and solve a system of equations to determine the number of each item built last week.

T = number of tables

C = number of chairs

S = number of stools

$$C + T + S = 275$$

$$C = 4(T + S) \rightarrow C - 4T - 4S = 0$$

$$150C + 200T + 75S = 42125$$

$$\begin{bmatrix} C \\ T \\ S \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -4 & -4 \\ 150 & 200 & 75 \end{bmatrix}^{-1} \begin{bmatrix} 275 \\ 0 \\ 42125 \end{bmatrix} = \begin{bmatrix} 220 \\ 40 \\ 15 \end{bmatrix}$$

220 chairs
40 tables
15 stools

8. A sports equipment company took out three different loans totaling \$350,000 from a bank to buy treadmills. The interest rates for each are: Loan 1 = 6.5%, Loan 2 = 7%, and Loan 3 = 9%. After one year the interest paid was \$24,950.00. The amount borrowed at the ^{Loan 1} rate was \$50,000 less than the amounts borrowed at the other two ^{loans} combined. Interpret the solution.

x = amt in Loan 1

y = amt in Loan 2

z = amt in Loan 3

$$x + y + z = 350000$$

$$.065x + .07y + .09z = 24950$$

$$x = (y+z) - 50000 \rightarrow x - y - z = -50000$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ .065 & .07 & .09 \\ 1 & -1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 350000 \\ 24950 \\ -50000 \end{bmatrix} = \begin{bmatrix} 150000 \\ 140000 \\ 60000 \end{bmatrix}$$

\$150,000 in Loan 1
\$140,000 in Loan 2
\$60,000 in Loan 3