5.11 Matrices Test Review Date: _____

Solve for X, given the following matrices. If not possible, state the reason why. *Show work!!*

$A = \begin{bmatrix} -2 & 0 \\ 4 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & a & -5 \\ 2 & 0 & -3 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 6 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 12 \\ 6 \end{bmatrix}$

1. X − A = D $X = A + D$

Not possible, dimensions not alike.

2. X = C − B

$= \begin{bmatrix} 1 & 0 & 2 \\ -3 & 6 & 1 \end{bmatrix} - \begin{bmatrix} 4 & a & -5 \\ 2 & 0 & -3 \end{bmatrix}$

$= \begin{bmatrix} -3 & -a & 7 \\ -5 & 6 & 4 \end{bmatrix}$

3. -2X = A

$X = -\frac{1}{2}[A] = -\frac{1}{2}\begin{bmatrix} -2 & 0 \\ 4 & 1 \end{bmatrix}$

$X = \begin{bmatrix} 1 & 0 \\ -2 & -1/2 \end{bmatrix}$

4. 3C = X − 2B $3C + 2B = X$

$3\begin{bmatrix} 1 & 0 & 2 \\ -3 & 6 & 1 \end{bmatrix} + 2\begin{bmatrix} 4 & a & -5 \\ 2 & 0 & -3 \end{bmatrix}$

$\begin{bmatrix} 3 & 0 & 6 \\ -9 & 18 & 3 \end{bmatrix} + \begin{bmatrix} 8 & 2a & -10 \\ 4 & 0 & -6 \end{bmatrix} = \begin{bmatrix} 11 & 2a & -4 \\ -5 & 18 & -3 \end{bmatrix}$

5. X = AB 2×2 and 2×3

$\begin{bmatrix} -2 & 0 \\ 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & a & -5 \\ 2 & 0 & -3 \end{bmatrix}$

$\begin{bmatrix} -2(4)+0(2) & -2a+0 & 10+0 \\ 16+2 & 4a+0 & -20-3 \end{bmatrix} = \begin{bmatrix} -8 & -2a & 10 \\ 18 & 4a & -23 \end{bmatrix}$

6. X = 2CD

2×3 and 2×1

not equal

Not possible since inner dimensions are not alike

7. X = A^{-1} $A^{-1} = \frac{1}{\det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\det(A) = -2(1) - (0)(4) = -2$

$A^{-1} = \frac{1}{-2}\begin{bmatrix} 1 & 0 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1/2 & 0 \\ 2 & 1 \end{bmatrix}$

8. AX = D $X = [A^{-1}][D]$

$\begin{bmatrix} -1/2 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 6 \end{bmatrix}$

$\begin{bmatrix} 12(-1/2)+6(0) \\ 12(2)+6(1) \end{bmatrix}$

$X = \begin{bmatrix} -6 \\ 30 \end{bmatrix}$

9. The **determinant** of $\begin{bmatrix} 1 & 1 & 0 \\ 3 & 4 & -2 \\ -5 & -6 & x \end{bmatrix}$ is 12. **Solve for x.**

* 1 1 0 1 1 / 3 4 −2 3 4 / −5 −6 x −5 −6

$4x + 10 + 0 - (0 + 12 + 3x) = 12$

$4x + 10 - 12 - 3x = 12$

$1x - 2 = 12$

$x = 14$

Write each matrix equation, then solve the system of equations using an Inverse Matrix. ***Show work!!***

10. $2x + y = 3$
$5x + 6y = 4$

$$\begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\frac{1}{12-5}\begin{bmatrix} 6 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 14 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} \quad \begin{matrix} 6(3)+4(-1) \\ -5(3)+2(4) \end{matrix} \rightarrow \begin{bmatrix} 14 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$(2, -1)$

11. $4x - 18 = 3y$
$8x - 7y = 34$

$4x - 3y = 18$
$8x - 7y = 34$

$$\begin{bmatrix} 4 & -3 \\ 8 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ 34 \end{bmatrix}$$

$$\det = 4(-7) - 8(-3) = -4$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 8 & -7 \end{bmatrix}^{-1}\begin{bmatrix} 18 \\ 34 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} -7 & 3 \\ -8 & 4 \end{bmatrix}\begin{bmatrix} 18 \\ 34 \end{bmatrix}$$

$$\begin{bmatrix} -7 & 3 \\ -8 & 4 \end{bmatrix}\begin{bmatrix} 18 \\ 34 \end{bmatrix} \quad \begin{matrix} -7(18)+3(34) \\ -8(18)+4(34) \end{matrix} \rightarrow \begin{bmatrix} -24 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} -24 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

$(6, 2)$

Write each as a matrix equation, then solve the 3 variable system of equations using a calculator.

12. $-x + 2y + 7z = 13$
$2x - y - 2z = -2$
$3x + 5y + 2z = -14$

$$\begin{bmatrix} -1 & 2 & 7 \\ 2 & -1 & -2 \\ 3 & 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ -2 \\ -14 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 2 & 7 \\ 2 & -1 & -2 \\ 3 & 5 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 13 \\ -2 \\ -14 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \\ 3 \end{bmatrix}$$

$(0, -4, 3)$

13. A doctor's prescription calls for a daily intake containing 40 mg of vitamin C and 30 mg of vitamin D. Your pharmacy stocks 2 liquids that can be used: one contains 20% vitamin C and 30% vitamin D, the other contains 40% vitamin C and 20% vitamin D. How many milligrams of each compound should be mixed to fill the prescription?

x = amt of compound 1
y = amt of compound 2

vit.C → $0.20x + 0.40y = 40$
vit.D → $0.30x + 0.20y = 30$

$$\begin{bmatrix} 0.2 & 0.4 \\ 0.3 & 0.2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 30 \end{bmatrix}$$

$$\det = 0.2(0.2) - (0.4)(0.3) = -0.08$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.2 & 0.4 \\ 0.3 & 0.2 \end{bmatrix}^{-1}\begin{bmatrix} 40 \\ 30 \end{bmatrix} \rightarrow = \frac{1}{-0.08}\begin{bmatrix} 0.2 & -0.4 \\ -0.3 & 0.2 \end{bmatrix}\begin{bmatrix} 40 \\ 30 \end{bmatrix} = \frac{-1}{0.08}\begin{bmatrix} 8-12 \\ -12+6 \end{bmatrix} \rightarrow \begin{bmatrix} 50 \\ 75 \end{bmatrix}$$

14. John has $20,000 to invest. As his financial consultant, you recommend that he invest in Treasury bills that yield 5%, Treasury bonds that yield 7%, and corporate bonds that yield 9%. John wants to have an annual income of $1280, and the amount invested in Treasury bills must be two times the amount invested in corporate bonds. Find the amount in each investment.

x = Amt of Tbills
y = Amt of Tbonds
z = Amt of corp. bonds

$$x + y + z = 20000$$
$$0.05x + 0.07y + 0.09z = 1280$$
$$1x + 0y - 2z = 0$$

$x = 2z$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.05 & 0.07 & 0.09 \\ 1 & 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20000 \\ 1280 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0.05 & 0.07 & 0.09 \\ 1 & 0 & -2 \end{bmatrix}^{-1}\begin{bmatrix} 20000 \\ 1280 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \$12,000 \\ \$2,000 \\ \$6,000 \end{bmatrix}$$

$12,000 in Tbills
$2000 in Tbonds
$6000 in C bonds