Calculus Ch. 5.1 Natural Log Function

Natural Log graph: Domain: _____ Range: _____

Graph characteristics: _____

Ex. 1: Sketch graph of ln(x – 3) and state domain:

Ex. 2 Draw the function and answer the examples.

a) $\lim_{x\to 0^+} \ln(x) =$ b) $\lim_{x\to 0^-} \ln(x) =$ c) $\lim_{x\to 0} \ln(x) =$ d) $\lim_{x\to \infty} \ln(x) =$

Properties: $\ln(1) = 0$ $\ln(a^n) = n\ln(a)$ $\ln(e) = 1$

$\ln(ab) = \ln(a) + \ln(b)$ $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

Ex. 3 Expand $\ln 3e^2$

Properties: $\ln(1) = 0$ $\ln(a^n) = n\ln(a)$ $\ln(e) = 1$

$\ln(ab) = \ln(a) + \ln(b)$ $\ln(a/b) = \ln(a) - \ln(b)$

Ex. 4 condense $2(\ln(x) - \ln(x + 1) - \ln(x - 1))$

Derivative of the Natural Logarithmic Function:

$$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$

Ex. 5: If $y = \ln(x)$, find y'

Ex. 6: if $y = \ln(x^2 - 5)$, find y'

Ex. 7: if $y = \ln\left(\frac{x^2}{\sqrt{2x^3}}\right)$, find y' (always simplify logs before taking the derivative)

Matching In Exercises 5–8, match the function with its graph. [The graphs are labeled (a), (b), (c), and (d).]

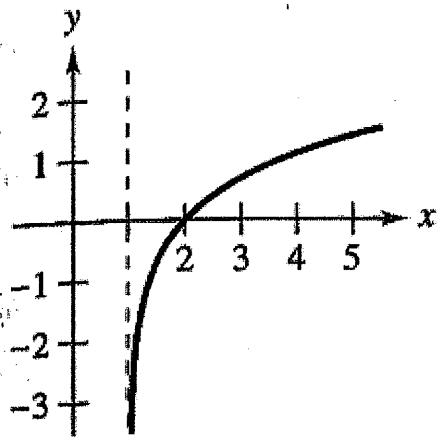
5. $f(x) = \ln x + 1$

6. $f(x) = -\ln x$

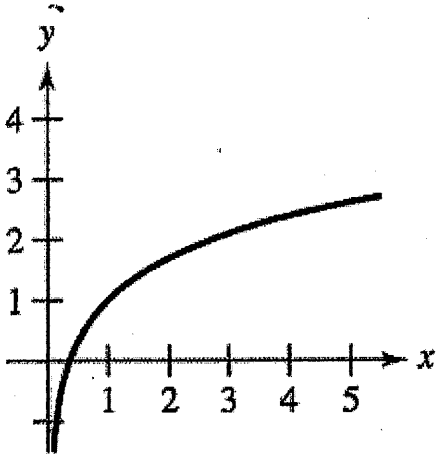
7. $f(x) = \ln(x - 1)$

8. $f(x) = -\ln(-x)$

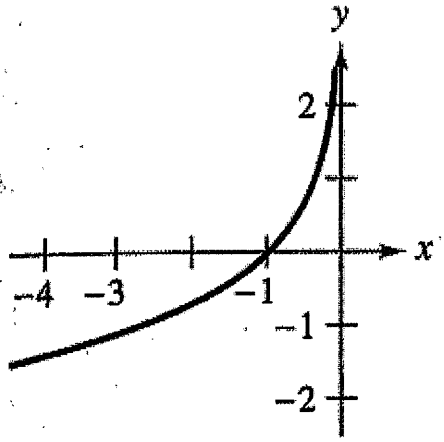
(a)



