Ch. 5.1-5.6 BC
Quiz Review
#2

1) Find $\frac{dy}{dx}$ $y = \left[\frac{x(x-1)^{3/2}}{\sqrt{x+1}}\right]^2$

2) Find $(f^{-1})'(2)$ for $f(x) = x^3 + 2x - 1$

3) Find $(f^{-1})'(6)$ for $f(x) = x^3 - \frac{4}{x}$

4) Find $\frac{dy}{dx}$ $y = 3(1+x)^{x+1}$

5) Use implicit rule to find and show that

$\frac{d}{dx} \arcsin u = \frac{u'}{\sqrt{1-u^2}}$

Solutions

1) $y = \left[\dfrac{x(x-1)^{3/2}}{\sqrt{x+1}}\right]^2$

$$\ln y = 2\ln\left[\frac{x(x-1)^{3/2}}{(x+1)^{1/2}}\right]$$

$$\ln y = 2\left[\ln x + \tfrac{3}{2}\ln(x-1) - \tfrac{1}{2}\ln(x+1)\right]$$

$$\ln y = 2\ln x + 3\ln(x-1) - \ln(x+1)$$

$$\frac{1}{y}\frac{dy}{dx} = 2\left(\frac{1}{x}\right) + 3\left(\frac{1}{x-1}\right) - \left(\frac{1}{x+1}\right)$$

$$\frac{dy}{dx} = \left[\frac{x(x-1)^{3/2}}{\sqrt{x+1}}\right]^2\left[\frac{2}{x} + \frac{3}{x-1} - \frac{1}{x+1}\right]$$

2) Find $(f^{-1})'(2)$ for $f(x) = x^3 + 2x - 1$ let $g(x) = (f^{-1})(x)$

| | |
|---|---|
| $f(\underline{1}) = 2$ | $g(2) = \underline{1}$ |
| $f'(1) = \underline{5}$ | $g'(2) = \boxed{1/5}$ |

$$f'(x) = 3x^2 + 2$$

$$f'(1) = 3(1)^2 + 2 = 5$$

3) $(f^{-1})'(6)$ for $f(x) = x^3 - \frac{4}{x}$

let $g(x) = (f^{-1})(x)$

| | |
|---|---|
| $f'(\underline{2}) = 6$ | $g(6) = \underline{2}$ |
| $f'(2) = \underline{13}$ | $g'(6) = \boxed{\frac{1}{13}}$ |

$f(x) = x^3 - 4x^{-1}$

$f'(x) = 3x^2 + 4x^{-2}$

$f'(2) = 12 + \frac{4}{4} = 13$

4) $y = 3(1+x)^{x+1}$

$\ln y = \ln 3(1+x)^{x+1}$

$\ln y = \ln 3 + \ln(1+x)^{x+1}$

$\ln y = \ln 3 + (x+1)\ln(1+x)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = 0 + (1)\ln(1+x) + (x+1)\left(\frac{1}{1+x}\right)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \ln(1+x) + 1$

$\frac{dy}{dx} = y\left[\ln(1+x)+1\right]$

$$\boxed{\frac{dy}{dx} = 3(1+x)^{x+1}\left[\ln(1+x)+1\right]}$$

5) $$\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$$

$$y = \arcsin u$$

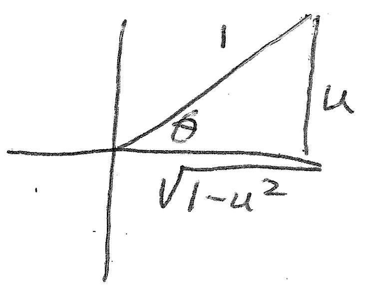
$$\sin y = \sin(\arcsin u)$$

$$\sin y = u$$

$$\cos y\left(\frac{dy}{dx}\right) = \frac{du}{dx}$$

$$\cos y\left(\frac{dy}{dx}\right) = u'$$

$$\frac{dy}{dx} = \frac{u'}{\cos y} = u'[\sec y] = u'[\sec(\arcsin u)]$$




$$\sec\theta = \frac{1}{\sqrt{1-u^2}}$$

$$\frac{dy}{dx} = u'\left[\frac{1}{\sqrt{1-u^2}}\right] = \boxed{\frac{u'}{\sqrt{1-u^2}}}$$