Calculus AB **Ch. 5.1-5.3 Natural Logs Quiz Review**

1. Find the domain for $y = \ln(2 + 3x) - 1$

2. Find $\frac{dy}{dx}$ $y = ln\sqrt{\frac{3-2x}{4x}}$

3. Find $\frac{dy}{dx}$ $y = x^{\sqrt{x+3}}$

4. $f(x) = \sqrt{5x-1} - 4$
   a. Find $(f^{-1})(x)$
   b. Find the domain for $(f^{-1})(x)$

5. $f(x) = x^3 + 2x^2 - 3$ Find $(f^{-1})'(13)$

Identify the domain and range of each.

6) $y = \ln(2x - 3) + 5$

7) $y = \ln(3x + 17) - 5$

Expand each logarithm.

8) $\ln(a \cdot b \cdot c^3)$

Condense each expression to a single logarithm.

9) $\dfrac{\ln u}{2} + \dfrac{\ln v}{2} + \dfrac{\ln w}{2}$

Differentiate each function with respect to x.

10) $f(x) = \ln \sqrt[4]{\dfrac{2x^3}{3x^2 - 4}}$

11) $f(x) = \ln\left(\dfrac{4x^2}{5x^3 - 3}\right)^5$

Use logarithmic differentiation to differentiate each function with respect to x.

12) $y = \sqrt[3]{x^2 + 1}$

13) $y = x^{2x}$

For each problem, find $(f^{-1})'(a)$

14) $f(x) = 3x^5 + 2x + 5,\ a = 5$

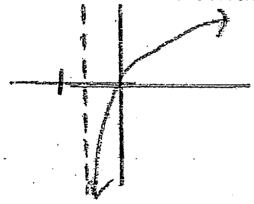
15) $f(x) = 2x^3 + 4x + 5,\ a = 5$

Calculus AB **Ch. 5.1-5.3 Natural Logs Quiz Review**

$\boxed{\text{Solution Key}}$

1. Find the domain for y = ln(2 + 3x) – 1

VA: $x = -\frac{2}{3}$



$2+3x=0$

$x=-\frac{2}{3}$

Domain: $\left(-\frac{2}{3}, \infty\right)$

2. Find $\frac{dy}{dx}$ $\quad y = \ln\sqrt{\frac{3-2x}{4x}}$ *expand first using log properties

$y = \ln\left(\frac{3-2x}{4x}\right)^{1/2}$

$y = \frac{1}{2}\ln\left(\frac{3-2x}{4x}\right)$

$y = \frac{1}{2}\left[\ln(3-2x) - \ln(4x)\right]$

$y' = \frac{1}{2}\left(\frac{-2}{3-2x}\right) - \frac{1}{2}\left(\frac{4}{4x}\right)$

$\boxed{y' = \frac{-1}{3-2x} - \frac{1}{2x}}$

3. Find $\frac{dy}{dx}$ $\quad y = x^{\sqrt{x+3}}$ *use log differentiation

$\ln y = \ln x^{(x+3)^{1/2}}$

$\ln y = (x+3)^{1/2}\cdot \ln x$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{2}(x+3)^{-1/2}(1)\ln x + (x+3)^{1/2}\left(\frac{1}{x}\right)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{\ln x}{2\sqrt{x+3}} + \frac{\sqrt{x+3}}{x}$

$\frac{dy}{dx} = y\left[\frac{\ln x}{2\sqrt{x+3}} + \frac{\sqrt{x+3}}{x}\right]$

$\boxed{\frac{dy}{dx} = x^{\sqrt{x+3}}\left[\frac{\ln x}{2\sqrt{x+3}} + \frac{\sqrt{x+3}}{x}\right]}$

4. f(x) = $\sqrt{5x-1} - 4$ $\quad y = \sqrt{5x-1} - 4$
   a. Find $(f^{-1})(x)$
   b. Find the domain for $(f^{-1})(x)$

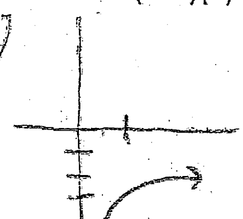
$x = \sqrt{5y-1} - 4$

$x+4 = \sqrt{5y-1}$

$(x+4)^2 = 5y-1$

$\frac{(x+4)^2+1}{5} = y$

$\boxed{f^{-1}(x) = \frac{(x+4)^2+1}{5}}$



D: $[1/5, \infty)$

R: $[-4, \infty)$

Domain for $(f^{-1})(x)$

D: $[-4, \infty)$

5. f(x) = $x^3 + 2x^2 - 3$ $\quad$ Find $(f^{-1})'(13)$

$f(\_) = 13$	$(f^{-1})(13) =$
$f'(2) = 20$	$(f^{-1})'(13) = \boxed{\frac{1}{20}}$

$x^3 + 2x^2 - 3 = 13$

$x^3 + 2x^2 = 16$

$x = 2$ ✓

$(2)^3 + 2(2)^2 = 16$ ✓

$f'(x) = 3x^2 + 4x$

$f'(2) = 3(2)^2 + 4(2)$

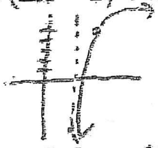
$= 12 + 8$

$= 20$

Identify the domain and range of each.