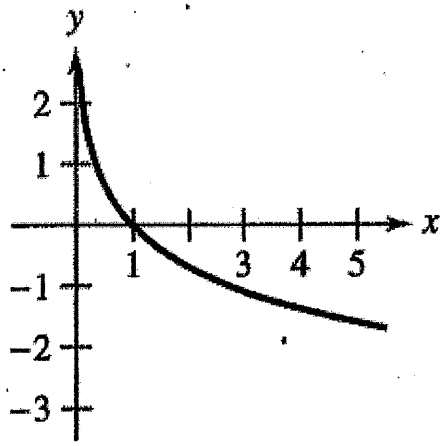
(b)



(c)



(d)



Using Properties of Logarithms **In Exercises 17 and 18, use the properties of logarithms to approximate the indicated logarithms, given that $\ln 2 \approx 0.6931$ and $\ln 3 \approx 1.0986$.**

1. $\ln(1) = 0$

2. $\ln(ab) = \ln a + \ln b$

3. $\ln(a^n) = n \ln a$

4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

17. (a) $\ln 6$	(b) $\ln \frac{2}{3}$
(c) $\ln 81$	(d) $\ln \sqrt{3}$
18. (a) $\ln 0.25$	(b) $\ln 24$
(c) $\ln \sqrt[3]{12}$	(d) $\ln \frac{1}{72}$

Natural Log Properties

1. $\ln(1) = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln(a^n) = n \ln a$
4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

Expanding a Logarithmic Expression In Exercises 19–28, use the properties of logarithms to expand the logarithmic expression.

19. $\ln \frac{x}{4}$

20. $\ln \sqrt{x^5}$

21. $\ln \frac{xy}{z}$

23. $\ln\left(x\sqrt{x^2 + 5}\right)$

25. $\ln \sqrt{\frac{x - 1}{x}}$

26. $\ln(3e^2)$

27. $\ln z(z - 1)^2$

28. $\ln \frac{1}{e}$

Condensing a Logarithmic Expression. In exercises #29-34, write the expression as a logarithm of a single quantity

Natural Log Properties

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4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

29. $\ln(x - 2) - \ln(x + 2)$

30. $3 \ln x + 2 \ln y - 4 \ln z$

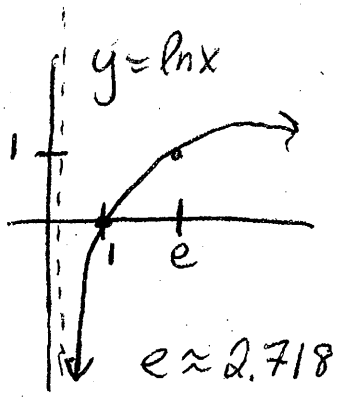
31. $\frac{1}{3}[2 \ln(x + 3) + \ln x - \ln(x^2 - 1)]$

32. $2[\ln x - \ln(x + 1) - \ln(x - 1)]$

33. $2 \ln 3 - \frac{1}{2} \ln(x^2 + 1)$

34. $\frac{3}{2}[\ln(x^2 + 1) - \ln(x + 1) - \ln(x - 1)]$

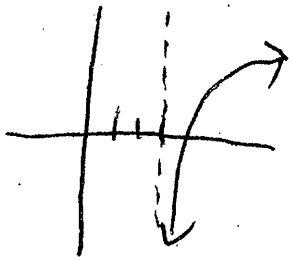
Calculus Ch. 5.1 Natural Log Function



Natural Log graph: Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

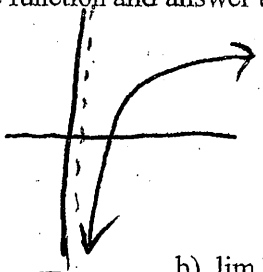
Graph characteristics: always continuous, always increasing
always concave down

Ex. 1: Sketch graph of $\ln(x-3)$ and state domain:



D: $(3, \infty)$

Ex. 2 Draw the function and answer the examples.



