Non-AP Calculus 5.1/5.3 Natural Logs Derivatives Quiz Review WS #3

Log Properties

1. $\ln(1) = 0$ 2. $\ln(ab) = \ln a + \ln b$	3. $\ln(a^n) = n \ln a$ 4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ 5. $\ln(e) = 1$	$\frac{d}{dx}[\ln u] = \frac{u'}{u}$

Expanding a Logarithmic Expression: Use properties of logarithms to expand the logarithmic expression

1. $y = \ln\left(x\left(\sqrt[7]{3x - 5x^3}\right)\right)$

2. $f(x) = ln\sqrt{\left(\frac{2-3x^2}{\sqrt[3]{6-x^4}}\right)^5}$

Condensing a Log Expression: Use properties of logarithms to write expression as a logarithm of a single quantity

3. $\frac{1}{4}[5\ln(5x - 6) - 3\ln(2 - x^2) - \frac{3}{2}\ln(12 - 7x)]$

Find the derivative of the functions below:

4) $y = ln\left(\frac{3x\sqrt{2x - 5x^3}}{\sqrt[3]{1 - 9x}}\right)$

5) $f(x) = ln\left(\frac{\sqrt{x - 2x^4}}{7 - \pi x}\right)^5$

6) $y = \sqrt[3]{x} + 2x^4 \ln\left(\sqrt{x^3 - 1}\right)$ Find y'(x)

Use Log differentiation to find the derivative of the function:

7) $y = \frac{2x^2\left(\sqrt[3]{7-3x}\right)}{\sqrt{(9-2x^4)^3}}$

Use Implicit Differentiation to find $\frac{dy}{dx}$:

8) $ln\left(\frac{x}{\sqrt{y}}\right) - 7y + 2x^3 = 20 - 2y^4$

9) Find an inverse function for f(x): $f(x) = 2(5 - x)^3$

Use function f(x) and the given real number a to find $(f^{-1})'(a)$

10) $f(x) = 2x^3 - 5x - 3$ a = -9

Key

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Log Properties		
1. $\ln(1) = 0$ 2. $\ln(ab) = \ln a + \ln b$	3. $\ln(a^n) = n \ln a$ 4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ 5. $\ln(e) = 1$	$\frac{d}{dx}[\ln u] = \frac{u'}{u}$

Expanding a Logarithmic Expression: Use properties of logarithms to expand the logarithmic expression

1. $y = \ln\left(x\left(\sqrt[7]{3x - 5x^3}\right)\right)$

$y = \ln x + \ln(3x-5x^3)^{1/7}$

$\boxed{y = \ln x + \frac{1}{7}\ln(3x-5x^3)}$

2. $f(x) = \ln\sqrt{\left(\frac{2-3x^2}{\sqrt[3]{6-x^4}}\right)^5}$

$y = \ln\left(\frac{2-3x^2}{(6-x^4)^{1/3}}\right)^{5/2}$

$y = \frac{5}{2}\ln\left(\frac{2-3x^2}{(6-x^4)^{1/3}}\right)$

$y = \frac{5}{2}\ln(2-3x^2) - \frac{5}{2}\ln(6-x^4)^{1/3}$

$y = \frac{5}{2}\ln(2-3x^2) - \frac{5}{2}\cdot\frac{1}{3}\ln(6-x^4)$

$\boxed{y = \frac{5}{2}\ln(2-3x^2) - \frac{5}{6}\ln(6-x^4)}$

Condensing a Log Expression: Use properties of logarithms to write expression as a logarithm of a single quantity

3. $\frac{1}{4}[5\ln(5x-6) - 3\ln(2-x^2) - \frac{3}{2}\ln(12-7x)]$

$\frac{1}{4}\left[\ln(5x-6)^5 - \ln(2-x^2)^3 - \ln(12-7x)^{3/2}\right]$

$\frac{1}{4}\ln\left[\frac{(5x-6)^5}{(2-x^2)^3(12-7x)^{3/2}}\right]$

$\boxed{\ln\left[\frac{(5x-6)^5}{(2-x^2)^3(12-7x)^{3/2}}\right]^{1/4}}$

Find the derivative of the functions below:

4) $y = \ln\left(\frac{3x\sqrt{2x-5x^3}}{\sqrt[3]{1-9x}}\right)$

$y = \ln(3x) + \ln(2x-5x^3)^{1/2} - \ln(1-9x)^{1/3}$

$y = \ln(3x) + \frac{1}{2}\ln(2x-5x^3) - \frac{1}{3}\ln(1-9x)$

$y' = \frac{3}{3x} + \frac{1}{2}\cdot\frac{2-15x^2}{2x-5x^3} - \frac{1}{3}\cdot\frac{-9}{1-9x}$

