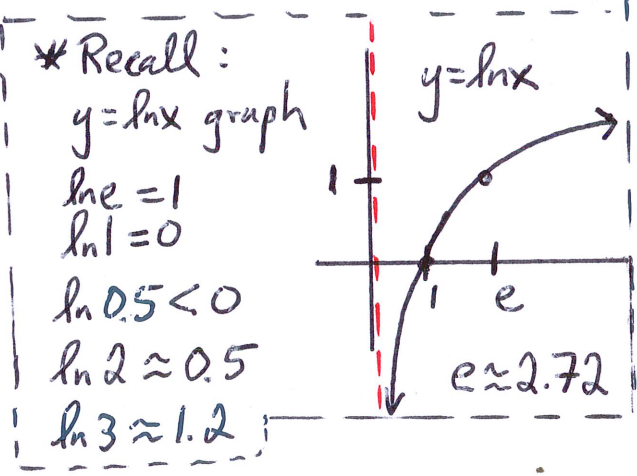


Ex. 4 Find Relative extrema, POI's for $y = \frac{x}{\ln x}$

* Domain: $(0, 1) \cup (1, \infty)$

$$\left. \begin{array}{l} \ln x = 0 \\ \log_e x = 0 \\ e^0 = 1 \end{array} \right| x \neq 1$$



$$y'(x) = \frac{(1)(\ln x) - x(\frac{1}{x})}{(\ln x)^2}$$

$$y'(x) = \frac{\ln x - 1}{(\ln x)^2}$$

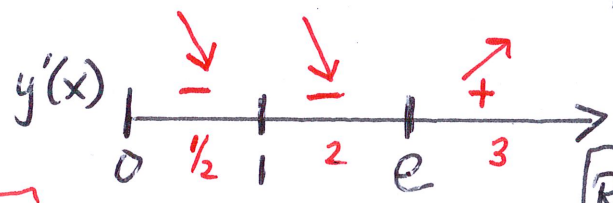
* find critical points

$$\ln x - 1 = 0 \quad | \quad (\ln x)^2 = 0$$

$$\ln x = 1 \quad | \quad \ln x = 0$$

$$\log_e x = 1 \quad | \quad \log_e x = 0$$

$$e^1 = x \quad | \quad e^0 = x$$

$$\underline{x = e} \quad | \quad \underline{x = 1}$$


Rel. min. at $x = e$

$$y''(x) = \frac{\overbrace{\left(\frac{1}{x}\right)(\ln x)^2}^{f'} - \overbrace{(\ln x - 1) \cdot 2(\ln x)\left(\frac{1}{x}\right)}^{g'}}{\underbrace{(\ln x)^4}_{g^2}}$$

* chain rule
 out: $()^2$
 in: $\ln x$

$$y''(x) = \frac{\frac{1}{x}(\ln x)^2 - \frac{2}{x} \ln x (\ln x - 1)}{(\ln x)^4}$$

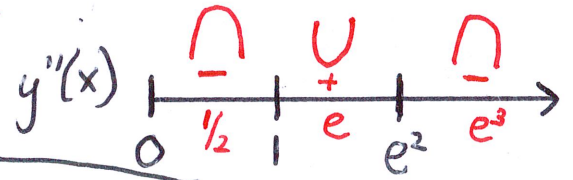
$$y''(x) = \frac{\frac{1}{x}(2 - \ln x)}{(\ln x)^3} = \frac{2 - \ln x}{x(\ln x)^3} = y''(x)$$

$$y''(x) = \frac{\frac{1}{x} \ln x (\ln x - 2(\ln x - 1))}{(\ln x)^4}$$

* critical pts:

$$\begin{array}{l|l|l} 2 - \ln x = 0 & x = 0 & (\ln x)^3 = 0 \\ 2 = \ln x & \underline{x = 0} & \ln x = 0 \\ 2 = \log_e x & & e^0 = x, \underline{x = 1} \\ \underline{e^2 = x} & & \end{array}$$

$$y''(x) = \frac{\frac{1}{x} \ln x (\ln x - 2 \ln x + 2)}{(\ln x)^4}$$



POI at $x = e^2$