5.1 Exercise Problems Max and Min Values (Extreme Value Theorem)

p.316-319 # 7, 39, 43, 47, 49, 51, 53

7) x_1 : neither
x_2 : relative (local) max
x_3 : local min and absolute min
x_4 : neither
x_5 : local max (relative max)
x_6 : neither
x_7 : relative (local) min
x_8 : absolute max

$\boxed{\text{Find Abs max and Absolute Min on interval (EVT)}}$

39) $f(x) = x^3 - 3x^2$ on $[1, 4]$

f(x) continuous on $[1, 4]$

$f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

$\cancel{x=0}$, $x = 2$

outside interval

$f(1) = -2$
$f(2) = -4$
$f(4) = 16$

Absolute maximum value is 16
Absolute minimum value is −4

43) $f(x) = x^{2/3}$ on $[-1, 1]$

f(x) continuous on $[-1, 1]$

$f'(x) = \frac{2}{3}x^{-1/3}$

$f'(x) = \frac{2}{3x^{1/3}}$ *set $3x^{1/3} = 0$

$x = 0$

$f(-1) = (-1)^{2/3} = 1$
$f(0) = 0$
$f(1) = (1)^{2/3} = 1$

Abs max value is 1
Abs min value is 0

47) $f(x) = x + \sin x$ $[0, \pi]$

f(x) continuous $[0, \pi]$

$f'(x) = 1 + \cos x$

$0 = 1 + \cos x$

$\cos x = -1$

$x = \cancel{-\pi}, \pi, \cancel{3\pi}, \ldots$

*no critical points to test

$f(0) = 0 + \sin(0) = 0$
$f(\pi) = \pi + \sin(\pi) = \pi$

Abs max value is π
Abs min value is 0

$\boxed{5.1}$

51) $f(x) = \dfrac{x^2}{x-1}$ $\quad [-1, 1/2]$

$f(x)$ continuous $[-1, 1/2]$. * There is a vertical asymptote at $x=1$ but this does not interfere with interval $[-1, 1/2]$

$$f'(x) = \frac{\overbrace{2x}^{f'}\overbrace{(x-1)}^{g} - \overbrace{x^2}^{f}\overbrace{(1)}^{g'}}{\underbrace{(x-1)^2}_{g^2}} \rightarrow f'(x) = \frac{2x^2-2x-x^2}{(x-1)^2} \rightarrow \frac{x^2-2x}{(x-1)^2}$$

$f'(x) = \dfrac{x^2-2x}{(x-1)^2}$ * To find critical points, set numerator and denominator of $f'(x) = 0$, separately

$x^2-2x=0$
$x(x-2)=0$
$x=0, \cancel{x=2}$ outside interval

$f(-1) = \dfrac{(-1)^2}{-1-1} = \dfrac{1}{-2}$

$f(0) = \dfrac{0}{0-1} = 0$

$f(1/2) = \dfrac{(1/2)^2}{1/2 - 1} \rightarrow \dfrac{1/4}{-1/2} \rightarrow \dfrac{-1}{2}$

Abs max value is 0
Abs min value is $-1/2$

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53) $f(x) = (x+3)^2(x-1)^{2/3}$ on $[-4, 5]$ $\quad$ $f(x)$ is continuous $[-4,5]$

$$f'(x) = \overbrace{2(x+3)(1)}^{f'} \cdot \overbrace{(x-1)^{2/3}}^{g} + \overbrace{(x+3)^2}^{f} \cdot \overbrace{\tfrac{2}{3}(x-1)^{-1/3}(1)}^{g'}$$

$$f'(x) = \frac{2(x+3)(x-1)^{2/3}}{1} + \frac{2(x+3)^2}{3(x-1)^{1/3}}$$

$$f'(x) = \frac{2(x+3)(x-1)\cdot 3}{3(x-1)^{1/3}} + \frac{2(x+3)^2}{3(x-1)^{1/3}}$$

$$f'(x) = \frac{6(x+3)(x-1) + 2(x+3)^2}{3(x-1)^{1/3}}$$

$$= \frac{2(x+3)\left[\overbrace{3(x-1) + (x+3)}^{3x-3\ +\ x+3}\right]}{3(x-1)^{1/3}}$$

$$f'(x) = \frac{2(x+3)(4x)}{3(x-1)^{1/3}}$$

$$f'(x) = \frac{8x(x+3)}{3(x-1)^{1/3}}$$

*set numerator $=0$, set denominator $=0$

$8x(x+3)=0$ $\quad|\quad$ $3(x-1)^{1/3}=0$
$x=0, x=-3$ $\quad|\quad$ $x=1$

$f(-4) = \sqrt[3]{25} \approx 2.924$
$f(-3) = 0$
$f(0) = 9$
$f(1) = 0$
$f(5) = 128\sqrt[3]{2} \approx 161.270$

Abs max value is $128\sqrt[3]{2}$
Abs min value is 0