Non-AP Calculus 5.1/5.3 Natural Logs Derivatives Quiz Review WS #2

Log Properties		
1. $\ln(1) = 0$	3. $\ln(a^n) = n \ln a$	$\frac{d}{dx}[\ln u] = \frac{u'}{u}$
2. $\ln(ab) = \ln a + \ln b$	4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$	
	5. $\ln(e) = 1$	

Expanding a Logarithmic Expression: Use properties of logarithms to expand the logarithmic expression

1. $y = \ln\left(x^3(5-2x)^{\frac{1}{7}}\right)$

2. $f(x) = ln\sqrt{\left(\frac{2-4x^2}{x^3+5}\right)^3}$

3. $y = ln\left(\frac{6-x^4}{x\sqrt{3-x^2}}\right)$

Condensing a Logarithmic Expression: Use properties of logarithms to write expression as a logarithm of a single quantity

4. $\frac{1}{3}[2\ln(x-4) + 3\ln(7-x^2) - \frac{3}{4}\ln(8-3x)]$

Find the derivative of the functions below:

5) $y = ln\left(3x\sqrt{2x-5x^3}\right)$

6) $f(x) = ln\left(\frac{\sqrt{3-2x^4}}{5-x}\right)$

7) $y = x^3 \ln(\sqrt[3]{x})$ Find y'(x)

Use Log differentiation to find the derivative of the function:

8) $y = \frac{x^3(\sqrt{3-7x^2})}{(x-1)^{\frac{2}{5}}}$

Use Implicit Differentiation to find $\frac{dy}{dx}$:

9) $y + ln\left(\frac{y}{x}\right) - 7 = 2x^4$

10) Find an inverse function for f(x): $f(x) = \sqrt{x^3 - 7}$

Use function f(x) and the given real number a to find $(f^{-1})'(a)$

11) $f(x) = 3x^4 - x^3 + 2$ a = 6

Key

Non-AP Calculus 5.1/5.3 Natural Logs Derivatives Quiz Review WS #2

Log Properties

1. $\ln(1) = 0$ 2. $\ln(ab) = \ln a + \ln b$	3. $\ln(a^n) = n \ln a$ 4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ 5. $\ln(e) = 1$	$\frac{d}{dx}[\ln u] = \frac{u'}{u}$

Expanding a Logarithmic Expression: Use properties of logarithms to expand the logarithmic expression

1. $y = \ln\left(x^3(5-2x)^{\frac{1}{7}}\right)$

$y = \ln x^3 + \ln(5-2x)^{1/7}$

$\boxed{y = 3\ln x + \frac{1}{7}\ln(5-2x)}$

2. $f(x) = \ln\sqrt{\left(\frac{2-4x^2}{x^3+5}\right)^3}$

$f(x) = \ln\left(\frac{2-4x^2}{x^3+5}\right)^{3/2}$

$f(x) = \frac{3}{2}\ln\left(\frac{2-4x^2}{x^3+5}\right)$

$\boxed{f(x) = \frac{3}{2}\ln(2-4x^2) - \frac{3}{2}\ln(x^3+5)}$

3. $y = \ln\left(\frac{6-x^4}{x\sqrt{3-x^2}}\right)$

$y = \ln(6-x^4) - \ln x - \ln(3-x^2)^{1/2}$

$\boxed{y = \ln(6-x^4) - \ln x - \frac{1}{2}\ln(3-x^2)}$

Condensing a Logarithmic Expression: Use properties of logarithms to write expression as a logarithm of a single quantity

4. $\frac{1}{3}[2\ln(x-4) + 3\ln(7-x^2) - \frac{3}{4}\ln(8-3x)]$

$\frac{1}{3}\left[\ln(x-4)^2 + \ln(7-x^2)^3 - \ln(8-3x)^{3/4}\right]$

$\frac{1}{3}\ln\left[\frac{(x-4)^2(7-x^2)^3}{(8-3x)^{3/4}}\right] = \boxed{\ln\left[\frac{(x-4)^2(7-x^2)^3}{(8-3x)^{3/4}}\right]^{1/3}}$

Find the derivative of the functions below:

5) $y = \ln\left(3x\sqrt{2x-5x^3}\right)$

$y = \ln(3x) + \ln\sqrt{2x-5x^3}$

$y = \ln(3x) + \ln(2x-5x^3)^{1/2}$

$y = \ln(3x) + \frac{1}{2}\ln(2x-5x^3)$

$y'(x) = \frac{3}{3x} + \frac{1}{2}\cdot\frac{2-15x^2}{2x-5x^3}$

$\boxed{y'(x) = \frac{1}{x} + \frac{2-15x^2}{2(2x-5x^3)}}$

6) $f(x) = \ln\left(\frac{\sqrt{3-2x^4}}{5-x}\right)$

$f(x) = \ln\sqrt{3-2x^4} - \ln(5-x)$

$f(x) = \ln(3-2x^4)^{1/2} - \ln(5-x)$

$f(x) = \frac{1}{2}\ln(3-2x^4) - \ln(5-x)$

$f'(x) = \frac{1}{2}\cdot\frac{-8x^3}{3-2x^4} - \frac{-1}{5-x}$

$\boxed{f'(x) = \frac{-4x^3}{3-2x^4} + \frac{1}{5-x}}$

7) $y = x^3 \ln(\sqrt[3]{x})$ Find y'(x)

$y = x^3 \cdot \ln x^{1/3}$

$y = x^3 \cdot \frac{1}{3}\ln x$

*product rule

$y = \underbrace{x^3}_{f} \cdot \underbrace{\frac{1}{3}\ln x}_{g}$

$y' = \overbrace{3x^2}^{f'} \cdot \overbrace{\frac{1}{3}\ln x}^{g} + \overbrace{x^3}^{f} \cdot \overbrace{\frac{1}{3}\left(\frac{1}{x}\right)}^{g'}$

$y' = \frac{3x^2\ln x}{3} + \frac{x^3}{3x}$

$\boxed{y' = x^2\ln x + \frac{x^2}{3}}$

Use Log differentiation to find the derivative of the function:

8) $y = \frac{x^3(\sqrt{3-7x^2})}{(x-1)^{\frac{2}{5}}}$

$\ln y = \ln\left[\frac{x^3(3-7x^2)^{1/2}}{(x-1)^{2/5}}\right]$

$\ln y = \ln x^3 + \ln(3-7x^2)^{1/2} - \ln(x-1)^{2/5}$

$\ln y = 3\ln x + \frac{1}{2}\ln(3-7x^2) - \frac{2}{5}\ln(x-1)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = 3 \cdot \frac{1}{x} + \frac{1}{2} \cdot \frac{-14x}{3-7x^2} - \frac{2}{5} \cdot \frac{1}{x-1}$

$\frac{dy}{dx} = y\left[\frac{3}{x} - \frac{7x}{3-7x^2} - \frac{2}{5(x-1)}\right]$

$\boxed{\frac{dy}{dx} = \frac{x^3\sqrt{3-7x^2}}{(x-1)^{2/5}}\left[\frac{3}{x} - \frac{7x}{3-7x^2} - \frac{2}{5(x-1)}\right]}$

Use Implicit Differentiation to find $\frac{dy}{dx}$:

9) $y + \ln\left(\frac{y}{x}\right) - 7 = 2x^4$

$y + \ln y - \ln x - 7 = 2x^4$

$1\frac{dy}{dx} + \frac{1}{y}\left(\frac{dy}{dx}\right) - \frac{1}{x} = 8x^3$

$\frac{dy}{dx}\left(1 + \frac{1}{y}\right) = 8x^3 + \frac{1}{x}$

$\boxed{\frac{dy}{dx} = \frac{8x^3 + \frac{1}{x}}{1 + \frac{1}{y}}}$

10) Find an inverse function for f(x): $f(x) = \sqrt{x^3 - 7}$

$y = \sqrt{x^3 - 7}$

$x = \sqrt{y^3 - 7}$

$(x)^2 = (\sqrt{y^3 - 7})^2$

$x^2 = y^3 - 7$

$x^2 + 7 = y^3$

$\sqrt[3]{x^2+7} = y$

$\boxed{f^{-1}(x) = \sqrt[3]{x^2+7}}$

Use function f(x) and the given real number a to find $(f^{-1})'(a)$

11) $f(x) = 3x^4 - x^3 + 2$ a = 6

$f(-1) = 6$	$(f^{-1})(6) = $ ___
$f'(-1) = $ ___	$(f^{-1})'(6) = $ ___

$6 = 3x^4 - x^3 + 2$

$0 = 3x^4 - x^3 - 4$

$0 = 3(-1)^4 - (-1)^3 - 4$ ✓

$x = -1$

$f'(x) = 12x^3 - 3x^2$

$f'(-1) = 12(-1)^3 - 3(-1)^2$

$f'(-1) = -12 - 3$

$f'(-1) = -15$

$\boxed{(f^{-1})'(6) = -\frac{1}{15}}$