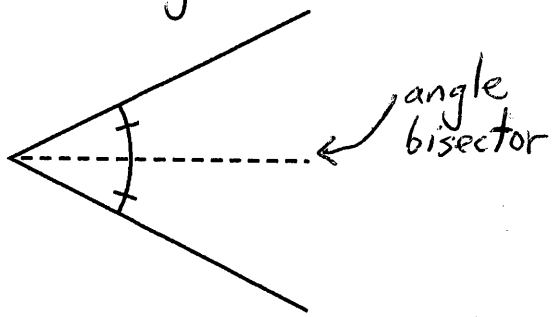
Geometry
Points of Concurrency Notes
Incenter and Circumcenter

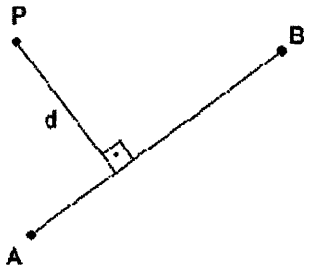
Key

Essential Question: What are the properties of an angle bisector?

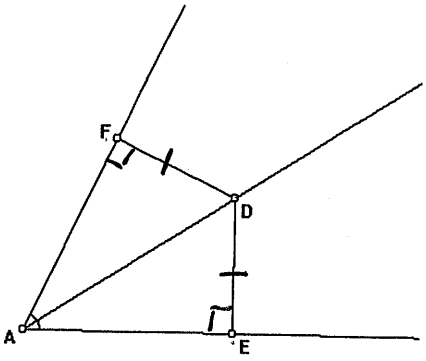
- An angle bisector is a segment, ray, line or plane that intersects an angle to form two adjacent congruent angles.



- The **distance from a point to a line** is the length of the perpendicular segment from the point to the line.

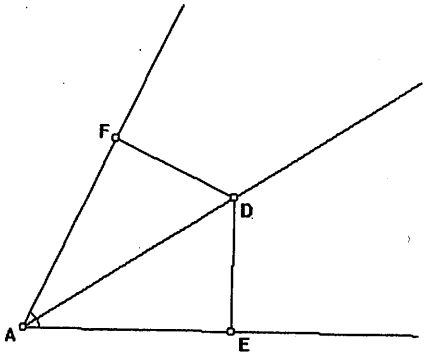


Angle Bisector Theorem: If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.



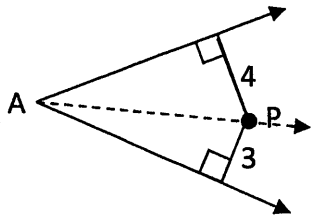
In other words:
If $\angle FAD \cong \angle EAD$, then $FD \cong ED$.

Converse of the Angle Bisector Theorem: If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.



In other words:
If FD = ED, then $\angle FAD \cong \angle EAD$.

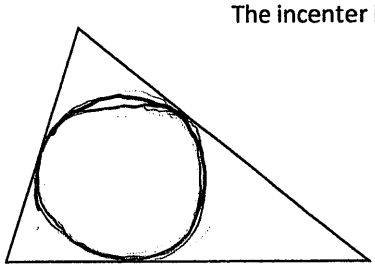
Example 1: Can you conclude that P is on the bisector of $\angle A$? Explain.



No, Distance from bisector to the sides of the angle are not equal.

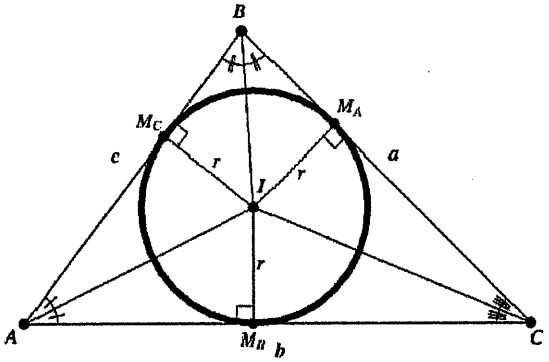
- When three or more lines (or rays or segments) intersect in the same point, they are called **concurrent lines**. The point of intersection of the lines is called the point of concurrency.
- The point of concurrency of the **angle bisectors** of a triangle is called the incenter of the triangle.

Sketch a picture of the incenter.



The incenter is always inside a triangle.

Theorem: The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.



Since the incenter is equidistant from the sides of a triangle, then an inscribed circle can be drawn. An inscribed circle is a circle with a center at the incenter and a radius that is the distance to the sides

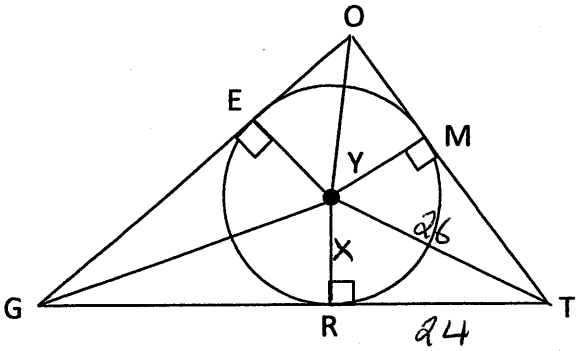
Example 2: If point Y is the incenter, find YR and MT if TY = 26 and RT = 24.

