Non-AP Calculus 5.1/5.3 Natural Logs Derivatives Quiz Review WS #4

Log Properties		
1. $\ln(1) = 0$	3. $\ln(a^n) = n\ln a$	$\frac{d}{dx}[\ln u] = \frac{u'}{u}$
2. $\ln(ab) = \ln a + \ln b$	4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$	
	5. $\ln(e) = 1$	

Expanding a Logarithmic Expression: Use properties of logarithms to expand the logarithmic expression

1. $y = \ln\left(x\left(\sqrt[5]{(3x-5x^3)^4}\right)\right)^7$

2. $f(x) = \ln\sqrt{\left(\frac{\sqrt[9]{6-x^4}}{5-\pi x^4}\right)^3}$

Condensing a Log Expression: Use properties of logarithms to write expression as a logarithm of a single quantity

$$\frac{2}{3}\left[-2\ln(5-x^3) - \frac{3}{2}\ln(12-7x) - 8\ln(x-3x^3)\right]$$

Find the derivative of the functions below:

4) $y = \ln\left(\frac{\sqrt{4x-2x^3}}{2x(\sqrt[5]{2x-5})}\right)^4$

5) $f(x) = \ln\left[(3-\pi x)(\sqrt{(2x-5x^4)^3}\right]^5$

6) $y = \sqrt{x^7} + 3x^5 \ln(\sqrt{x^4 - 1})$ Find y'(x)

Use Log differentiation to find the derivative of the function:

7) $y = \frac{(\sqrt[4]{7-2x})}{(2x^2)\sqrt{(3-2x^3)^5}}$

Use Implicit Differentiation to find $\frac{dy}{dx}$:

8) $ln\left(\frac{\sqrt{y}}{x}\right) - y + 4x^3 + x = 5\pi - 3y^2$

9) Find an inverse function for f(x): $f(x) = 3(\sqrt[3]{5 - x})$

Use function f(x) and the given real number a to find $(f^{-1})'(a)$

10) $f(x) = 3x^3 - 2x - 3$ a = 17

Key

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Log Properties

1. $\ln(1) = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln(a^n) = n \ln a$
4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
5. $\ln(e) = 1$

$$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$

Expanding a Logarithmic Expression: Use properties of logarithms to expand the logarithmic expression

1. $y = \ln\left(x\left(\sqrt[5]{(3x-5x^3)^4}\right)\right)^7$

$y = 7\ln x + 7\ln(3x-5x^3)^{4/5}$

$y = 7\ln x + 7 \cdot \frac{4}{5}\ln(3x-5x^3)$

$\boxed{y = 7\ln x + \frac{28}{5}\ln(3x-5x^3)}$

2. $f(x) = \ln\sqrt{\left(\frac{\sqrt[9]{6-x^4}}{5-\pi x^4}\right)^3}$

$f(x) = \ln\left(\frac{\sqrt[9]{6-x^4}}{5-\pi x^4}\right)^{3/2}$

$f(x) = \frac{3}{2}\ln(6-x^4)^{1/9} - \frac{3}{2}\ln(5-\pi x^4)$

$f(x) = \frac{3}{2}\cdot\frac{1}{9}\ln(6-x^4) - \frac{3}{2}\ln(5-\pi x^4)$

$\boxed{f(x) = \frac{1}{6}\ln(6-x^4) - \frac{3}{2}\ln(5-\pi x^4)}$

Condensing a Log Expression: Use properties of logarithms to write expression as a logarithm of a single quantity

$\frac{2}{3}[-2\ln(5-x^3) - \frac{3}{2}\ln(12-7x) - 8\ln(x-3x^3)]$

$\frac{2}{3}\left[-\ln(5-x^3)^2 - \ln(12-7x)^{3/2} - \ln(x-3x^3)^8\right]$

$\frac{2}{3}\ln\left[\frac{1}{(5-x^3)^2(12-7x)^{3/2}(x-3x^3)^8}\right]$

$\boxed{\ln\left[\frac{1}{(5-x^3)^2(12-7x)^{3/2}(x-3x^3)^8}\right]^{2/3}}$

Find the derivative of the functions below:

4) $y = \ln\left(\frac{\sqrt{4x-2x^3}}{2x(\sqrt[5]{2x-5})}\right)^4$

$y = 4\ln(4x-2x^3)^{1/2} - 4\ln(2x) - 4\ln(2x-5)^{1/5}$

$y = 4\cdot\frac{1}{2}\ln(4x-2x^3) - 4\ln(2x) - 4\cdot\frac{1}{5}\ln(2x-5)$

$y' = 2\cdot\frac{4-6x^2}{4x-2x^3} - 4\cdot\frac{2}{2x} - \frac{4}{5}\cdot\frac{2}{2x-5}$

