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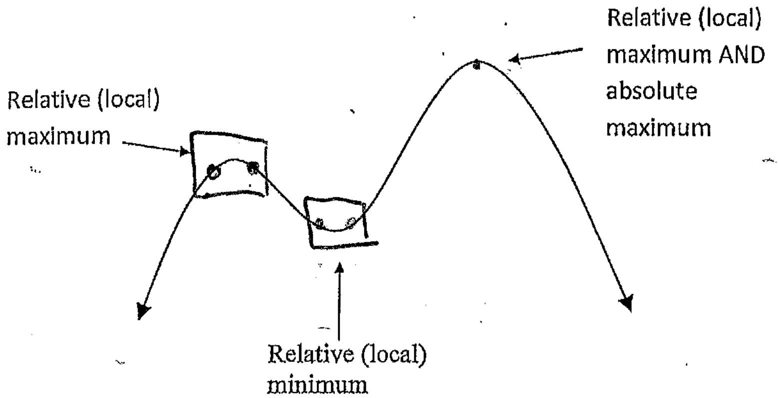
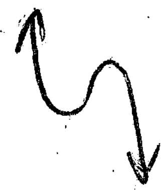
Calculus ~~Ch4.3A~~ 5.1 Notes: Extrema and Critical Points

Key

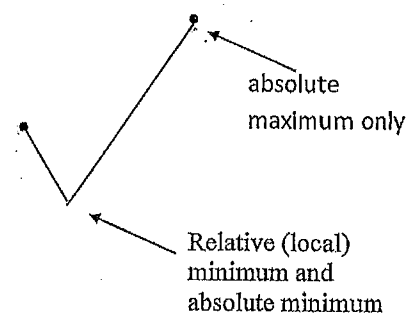
1

Extrema : maximums and minimums

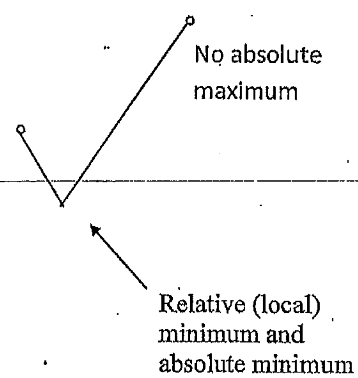




Closed interval



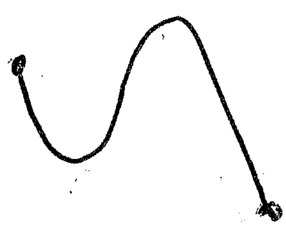
Open Interval



Relative (local) extrema: any "hills and valleys" of graph

Absolute (global) extrema: highest or lowest points on the entire graph

*holes and $\pm\infty$ can **not** be considered as absolute extrema.



(EVT)

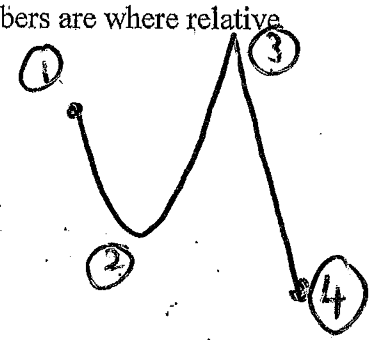
Extreme Value Theorem: If a function is continuous on a closed interval, then it has **both** an (absolute) minimum and an (absolute) maximum on that interval.

Fermat's Theorem: If a function is continuous on a closed interval, then the absolute extreme will **either** be at the **a)** critical numbers or **b)** at an endpoint.

Critical numbers (values) : x-values in the domain of a function where the derivative of a function is either 0 or undefined.

*Relative extrema ONLY occur at critical numbers, but not all critical numbers are where relative extrema occur.

*Maximum and minimum values refer to the **y-values** of the point.



(2)

Steps:

1. Find critical points
   a. Set f'(x) = 0
   b. Find where f'(x) is undefined (Set denominator of f'(x) = 0)
2. Plug all critical points and endpoints into f(x)
3. Compare y-values to determine absolute maximum(s) and absolute minimum(s)

Find all critical numbers for each. What are the values of the absolute extrema?

Example 1: $f(x) = 3x^4 - 4x^3$ on [0, 2]

$f'(x) = 12x^3 - 12x^2$

$0 = 12x(x-1)$

$x = 0, 1$

$f(0) = 0$

$f(1) = -1$ (min)

$f(2) = 16$ (max)

Example 2: $f(x) = (x-1)^{\frac{2}{3}}$ on [-1, 0] f(x) continuous on [-1, 0]

$f'(x) = \frac{2}{3}(x-1)^{-1/3}(1)$

$f'(x) = \frac{2}{3(x-1)^{1/3}}$

$x = 1$ (critical pt.)