a) $\lim_{x \to 0^+} \ln(x) =$ $-\infty$

b) $\lim_{x \to 0^-} \ln(x) =$ DNE

c) $\lim_{x \to 0} \ln(x) =$ DNE

d) $\lim_{x \to \infty} \ln(x) = +\infty$

Properties: $\ln(1) = 0$ $\ln(a^n) = n\ln(a)$ $\ln(e) = 1$

$\ln(ab) = \ln(a) + \ln(b)$ $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

Ex. 3 Expand $\ln(3e^2)$

$\ln 3 + \ln e^2$

$\ln 3 + 2\ln e$

$\ln 3 + 2(1)$ $\ln 3 + 2$

Properties: $\ln(1) = 0$ $\ln(a^n) = n\ln(a)$ $\ln(e) = 1$

$\ln(ab) = \ln(a) + \ln(b)$ $\ln(a/b) = \ln(a) - \ln(b)$

Ex. 4 condense $2[\ln(x) - \ln(x+1) - \ln(x-1)]$

$2\left[\ln x - (\ln(x+1) + \ln(x-1))\right]$

$2\left[\ln x - \ln(x^2-1)\right]$

$2\left[\ln\left(\frac{x}{x^2-1}\right)\right]$

$\ln\left(\frac{x}{x^2-1}\right)^2$

Derivative of the Natural Logarithmic Function:

$\frac{d}{dx}[\ln u] = \frac{u'}{u}$

Ex. 5: If $y = \ln(x)$, find y'

$y' = \frac{1}{x}$

Ex. 6: if $y = \ln(x^2 - 5)$, find y'

$y' = \frac{2x}{x^2-5}$

Ex. 8 Find $\frac{dy}{dx}$ $4xy + \ln(x^2y) = 7$

$4xy + \ln x^2 + \ln y = 7$

$4xy + 2\ln x + \ln y = 7$

$4y + 4x\left(\frac{dy}{dx}\right) + 2\left(\frac{1}{x}\right) + \frac{1}{y}\left(\frac{dy}{dx}\right) = 0$

$4x\left(\frac{dy}{dx}\right) + \frac{1}{y}\left(\frac{dy}{dx}\right) = -\frac{2}{x} - 4y$

$\frac{dy}{dx}\left(4x + \frac{1}{y}\right) = -\frac{2}{x} - 4y$

$\frac{dy}{dx} = \frac{\left(-\frac{2}{x} - 4y\right)}{\left(4x + \frac{1}{y}\right)}\left(\frac{xy}{xy}\right)$

$\frac{dy}{dx} = \frac{-2y - 4xy^2}{4xy^2 + x}$

$y' = \frac{-4xy^2 - 2y}{4x^2y + x}$

Ex. 7: if $y = \ln\left(\frac{x^2}{\sqrt{2x^3}}\right)$, find y' (always simplify logs before taking the derivative)

$y = \ln x^2 - \ln(2x^3)^{1/2}$

$y = 2\ln x - \frac{1}{2}\ln(2x^3)$

$y' = 2\left(\frac{1}{x}\right) - \frac{1}{2}\left(\frac{6x^2}{2x^3}\right)$

$y' = \frac{2}{x} - \frac{3}{2x}$

$y' = \frac{4-3}{2x} = \frac{1}{2x}$

Matching In Exercises 5–8, match the function with its graph. [The graphs are labeled (a), (b), (c), and (d).]

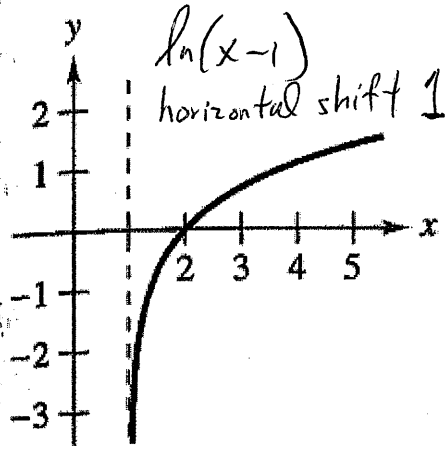
5. $f(x) = \ln x + 1$ [b]

