Key

5.1 AP Practice Problems (p. 319)

1. The critical numbers of $g(x) = \sin x + \cos x$ on the open interval $(0, 2\pi)$ are

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$

$\boxed{\text{(C)}}$ $\frac{\pi}{4}$ and $\frac{5\pi}{4}$ (D) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ and $\frac{7\pi}{4}$

*set $g'(x)=0$ to find critical numbers

$g'(x) = \cos x + (-\sin x)$

$0 = \cos x - \sin x$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$\tan x = 1$

$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

2. On the closed interval $[0, 2\pi]$, the absolute minimum of $f(x) = e^{\sin x}$ occurs at

(A) 0 (B) $\frac{\pi}{2}$ $\boxed{\text{(C)}}$ $\frac{3\pi}{2}$ (D) 2π

*EVT (candidates test)

i) endpoints

ii) critical points ($f'(x)=0$)

$f'(x) = e^{\sin x} \cdot \cos x$

$0 = (e^{\sin x})(\cos x)$

$e^{\sin x} = 0$ — none

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$f(0) = e^{\sin 0} = e^0 = 1$

$f(\frac{\pi}{2}) = e^{\sin(\pi/2)} = e^1 = e$

$f(\frac{3\pi}{2}) = e^{\sin(\frac{3\pi}{2})} = e^{-1} = \frac{1}{e}$

$f(2\pi) = e^{\sin(2\pi)} = e^0 = 1$

$\boxed{\text{Absolute minimum is } \frac{1}{e} \text{ at } x = \frac{3\pi}{2}}$

3. The maximum value of $f(x) = 2x^3 - 15x^2 + 36x$ on the closed interval $[0, 4]$ is

max y-value!!

(A) 28 (B) 30 $\boxed{\text{(C)}}$ 32 (D) 48

*test critical points and endpoints.

*EVT candidates test

$f'(x) = 6x^2 - 30x + 36$

$0 = 6(x^2 - 5x + 6)$

$0 = 6(x-3)(x-2)$

$x = 3, x = 2$

$f(0) = 0$

$f(2) = 2(2)^3 - 15(2)^2 + 36(2) = 28$

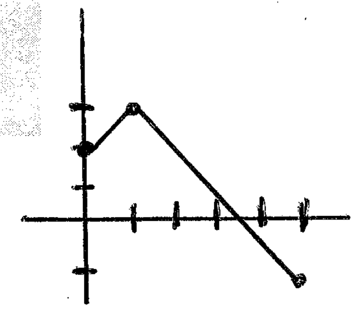
$f(3) = 2(3)^3 - 15(3)^2 + 36(3) = 27$

$f(4) = 2(4)^3 - 15(4)^2 + 36(4) = 32$

$\boxed{\text{Absolute maximum is } 32 \text{ at } x = 4}$

4. On the closed interval [0, 5], the function $f(x) = 3 - |x - 1|$ has:

$\boxed{\text{(A)}}$ both an absolute maximum and an absolute minimum.
(B) an absolute maximum but no absolute minimum.
(C) no absolute maximum but an absolute minimum.
(D) an absolute maximum and two absolute minima.



* sketch graph

$y = -|x-1| + 3$

vertex is $(1, 3)$

$f(0) = 2$

$f(5) = -4+3 = -1$

Abs max is $y = 3$

Abs min is $y = -1$

5. The critical numbers of the function

$$f(x) = \begin{cases} x^2 + 1 & \text{if } -2 \le x \le 1 \\ 3x^2 - 4x + 3 & \text{if } 1 < x \le 3 \end{cases} \text{ are}$$

(A) 0 and 1 (B) 0 and $\frac{2}{3}$ (C) 0, $\frac{2}{3}$, and 1 $\boxed{\text{(D) } 0}$

$$f'(x) = \begin{cases} 2x & , -2 \le x \le 1 \\ 6x - 4 & , 1 < x \le 3 \end{cases}$$

* critical number is where $f'(x) = 0$ or where $f'(x)$ does not exist

$2x = 0$

$\boxed{x = 0}$

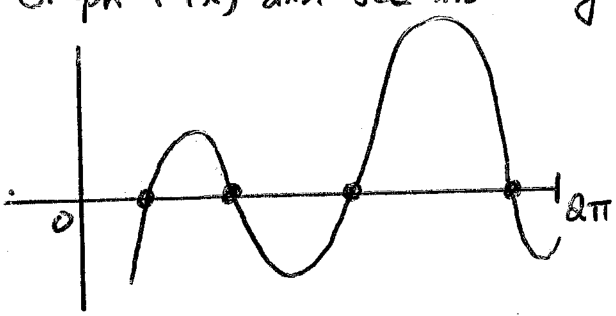
$6x - 4 = 0$

$x = 4/6 = \boxed{2/3}$ is not on the interval of $1 < x < 3$

6. $f'(x) = x \sin^2 x - \frac{1}{x}$ is the derivative of a function f. How many critical numbers does f have on the open interval $(0, 2\pi)$?

(A) 1 (B) 3 $\boxed{\text{(C)}}$ 4 (D) 5

* Graph $f'(x)$ and see how many times $f'(x) = 0$ (how many times does $f'(x)$ cross the x-axis)



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