**Non-AP Calculus** **5.1/5.3 Natural Logs Properties and Derivatives Quiz Review WS #1**

**Log Properties**

1. $\ln(1) = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln(a^n) = n \ln a$
4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
5. $\ln(e) = 1$

$$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$

**Expanding a Logarithmic Expression: Use properties of logarithms to expand the logarithmic expression**

1. $y = \ln\left(x\sqrt[3]{6 - x^4}\right)$

2. $f(x) = ln\sqrt{\frac{3-2x}{4x}}$

3. $y = ln\left(\frac{2x^4}{x\sqrt{3x^5-1}}\right)$

**Condensing a Logarithmic Expression: Use properties of logarithms to write expression as a logarithm of a single quantity**

4. $5\ln x + 3\ln y - 5\ln w - 6\ln z$

5. $\frac{1}{5}[\ln(x-4) - 3\ln(7-x^2) - \ln(8-x)]$

Find the derivative of the functions below:

6) $y = ln\left(x\sqrt{6 - x^3}\right)$

Find the derivative of the functions below:

7) $y = \ln\sqrt{\left(\frac{3-2x}{x^3}\right)^5}$

8) $y = x^3 \ln(x^2)$

Use Log differentiation to find the derivative of the function:

9) $y = \frac{x^3\left(\sqrt{5-4x^5}\right)}{(x-1)^2}$

Use Implicit Differentiation to find $\frac{dy}{dx}$:

10) $x + \ln(xy) - 2y = x^3$

11) Find an inverse function for f(x): $f(x) = 2x^3 - 1$

Use function f(x) and the given real number a to find $(f^{-1})'(a)$

12) $f(x) = 2x^3 - 3x + 1$ a = 11

Key

**Non-AP Calculus 5.1/5.3 Natural Logs Properties and Derivatives Quiz Review WS #1**

**Log Properties**

1. $\ln(1) = 0$
2. $\ln(ab) = \ln a + \ln b$
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$$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$

**Expanding a Logarithmic Expression: Use properties of logarithms to expand the logarithmic expression**

1. $y = \ln\left(x\sqrt[3]{6 - x^4}\right)$

$y = \ln x + \ln \sqrt[3]{6-x^4}$

$y = \ln x + \ln(6-x^4)^{1/3}$

$y = \ln x + \frac{1}{3}\ln(6-x^4)$

$\boxed{y = \ln x + \frac{1}{3}\ln(6-x^4)}$

2. $f(x) = \ln\sqrt{\frac{3-2x}{4x}}$

$f(x) = \ln\left(\frac{3-2x}{4x}\right)^{1/2}$

$f(x) = \frac{1}{2}\ln\left(\frac{3-2x}{4x}\right)$

$\boxed{f(x) = \frac{1}{2}\ln(3-2x) - \frac{1}{2}\ln(4x)}$

3. $y = \ln\left(\frac{2x^4}{x\sqrt{3x^5-1}}\right)$

$y = \ln(2x^4) - \ln x - \ln\sqrt{3x^5-1}$

$y = \ln(2x^4) - \ln x - \ln(3x^5-1)^{1/2}$

$\boxed{y = \ln(2x^4) - \ln x - \frac{1}{2}\ln(3x^5-1)}$

**Condensing a Logarithmic Expression: Use properties of logarithms to write expression as a logarithm of a single quantity**

4. $5\ln x + 3\ln y - 5\ln w - 6\ln z$

$\ln x^5 + \ln y^3 - \ln w^5 - \ln z^6$

$\ln\left(\frac{x^5y^3}{w^5z^6}\right)$

5. $\frac{1}{5}[\ln(x-4) - 3\ln(7-x^2) - \ln(8-x)]$

$\frac{1}{5}\left[\ln(x-4) - \ln(7-x^2)^3 - \ln(8-x)\right]$

$\frac{1}{5}\ln\left(\frac{x-4}{(7-x^2)^3(8-x)}\right) = \boxed{\ln\left(\frac{x-4}{(7-x^2)^3(8-x)}\right)^{1/5}}$

Find the derivative of the functions below:

6) $y = \ln\left(x\sqrt{6 - x^3}\right)$

*expand equation first:

$y = \ln x + \ln\sqrt{6-x^3}$

$y = \ln x + \ln(6-x^3)^{1/2}$

$y = \ln x + \frac{1}{2}\ln(6-x^3)$

$y' = \frac{1}{x} + \frac{1}{2}\cdot\frac{-3x^2}{6-x^3}$

$\boxed{y' = \frac{1}{x} - \frac{3x^2}{2(6-x^3)}}$

Find the derivative of the functions below:

7) $y = \ln\sqrt{\left(\frac{3-2x}{x^3}\right)^5}$

$y = \ln\left(\frac{3-2x}{x^3}\right)^{5/2}$

$y = \frac{5}{2}\ln\left(\frac{3-2x}{x^3}\right)$

$y = \frac{5}{2}\ln(3-2x) - \frac{5}{2}\ln(x^3)$

$y' = \frac{5}{2}\cdot\frac{-2}{3-2x} - \frac{5}{2}\cdot\frac{3x^2}{x^3}$

$\boxed{y' = \frac{-5}{3-2x} - \frac{15}{2x}}$

8) $y = x^3\ln(x^2)$ *product rule

$y' = \overbrace{3x^2}^{f'}\cdot\overbrace{\ln x^2}^{g} + \overbrace{x^3}^{f}\cdot\overbrace{\frac{2x}{x^2}}^{g'}$

$\boxed{y' = 3x^2\ln x^2 + 2x^2}$

Use Log differentiation to find the derivative of the function:

9) $y = \frac{x^3\left(\sqrt{5-4x^5}\right)}{(x-1)^2}$

$\ln y = \ln\left[\frac{x^3(5-4x^5)^{1/2}}{(x-1)^2}\right]$

$\ln y = \ln x^3 + \ln(5-4x^5)^{1/2} - \ln(x-1)^2$

$\ln y = 3\ln x + \frac{1}{2}\ln(5-4x^5) - 2\ln(x-1)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = 3\cdot\frac{1}{x} + \frac{1}{2}\cdot\frac{-20x^4}{5-4x^5} - 2\cdot\frac{1}{x-1}$

$\frac{dy}{dx} = \left[\frac{x^3\sqrt{5-4x^5}}{(x-1)^2}\right]\left[\frac{3}{x} - \frac{10x^4}{5-4x^5} - \frac{2}{x-1}\right]$

Use Implicit Differentiation to find $\frac{dy}{dx}$:

10) $x + \ln(xy) - 2y = x^3$

$x + \ln x + \ln y - 2y = x^3$

$1 + \ln x + \frac{1}{y}\left(\frac{dy}{dx}\right) - 2\left(\frac{dy}{dx}\right) = 3x^2$

$\frac{1}{y}\left(\frac{dy}{dx}\right) - 2\left(\frac{dy}{dx}\right) = -3x^2 - \ln x - 1$

$\frac{dy}{dx}\left(\frac{1}{y} - 2\right) = -3x^2 - \ln x - 1$

$\boxed{\frac{dy}{dx} = \frac{-3x^2 - \ln x - 1}{\frac{1}{y} - 2}}$

11) Find an inverse function for f(x): $f(x) = 2x^3 - 1$

$y = 2x^3 - 1$ | $x = 2y^3 - 1$ | $x + 1 = 2y^3$ | $\frac{x+1}{2} = y^3$ | $y = \sqrt[3]{\frac{x+1}{2}}$

$\boxed{f^{-1}(x) = \sqrt[3]{\frac{x+1}{2}}}$

Use function f(x) and the given real number a to find $(f^{-1})'(a)$

12) $f(x) = 2x^3 - 3x + 1$ a = 11

| | |
|---|---|
| $f(b) = a$ | $f^{-1}(a) = b$ |
| $f'(b) = n$ | $f^{-1\prime}(a) = \frac{1}{n}$ |

| | |
|---|---|
| $f(2) = 11$ | $f^{-1}(11) = \underline{2}$ |
| $f'(2) = \_\_$ | $(f^{-1})'(11) = \_\_$ |

$11 = 2x^3 - 3x + 1$

$0 = 2x^3 - 3x - 10$

$x = 2$

$f'(x) = 6x^2 - 3$

$f'(2) = 6(2)^2 - 3$

$f'(2) = 24 - 3$

$f'(2) = 21$

$\boxed{(f^{-1})'(11) = \frac{1}{21}}$