$\boxed{y' = \frac{1}{x} + \frac{2-15x^2}{2(2x-5x^3)} + \frac{3}{1-9x}}$

5) $f(x) = \ln\left(\frac{\sqrt{x-2x^4}}{7-\pi x}\right)^5$

$f(x) = 5\ln(x-2x^4)^{1/2} - 5\ln(7-\pi x)$

$f(x) = 5\cdot\frac{1}{2}\ln(x-2x^4) - 5\ln(7-\pi x)$

$f'(x) = \frac{5}{2}\cdot\frac{1-8x^3}{x-2x^4} - 5\cdot\frac{-\pi}{7-\pi x}$

$\boxed{f'(x) = \frac{5(1-8x^3)}{2(x-2x^4)} + \frac{5\pi}{7-\pi x}}$

6) $y = \sqrt[3]{x} + 2x^4 \ln(\sqrt{x^3 - 1})$ Find y'(x)

$y = x^{1/3} + 2x^4 \cdot \ln(x^3-1)^{1/2}$

$y = x^{1/3} + 2x^4 \cdot \frac{1}{2}\ln(x^3-1)$

$y = x^{1/3} + \overset{f}{x^4} \cdot \overset{g}{\ln(x^3-1)}$

$y' = \frac{1}{3}x^{-2/3} + \overset{f'}{4x^3} \cdot \overset{g}{\ln(x^3-1)} + \overset{f}{x^4} \cdot \overset{g'}{\frac{3x^2}{x^3-1}}$

$$\boxed{y' = \frac{1}{3x^{2/3}} + 4x^3\ln(x^3-1) + \frac{3x^6}{x^3-1}}$$

Use Log differentiation to find the derivative of the function:

7) $y = \frac{2x^2(\sqrt[3]{7-3x})}{\sqrt{(9-2x^4)^3}}$

$\ln y = \ln\left[\frac{2x^2 \cdot (7-3x)^{1/3}}{(9-2x^4)^{3/2}}\right]$

$\ln y = \ln(2x^2) + \ln(7-3x)^{1/3} - \ln(9-2x^4)^{3/2}$

$\ln y = \ln(2x^2) + \frac{1}{3}\ln(7-3x) - \frac{3}{2}\ln(9-2x^4)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{4x}{2x^2} + \frac{1}{3} \cdot \frac{-3}{7-3x} - \frac{3}{2} \cdot \frac{-8x^3}{9-2x^4}$

$\frac{dy}{dx} = y\left[\frac{2}{x} - \frac{1}{7-3x} + \frac{12x^3}{9-2x^4}\right]$

$$\boxed{\frac{dy}{dx} = \frac{2x^2(\sqrt[3]{7-3x})}{\sqrt{(9-2x^4)^3}}\left[\frac{2}{x} - \frac{1}{7-3x} + \frac{12x^3}{9-2x^4}\right]}$$

Use Implicit Differentiation to find $\frac{dy}{dx}$:

8) $ln\left(\frac{x}{\sqrt{y}}\right) - 7y + 2x^3 = 20 - 2y^4$

$\ln x - \ln y^{1/2} - 7y + 2x^3 = 20 - 2y^4$

$\ln x - \frac{1}{2}\ln y - 7y + 2x^3 = 20 - 2y^4$

$\frac{1}{x} - \frac{1}{2} \cdot \frac{1}{y}\left(\frac{dy}{dx}\right) - 7\left(\frac{dy}{dx}\right) + 6x^2 = 0 - 8y^3\left(\frac{dy}{dx}\right)$

$\frac{-1}{2y}\left(\frac{dy}{dx}\right) - 7\left(\frac{dy}{dx}\right) + 8y^3\left(\frac{dy}{dx}\right) = \frac{-1}{x} - 6x^2$

$\frac{dy}{dx}\left(\frac{-1}{2y} - 7 + 8y^3\right) = \frac{-1}{x} - 6x^2$

$$\boxed{\frac{dy}{dx} = \frac{\frac{-1}{x} - 6x^2}{\frac{-1}{2y} - 7 + 8y^3}}$$

9) Find an inverse function for f(x): $f(x) = 2(5-x)^3$

$y = 2(5-x)^3$

$x = 2(5-y)^3$

$\frac{x}{2} = (5-y)^3$

$\sqrt[3]{\frac{x}{2}} = \sqrt[3]{(5-y)^3}$

$\sqrt[3]{\frac{x}{2}} = 5 - y$

$y = 5 - \sqrt[3]{x/2}$

$$\boxed{(f^{-1})(x) = 5 - \sqrt[3]{\frac{x}{2}}}$$

Use function f(x) and the given real number a to find $(f^{-1})'(a)$

10) $f(x) = 2x^3 - 5x - 3$ a = −9

$-9 = 2x^3 - 5x - 3$

$0 = 2x^3 - 5x + 6$

$\underline{\underline{x = -2}}$

$f'(x) = 6x^2 - 5$

$f'(-2) = 6(-2)^2 - 5$

$f'(-2) = 19$

$$\boxed{(f^{-1})'(-9) = \frac{1}{19}}$$

$f(-2) = -9$	$(f^{-1})(-9) = -2$
$f'(-2) = 19$	$(f^{-1})'(-9) =$