6) $y = \ln(2x-3)+5$ $\quad$ D: $(3/2, \infty)$

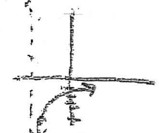
VA: $x = 3/2$ $\quad$ R: $(-\infty, \infty)$



$x = -17/3$

7) $y = \ln(3x+17)-5$ $\quad$ D: $(-17/3, \infty)$

R: $(-\infty, \infty)$



Expand each logarithm.

8) $\ln(a \cdot b \cdot c^3)$

$\boxed{\ln a + \ln b + 3\ln c}$

Condense each expression to a single logarithm.

9) $\dfrac{\ln u}{2} + \dfrac{\ln v}{2} + \dfrac{\ln w}{2}$

$\frac{1}{2}\ln u + \frac{1}{2}\ln v + \frac{1}{2}\ln w$

$\ln u^{1/2} + \ln v^{1/2} + \ln w^{1/2}$

$\ln u^{1/2} v^{1/2} w^{1/2}$

$= \boxed{\ln (uvw)^{1/2}}$

Differentiate each function with respect to x.

10) $f(x) = \ln \sqrt[4]{\dfrac{2x^3}{3x^2-4}} = \ln\left(\dfrac{2x^3}{3x^2-4}\right)^{1/4}$

$y = \frac{1}{4}\ln\left(\dfrac{2x^3}{3x^2-4}\right)$

$y = \frac{1}{4}\ln(2x^3) - \frac{1}{4}\ln(3x^2-4)$

$y' = \frac{1}{4}\left(\dfrac{6x^2}{2x^3}\right) - \frac{1}{4}\left(\dfrac{6x}{3x^2-4}\right)$

$\boxed{y' = \dfrac{3}{4x} - \dfrac{3x}{2(3x^2-4)}}$

11) $f(x) = \ln\left(\dfrac{4x^2}{5x^3-3}\right)^5$

$y = 5\ln\left(\dfrac{4x^2}{5x^3-3}\right)$

$y = 5\ln(4x^2) - 5\ln(5x^3-3)$

$y' = 5\left(\dfrac{8x}{4x^2}\right) - 5\left(\dfrac{15x^2}{5x^3-3}\right)$

$\boxed{y' = \dfrac{10}{x} - \dfrac{75x^2}{5x^3-3}}$

Use logarithmic differentiation to differentiate each function with respect to x.

12) $y = \sqrt[3]{x^2+1}$

$\ln y = \ln(x^2+1)^{1/3} = \frac{1}{3}\ln(x^2+1)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{3}\left(\dfrac{2x}{x^2+1}\right)$

$\frac{dy}{dx} = y\left(\dfrac{2x}{3(x^2+1)}\right) = \boxed{\sqrt[3]{x^2+1}\left(\dfrac{2x}{3(x^2+1)}\right)}$

13) $y = x^{2x}$

$\ln y = \ln x^{2x}$

$\ln y = 2x \ln x$

$\frac{1}{y}\frac{dy}{dx} = 2\ln x + 2x\left(\frac{1}{x}\right)$

$\boxed{\frac{dy}{dx} = x^{2x}\left[2\ln x + 2\right]}$

For each problem, find $(f^{-1})'(a)$

14) $f(x) = 3x^5 + 2x + 5,\ a = 5$

$f(0) = 5$ $\quad$ $(f^{-1})(5) = 0$

$f'(0) = 2$ $\quad$ $(f^{-1})'(5) = \boxed{\frac{1}{2}}$

$3x^5 + 2x + 5 = 5$

$3x^5 + 2x = 0$

$x = 0$

$f'(x) = 15x^4 + 2$

$f'(0) = 0 + 2 = 2$

15) $f(x) = 2x^3 + 4x + 5,\ a = 5$

$f(0) = 5$ $\quad$ $(f^{-1})(5) = 0$

$f'(0) = 4$ $\quad$ $(f^{-1})'(5) = \boxed{\frac{1}{4}}$

$5 = 2x^3 + 4x + 5$

$0 = 2x^3 + 4x$

$x = 0$

$f'(x) = 6x^2 + 4$

$f'(0) = 0 + 4 = 4$