$x^2 + 24^2 = 26^2$

$x^2 = 100$

$x = 10$

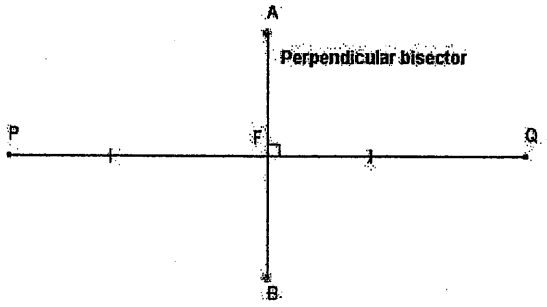
$\boxed{YR = 10}$



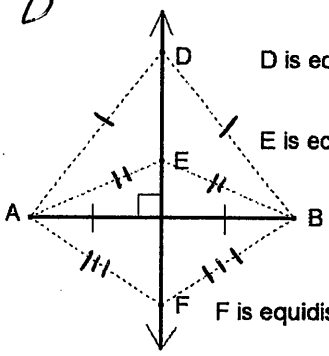
Key

Essential Question: What are the properties of a perpendicular bisector?

- A **perpendicular bisector** is a segment, ray, line or plane that is perpendicular to a segment at its midpoint.



Perpendicular Bisector Theorem: If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.



D is equidistant to A and B

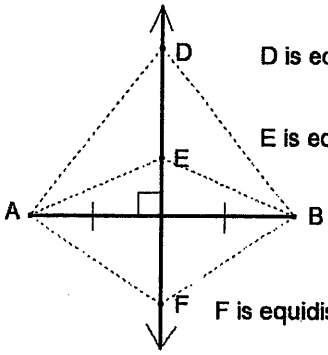
E is equidistant to A and B

F is equidistant to A and B

In other words:

If $\overleftrightarrow{DF}$ is the perpendicular bisector of $\overline{AB}$, then AD = BD, AE = BE, and AF = BF.

Converse of the Perpendicular Bisector Theorem: If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.



D is equidistant to A and B

E is equidistant to A and B

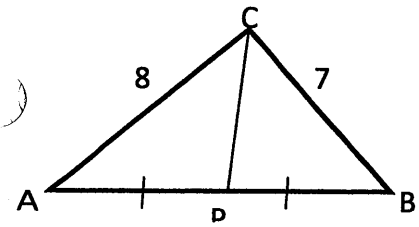
F is equidistant to A and B

In other words:

If AD = BD, AE = BE, and AF = BF, then, $\overleftrightarrow{DF}$ is the perpendicular bisector of $\overline{AB}$.

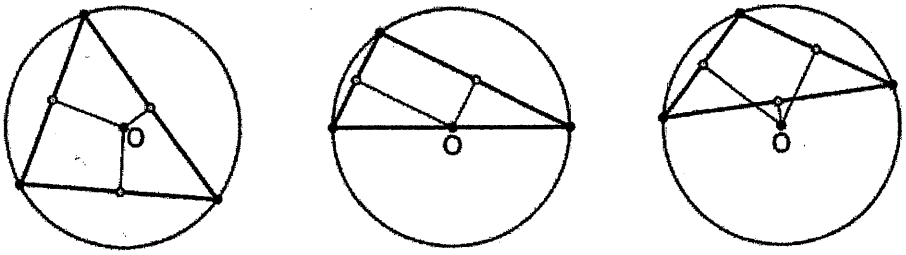
Example 1: Can you conclude that C is on the perpendicular bisector of $\overline{AB}$? Explain.

no, since $AC \neq BC$



- A **perpendicular bisector of a triangle** is a segment, ray, or line that is perpendicular to a side of the triangle at the midpoint of the side.

- The point of concurrency of the **perpendicular bisectors** of a triangle is called the circumcenter of the triangle.

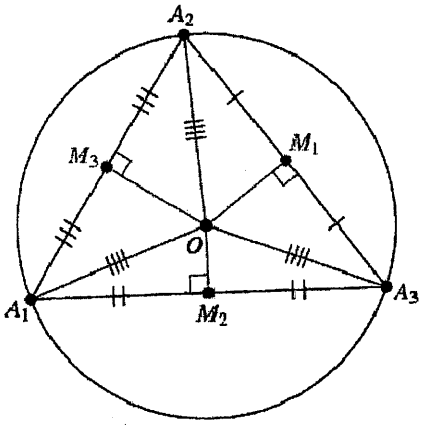


the circumcenter of an acute triangle is inside the triangle

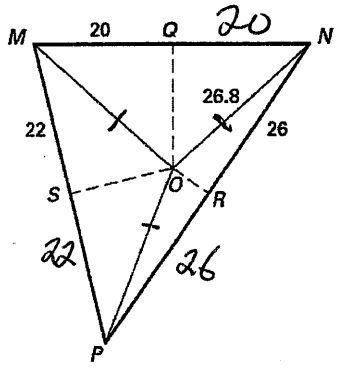
the circumcenter of a right triangle is on the hypotenuse

the circumcenter of an obtuse triangle is outside the triangle

Theorem: The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.



Example 2: If point O is the circumcenter, find MO, PO, PS, PR, and MN.



MO = 26.8
PO = 26.8
PS = 22
PR = 26
MN = 20

Example 3: Cassie, Jim, and Sal are old college buddies that have moved apart. They want to get together. To be fair, they want to find a location that is the same distance for each of them to travel. How could they locate that spot?

Find incenter or circumcenter