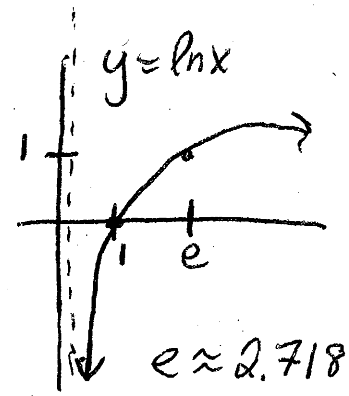
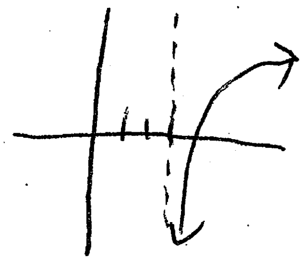
Calculus Ch. 5.1 Natural Log Function



Natural Log graph: Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

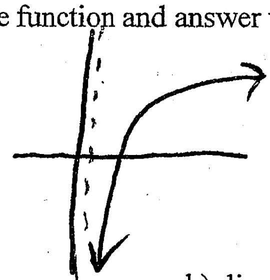
Graph characteristics: always continuous, always increasing, always concave down

Ex. 1: Sketch graph of ln(x – 3) and state domain:



D: $(3, \infty)$

Ex. 2 Draw the function and answer the examples.



a) $\lim_{x\to 0^+} \ln(x) = \boxed{-\infty}$ b) $\lim_{x\to 0^-} \ln(x) = \boxed{DNE}$ c) $\lim_{x\to 0} \ln(x) =$ DNE d) $\lim_{x\to \infty} \ln(x) = +\infty$

Properties: ln(1) = 0 ln(aⁿ) = nln(a) ln(e) = 1

ln(ab) = ln(a) + ln(b) $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

Ex. 3 Expand $\ln\left(3e^2\right)$

$\ln 3 + \ln e^2$

$\ln 3 + 2\ln e$

$\ln 3 + 2(1)$ $\boxed{\ln 3 + 2}$

Properties: $\ln(1) = 0$ $\ln(a^n) = n\ln(a)$ $\ln(e) = 1$

$\ln(ab) = \ln(a) + \ln(b)$ $\ln(a/b) = \ln(a) - \ln(b)$

Ex. 4 condense $2[\ln(x) - \ln(x + 1) - \ln(x - 1)]$

$$2\left[\ln x - \left(\ln(x+1) + \ln(x-1)\right)\right]$$

$$2\left[\ln x - \ln(x^2-1)\right]$$

$$2\left[\ln\left(\frac{x}{x^2-1}\right)\right]$$

$$\ln\left(\frac{x}{x^2-1}\right)^2$$

Derivative of the Natural Logarithmic Function:

$$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$

Ex. 5: If $y = \ln(x)$, find y'

$$y' = \frac{1}{x}$$

Ex. 6: if $y = \ln(x^2 - 5)$, find y'

$$y' = \frac{2x}{x^2-5}$$

Ex. 8 Find $\frac{dy}{dx}$ $4xy + \ln(x^2y) = 7$

$$4xy + \ln x^2 + \ln y = 7$$

$$4xy + 2\ln x + \ln y = 7$$

$$4y + 4x\left(\frac{dy}{dx}\right) + 2\left(\frac{1}{x}\right) + \frac{1}{y}\left(\frac{dy}{dx}\right) = 0$$

$$4x\left(\frac{dy}{dx}\right) + \frac{1}{y}\left(\frac{dy}{dx}\right) = -\frac{2}{x} - 4y$$

$$\frac{dy}{dx}\left(4x + \frac{1}{y}\right) = -\frac{2}{x} - 4y$$

$$\frac{dy}{dx} = \frac{\left(-\frac{2}{x} - 4y\right)}{\left(4x + \frac{1}{y}\right)} \frac{(xy)}{(xy)}$$