$\boxed{y' = \frac{4-6x^2}{2x-x^3} - \frac{4}{x} - \frac{8}{5(2x-5)}}$

5) $f(x) = \ln\left[(3-\pi x)(\sqrt{(2x-5x^4)^3}\right]^5$

$f(x) = 5\ln(3-\pi x) + 5\ln(2x-5x^4)^{3/2}$

$f(x) = 5\ln(3-\pi x) + 5\cdot\frac{3}{2}\ln(2x-5x^4)$

$f'(x) = 5\cdot\frac{-\pi}{3-\pi x} + \frac{15}{2}\cdot\frac{2-20x^3}{2x-5x^4}$

$\boxed{f'(x) = \frac{-5\pi}{3-\pi x} + \frac{15(1-10x^3)}{2x-5x^4}}$

6) $y = \sqrt{x^7} + 3x^5 \ln(\sqrt{x^4 - 1})$ Find y'(x)

$y = x^{7/2} + 3x^5 \cdot \ln(x^4-1)^{1/2}$

$y = x^{7/2} + 3x^5 \cdot \frac{1}{2}\ln(x^4-1)$

$y = x^{7/2} + \underbrace{\frac{3}{2}x^5}_{f} \cdot \underbrace{\ln(x^4-1)}_{g}$

$y' = \frac{7}{2}x^{5/2} + \underbrace{\frac{15}{2}x^4}_{f'} \underbrace{\ln(x^4-1)}_{g} + \underbrace{\frac{3}{2}x^5}_{f} \cdot \underbrace{\frac{4x^3}{x^4-1}}_{g'}$

$\boxed{y' = \frac{7}{2}x^{5/2} + \frac{15x^4\ln(x^4-1)}{2} + \frac{6x^8}{x^4-1}}$

Use Log differentiation to find the derivative of the function:

7) $y = \frac{(\sqrt[4]{7-2x})}{(2x^2)\sqrt{(3-2x^3)^5}}$

$\ln y = \ln\left[\frac{(7-2x)^{1/4}}{2x^2 \cdot (3-2x^3)^{5/2}}\right]$

$\ln y = \ln(7-2x)^{1/4} - \ln(2x^2) - \ln(3-2x^3)^{5/2}$

$\ln y = \frac{1}{4}\ln(7-2x) - \ln(2x^2) - \frac{5}{2}\ln(3-2x^3)$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{4}\cdot\frac{-2}{7-2x} - \frac{4x}{2x^2} - \frac{5}{2}\cdot\frac{-6x^2}{3-2x^3}$

$\frac{dy}{dx} = y\cdot\left[\frac{-1}{2(7-2x)} - \frac{2}{x} + \frac{15x^2}{3-2x^3}\right]$

$\boxed{\frac{dy}{dx} = \left[\frac{\sqrt[4]{7-2x}}{(2x^2)(\sqrt{3-2x^3})^5}\right]\left[\frac{-1}{2(7-2x} - \frac{2}{x} + \frac{15x^2}{3-2x^3}\right]}$

Use Implicit Differentiation to find $\frac{dy}{dx}$:

8) $ln\left(\frac{\sqrt{y}}{x}\right) - y + 4x^3 + x = 5\pi - 3y^2$

$\ln\sqrt{y} - \ln x - y + 4x^3 + x = 5\pi - 3y^2$

$\ln y^{1/2} - \ln x - y + 4x^3 + x = 5\pi - 3y^2$

$\frac{1}{2}\ln y - \ln x - y + 4x^3 + x = 5\pi - 3y^2$

$\frac{1}{2}\cdot\frac{1}{y}\left(\frac{dy}{dx}\right) - \frac{1}{x} - 1\left(\frac{dy}{dx}\right) + 12x^2 + 1 = 0 - 6y\left(\frac{dy}{dx}\right)$

$\frac{1}{2y}\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) + 6y\left(\frac{dy}{dx}\right) = \frac{1}{x} - 12x^2 - 1$

$\frac{dy}{dx}\left(\frac{1}{2y} - 1 + 6y\right) = \frac{1}{x} - 12x^2 - 1$

$\boxed{\frac{dy}{dx} = \frac{\frac{1}{x} - 12x^2 - 1}{\frac{1}{2y} - 1 + 6y}}$

9) Find an inverse function for f(x): $f(x) = 3(\sqrt[3]{5-x})$

$y = 3\sqrt[3]{5-x}$

$x = 3\sqrt[3]{5-y}$

$\frac{x}{3} = \sqrt[3]{5-y}$

$\left(\frac{x}{3}\right)^3 = 5-y$

$y = 5 - \left(\frac{x}{3}\right)^3$

$\boxed{(f^{-1})(x) = 5 - \left(\frac{x}{3}\right)^3}$

Use function f(x) and the given real number a to find $(f^{-1})'(a)$

10) $f(x) = 3x^3 - 2x - 3$ a = 17

$f(2) = 17$ | $(f^{-1})(17) = 2$

$f'(2) = 34$ | $(f^{-1})'(17) = \frac{1}{34}$

$17 = 3x^3 - 2x - 3$

$0 = 3x^3 - 2x - 20$

$x = 2$

$f'(x) = 9x^2 - 2$

$f'(2) = 9(2)^2 - 2$

$f'(2) = 34$

$\boxed{(f^{-1})'(17) = \frac{1}{34}}$