6. $f(x) = -\ln x$ [d]

7. $f(x) = \ln(x - 1)$ [a]

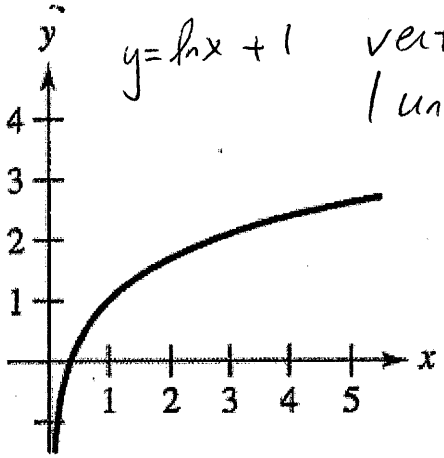
8. $f(x) = -\ln(-x)$ [c]

(a)



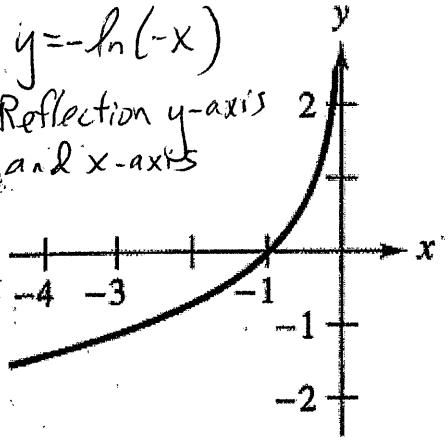
$\ln(x-1)$
horizontal shift 1 unit right

(b)



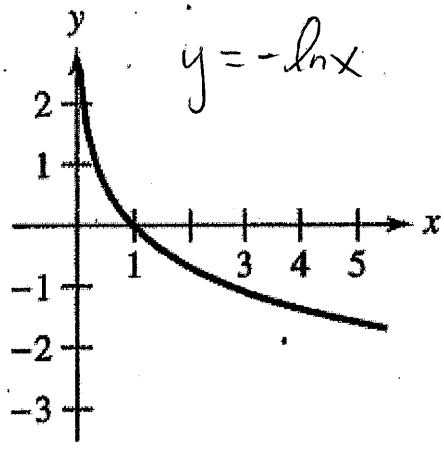
$y = \ln x + 1$ vertical shift 1 unit upward

(c)



$y = -\ln(-x)$
Reflection y-axis and x-axis

(d)



$y = -\ln x$ Reflection in the x-axis

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2. $\ln(ab) = \ln a + \ln b$