$$\boxed{\frac{dy}{dx} = \frac{-2y - 4xy^2}{4xy^2 + x}}$$

$$y' = \frac{-4xy^2 - 2y}{4x^2y + x}$$

Ex. 7: if $y = \ln\left(\frac{x^2}{\sqrt{2x^3}}\right)$, find y' (always simplify logs before taking the derivative)

$$y = \ln x^2 - \ln(2x^3)^{1/2}$$

$$y = 2\ln x - \frac{1}{2}\ln(2x^3)$$

$$y' = 2\left(\frac{1}{x}\right) - \frac{1}{2}\left(\frac{6x^2}{2x^3}\right)$$

$$\boxed{y' = \frac{2}{x} - \frac{3}{2x}}$$

$$\boxed{y' = \frac{4-3}{2x}} = \boxed{\frac{1}{2x}}$$

Calculus Ch. 5.1b, 5.3 Notes

Recall $\frac{d}{dx}\ln u = \frac{u'}{u}$

Warm-up Problem: If g(2) = 3 and g ' (2) = -4, find f ' (2) if f(x) = $x^2\ln(g(x))$.

$$f'(x) = 2x \cdot \ln(g(x)) + x^2\left(\frac{g'(x)}{g(x)}\right)$$

$$f'(2) = 2(2) \cdot \ln[g(2)] + 2^2 \cdot \frac{g'(2)}{g(2)}$$

$$f'(2) = 4 \cdot \ln(3) + 2^2\left(\frac{-4}{3}\right)$$

$$\boxed{f'(2) = 4\ln 3 - \frac{16}{3}}$$

Logarithmic Differentiation : Simplifying non-log functions using log properties to expand before differentiating.

Log differentiation steps:

1. Take the **ln** (natural log) of both sides
2. Simplify and expand using log properties
3. Use implicit differentiation
4. Substitute for y

Example 1: Find the derivative of $y = \frac{(x-2)^2}{\sqrt{x^2+1}}$.

$$\ln y = \ln\left[\frac{(x-2)^2}{(x^2+1)^{1/2}}\right]$$

$$\ln y = \ln(x-2)^2 - \ln(x^2+1)^{1/2}$$

$$\ln y = 2\ln(x-2) - \frac{1}{2}\ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = 2\left(\frac{1}{x-2}\right) - \frac{1}{2}\left(\frac{2x}{x^2+1}\right)$$

$$\frac{dy}{dx} = y\left[\frac{2}{x-2} - \frac{x}{x^2+1}\right]$$

$$\frac{dy}{dx} = \frac{(x-2)^2}{\sqrt{x^2+1}}\left[\frac{2}{x-2} - \frac{x}{x^2+1}\right]$$

Example 2: Find the derivative of y = x^{2x+3}

$$\ln y = \ln x^{2x+3}$$

$$\ln y = (2x+3)(\ln x)$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = (2)[\ln x] + (2x+3)\left(\frac{1}{x}\right)$$

$$\frac{dy}{dx} = y\left[2\ln x + \frac{2x+3}{x}\right]$$

$$\frac{dy}{dx} = x^{2x+3}\left[2\ln x + 2 + \frac{3}{x}\right]$$

Absolute Value Rule: $\frac{d}{dx}\ln|u| = \frac{u'}{u}$

Example 3: Find $\frac{d}{dx}\ln|x^2-5| = \frac{2x}{x^2-5}$

Example 4: Locate any relative extrema and inflection points for $y = \frac{x}{\ln x}$

$$y'(x) = \frac{1\ln x - x\left(\frac{1}{x}\right)}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2} \rightarrow$$

set $\ln x - 1 = 0$

$\ln x = 1$

$e^1 = x$

0 — 1 — e (signs: −, −, +)

Rel. min at $\left(e, \frac{e}{\ln e}\right) = (e, e)$

$$y''(x) = \frac{\left(\frac{1}{x}\right)(\ln x)^2 - (\ln x - 1)2(\ln x)\left(\frac{1}{x}\right)}{(\ln x)^4}$$

$$= \frac{\frac{1}{x}\ln x[\ln x - 2\ln x + 2]}{(\ln x)^4}$$

$$y''(x) = \frac{-\ln x + 2}{x(\ln x)^3}$$

$2 - \ln x = 0$

$2 = \ln x$

$e^2 = x$

0 — 1 — e^2 (signs: +, +, −; ∪, ∪, ∩)

POI at $\left(e^2, \frac{e^2}{2}\right)$ b/c $y''(x)$ change signs.