$f(-1) = \sqrt[3]{4}$ (Abs. max is $\sqrt[3]{4}$ at $x = -1$)

$f(0) = 1$ (Abs. min is 1 at $x = 0$)

f(x) continuous on [0, 3]

Example 3: $f(x) = \frac{4}{3}x\sqrt{3-x}$ on [0, 3] $f(x) = \frac{4}{3}x(3-x)^{1/2}$

$f'(x) = \frac{4}{3}(3-x)^{1/2} + \frac{4}{3}x \cdot \frac{1}{2}(3-x)^{-1/2}(-1)$

$0 = \frac{4\sqrt{3-x}}{3} - \frac{2x}{3\sqrt{3-x}}$

$= \frac{4(3-x) - 2x}{3\sqrt{3-x}} = \frac{12-4x-2x}{3\sqrt{3-x}}$

$f'(x) = \frac{12-6x}{3\sqrt{3-x}}$

$12 - 6x = 0$ $6x = 12$ $\underline{x = 2}$

$3\sqrt{3-x} = 0$ $\underline{x = 3}$

$f(0) = 0$] (Abs. min)

$f(3) = 0$

$f(2) = \frac{4}{3}(2)\sqrt{1} = \frac{8}{3}$ (Abs. max)

Key

3

Chapter 5 Curve Sketching 5.1 EVT Classwork Problems

Finding Extrema on a Closed Interval In Exercises 17–36, find the absolute extrema of the function on the closed interval.

19. $g(x) = 2x^2 - 8x, \ [0, 6]$

$g(x)$ continuous $[0,6]$

$g'(x) = 4x - 8$

$0 = 4x - 8$

$8 = 4x$

$x = 2$

$g(0) = 0$

$g(2) = -8$

$g(6) = 24$

Abs max is 24 at $x=6$

Abs min is -8 at $x=2$

21. $f(x) = x^3 - \frac{3}{2}x^2, \ [-1, 2]$

$f(x)$ continuous $[-1,2]$

$f'(x) = 3x^2 - 3x$

$0 = 3x(x-1)$

$x = 0, 1$

$f(-1) = -5/2$

$f(0) = 0$

$f(1) = -1/2$

$f(2) = 2$

Abs max is 2 at $x=2$

Abs min is $-5/2$ at $x=-1$

23. $y = 3x^{2/3} - 2x, \ [-1, 1]$

$f(x)$ continuous on $[-1,1]$

*set denom = 0

$x = 0$

$y'(x) = 3 \cdot \frac{2}{3}x^{-1/3} - 2$

$y'(x) = \frac{2}{x^{1/3}} - 2$

$2 = \frac{2}{x^{1/3}}$

$2x^{1/3} = 2$

$x^{1/3} = 1$

$x = 1$

$f(0) = 0$

$f(-1) = 5$

$f(1) = 1$

Abs max is 5 at $x=-1$

Abs min is 0 at $x=0$

24. $g(x) = \sqrt[3]{x}, \ [-8, 8]$

$g(x)$ continuous $[-8,8]$

$g(x) = x^{1/3}$

$g'(x) = \frac{1}{3}x^{-2/3}$

$g'(x) = \frac{1}{3x^{2/3}}$

$x = 0$

$g(-8) = \sqrt[3]{-8} = -2$

$g(0) = 0$

$g(8) = \sqrt[3]{8} = 2$

Abs max is 2 at $x=8$

Abs min is -2 at $x=-8$

26. $f(x) = \frac{2x}{x^2 + 1}, \ [-2, 2]$

$f(x)$ continuous $[-2,2]$

$f'(x) = \frac{\overbrace{2}^{f'}\overbrace{(x^2+1)}^{g} - \overbrace{2x}^{f}\overbrace{(2x)}^{g'}}{\underbrace{(x^2+1)^2}_{g^2}} = \frac{2x^2+2-4x^2}{(x^2+1)^2}$

$f'(x) = \frac{2-2x^2}{(x^2+1)^2}$

$2 - 2x^2 = 0$

$2 = 2x^2$

$x^2 = 1$

$x = \pm 1$

$f(-2) = -4/5 \quad f(1) = 1$

$f(-1) = -1 \quad f(2) = 4/5$

Abs max is 1 at $x=1$

Abs min is -1 at $x=-1$

28. $h(t) = \frac{t}{t + 3}, \ [-1, 6]$

VA: $t = -3$

$h(t)$ continuous $[-1,6]$

$h'(t) = \frac{(1)(t+3) - (t)(1)}{(t+3)^2}$

$h'(t) = \frac{t+3-t}{(t+3)^2} = \frac{3}{(t+3)^2}$

No critical point

$0 \neq \frac{+3}{(t+3)^2}$

$h(-1) = -1/2$

$h(6) = 2/3$

Abs max is $2/3$ at $t=6$

Abs min is $-1/2$ at $t=-1$