3. $\ln(a^n) = n \ln a$

4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

17. (a) $\ln 6$

$\ln(2 \cdot 3) = \ln 2 + \ln 3$

$= 1.7917$

(b) $\ln \frac{2}{3}$

$\ln\left(\frac{2}{3}\right) = \ln 2 - \ln 3$

$= -0.4055$

(c) $\ln 81$

$\ln(3^4) = 4 \ln 3$

$= 4.3944$

(d) $\ln \sqrt{3}$

$\ln 3^{1/2} = \frac{1}{2} \ln 3$

$= 0.5493$

18. (a) $\ln 0.25$ $\ln\left(\frac{1}{4}\right) = \ln\left(\frac{1}{2^2}\right)$

$\ln\left(\frac{1}{2}\right)^2 = 2 \ln\left(\frac{1}{2}\right)$

$2[\ln 1 - \ln 2] = -1.3862$

(b) $\ln 24$

$\ln(3 \cdot 2^3)$ | $\ln 3 + 3 \ln 2$

$= \ln 3 + \ln 2^3$ | $= 3.1779$

(c) $\ln \sqrt[3]{12}$

$\ln(12)^{1/3}$

$\frac{1}{3}[\ln(3 \cdot 2^2)]$

$\frac{1}{3} \ln 3 + \frac{1}{3} \ln 2^2$

$\frac{1}{3} \ln 3 + \frac{2}{3} \ln 2 = 0.8283$

(d) $\ln \frac{1}{72}$

$\ln\left(\frac{1}{8 \cdot 9}\right)$ | $\ln(2^3 \cdot 3^2)^{-1}$

$\ln\left(\frac{1}{2^3 \cdot 3^2}\right)$ | $-1[\ln 2^3 + \ln 3^2]$

$-3 \ln 2 - 2 \ln 3$

$= -4.2765$

Natural Log Properties

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Expanding a Logarithmic Expression In Exercises 19–28, use the properties of logarithms to expand the logarithmic expression.

19. $\ln \frac{x}{4}$

$\ln x - \ln 4$

$\ln x - \ln 2^2$

$\ln x - 2\ln 2$

20. $\ln \sqrt{x^5}$

$\ln x^{5/2}$

$\frac{5}{2}\ln x$

21. $\ln \frac{xy}{z}$

$\ln x + \ln y - \ln z$

23. $\ln\left(x\sqrt{x^2+5}\right)$

$\ln\left[x \cdot (x^2+5)^{1/2}\right]$

$\ln x + \ln(x^2+5)^{1/2}$

$\ln x + \frac{1}{2}\ln(x^2+5)$

25. $\ln \sqrt{\frac{x-1}{x}}$

$\ln\left[\frac{x-1}{x}\right]^{1/2}$

$\frac{1}{2}\ln\left[\frac{x-1}{x}\right]$

$\frac{1}{2}\left[\ln(x-1) - \ln x\right]$

$\frac{1}{2}\ln(x-1) - \frac{1}{2}\ln x$

26. $\ln(3e^2)$

$\ln 3 + \ln e^2$

$\ln 3 + 2\ln e$

$\ln 3 + 2$

27. $\ln z(z-1)^2$

$\ln z + \ln(z-1)^2$

$\ln z + 2\ln(z-1)$

28. $\ln \frac{1}{e}$

$\ln(e^{-1}) = -1 \ln e$

$= -1(1) = -1$

Condensing a Logarithmic Expression. In exercises #29-34, write the expression as a logarithm of a single quantity

Natural Log Properties

1. $\ln(1) = 0$
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4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

29. $\ln(x-2) - \ln(x+2)$

$\ln\left(\frac{x-2}{x+2}\right)$

30. $3\ln x + 2\ln y - 4\ln z$

$\ln x^3 + \ln y^2 - \ln z^4$

$\ln\left(\frac{x^3 y^2}{z^4}\right)$

31. $\frac{1}{3}[2\ln(x+3) + \ln x - \ln(x^2-1)]$

$\frac{1}{3}\left[\ln(x+3)^2 + \ln x - \ln(x^2-1)\right]$

$\frac{1}{3}\ln\left(\frac{(x+3)^2 \cdot x}{x^2-1}\right)$

$\ln\left[\frac{x(x+3)^2}{x^2-1}\right]^{1/3} = \ln\sqrt[3]{\frac{x(x+3)^2}{x^2-1}}$

32. $2[\ln x - \ln(x+1) - \ln(x-1)]$

$2\left[\ln\left(\frac{x}{(x+1)(x-1)}\right)\right] = \ln\left(\frac{x}{x^2-1}\right)^2$

33. $2\ln 3 - \frac{1}{2}\ln(x^2+1)$

$\ln 3^2 - \ln(x^2+1)^{1/2} = \ln\left(\frac{9}{\sqrt{x^2+1}}\right)$

34. $\frac{3}{2}[\ln(x^2+1) - \ln(x+1) - \ln(x-1)]$

$\frac{3}{2}\ln\left(\frac{x^2+1}{(x+1)(x-1)}\right)$

$\ln\left(\frac{x^2+1}{x^2-1}\right)^{3/2} = \ln\sqrt{\left(\frac{x^2+1}{x^2-1}\right)^3}$