Ch.5.3 Notes **Derivatives of Inverse Function**

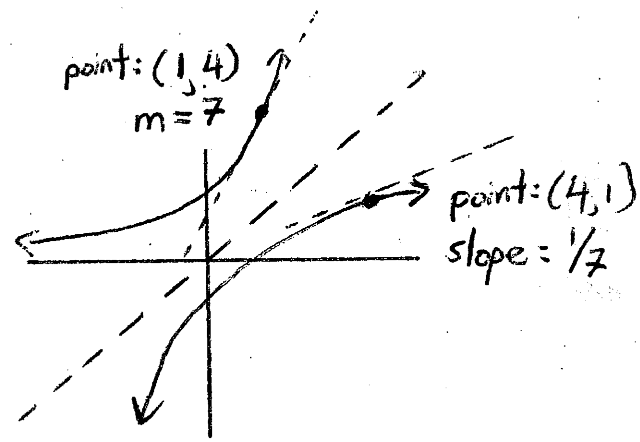
A. Inverse Functions

1) x's and y's are swapped
2) Domains and ranges are swapped
3) Graphs are symmetric over the line y = x
4) F(x) must be one-to-one (pass the horizontal line test) for its inverse to be a function
5) A function that is **monotonic** (always increasing or always decreasing) will always have an inverse that is a function
6) If f and g are inverses, then f(g(x)) = x and g(f(x)) = x

Example 4: find the inverse of f(x) = 6x + 2

*ALWAYS restrict the domain of the inverse function to the range of the original function

Ex. 5 f(x) = $\sqrt{x-5}$. Find the domain of the inverse function



* At their corresponding points, the slopes of tangent line will be reciprocals of each other

Evaluate derivative of inverse at a point: (find $(f^{-1})'(a)$)

$f(b) = a$	$(f^{-1})(a) = b$
$f'(b) = n$	$(f^{-1})'(a) = \frac{1}{n}$

Example 6: f(x) = x^3 + 4x + 2 find $(f^{-1})'(-3)$ $g'(-3)$

$f(-1) = -3$	$g(-3) = \underline{-1}$
$f'(-1) = 7$	$g'(3) = \frac{1}{7}$

$-3 = x^3 + 4x + 2$

$x = -1$

$f'(x) = 3x^2 + 4$

$f'(-1) = 3(-1)^2 + 4$

$= 3 + 4 = 7$

$\boxed{g'(3) = \frac{1}{7}}$

Example 7: f(x) = $\sqrt{x^3 - 7}$ find $(f^{-1})'(1)$ Find $g'(1)$

$f(2) = 1$	$g(1) = \underline{2}$
$f'(2) = 6$	$g'(1) = \frac{1}{6}$

$1 = \sqrt{x^3 - 7}$

$1 = x^3 - 7$ $x^3 = 8$, $x = 2$

$f(x) = (x^3 - 7)^{1/2}$

$f'(x) = \frac{1}{2}(x^3 - 7)^{-1/2}(3x^2)$

$= \frac{3x^2}{2\sqrt{x^3 - 7}}$

$f'(2) = \frac{12}{2(1)}$

Example 8: If g(f(x)) = x , g(7) = 2, and g'(7) = 10, then f'(2) is

$g(7) = 2$	$f(2) = 7$
$g'(7) = 10$	$f'(2) = \frac{1}{10}$

$\boxed{f'(2) = \frac{1}{10}}$

Example 9: If g(f(x)) = x , g(9) = 3, and g'(9) = -4, then f'(3) is

$g(9) = 3$	$f(3) = 9$
$g'(9) = -4$	$f'(3) = \frac{-1}{4}$

$\boxed{f'(3) = -\frac{1}{4}}$