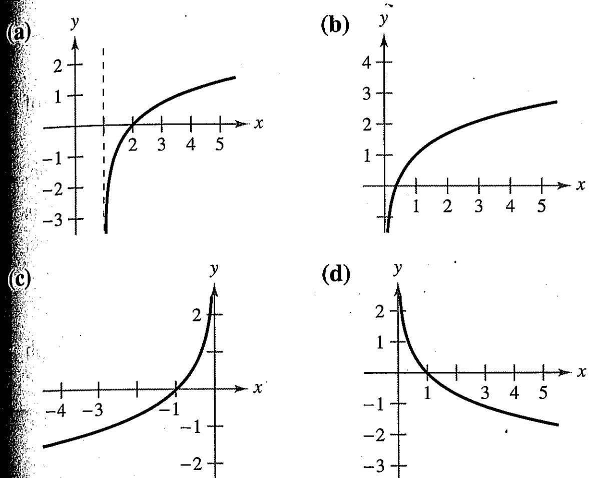
5.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Evaluating a Logarithm **In Exercises 1–4, use a graphing utility to evaluate the logarithm by (a) using the natural logarithm key and (b) using the integration capabilities to evaluate the integral $\int_1^x (1/t)\,dt$.**

1. $\ln 45$
2. $\ln 8.3$
3. $\ln 0.8$
4. $\ln 0.6$

Matching **In Exercises 5–8, match the function with its graph. [The graphs are labeled (a), (b), (c), and (d).]**



5. $f(x) = \ln x + 1$
6. $f(x) = -\ln x$
7. $f(x) = \ln(x - 1)$
8. $f(x) = -\ln(-x)$

Sketching a Graph **In Exercises 9–16, sketch the graph of the function and state its domain.**

9. $f(x) = 3 \ln x$
10. $f(x) = -2 \ln x$
11. $f(x) = \ln 2x$
12. $f(x) = \ln|x|$
13. $f(x) = \ln(x - 3)$
14. $f(x) = \ln x - 4$
15. $h(x) = \ln(x + 2)$
16. $f(x) = \ln(x - 2) + 1$

Using Properties of Logarithms **In Exercises 17 and 18, use the properties of logarithms to approximate the indicated logarithms, given that $\ln 2 \approx 0.6931$ and $\ln 3 \approx 1.0986$.**

17. (a) $\ln 6$ (b) $\ln \frac{2}{3}$ (c) $\ln 81$ (d) $\ln \sqrt{3}$
18. (a) $\ln 0.25$ (b) $\ln 24$ (c) $\ln \sqrt[3]{12}$ (d) $\ln \frac{1}{72}$

Expanding a Logarithmic Expression **In Exercises 19–28, use the properties of logarithms to expand the logarithmic expression.**

19. $\ln \dfrac{x}{4}$
20. $\ln \sqrt{x^5}$
21. $\ln \dfrac{xy}{z}$
22. $\ln(xyz)$
23. $\ln\left(x\sqrt{x^2 + 5}\right)$
24. $\ln \sqrt{a - 1}$
25. $\ln \sqrt{\dfrac{x - 1}{x}}$
26. $\ln(3e^2)$
27. $\ln z(z - 1)^2$
28. $\ln \dfrac{1}{e}$

Condensing a Logarithmic Expression **In Exercises 29–34, write the expression as a logarithm of a single quantity.**

29. $\ln(x - 2) - \ln(x + 2)$
30. $3 \ln x + 2 \ln y - 4 \ln z$
31. $\frac{1}{3}[2 \ln(x + 3) + \ln x - \ln(x^2 - 1)]$
32. $2[\ln x - \ln(x + 1) - \ln(x - 1)]$
33. $2 \ln 3 - \frac{1}{2} \ln(x^2 + 1)$
34. $\frac{3}{2}[\ln(x^2 + 1) - \ln(x + 1) - \ln(x - 1)]$

Verifying Properties of Logarithms **In Exercises 35 and 36, (a) verify that $f = g$ by using a graphing utility to graph f and g in the same viewing window and (b) verify that $f = g$ algebraically.**

35. $f(x) = \ln \dfrac{x^2}{4}, \quad x > 0, \quad g(x) = 2 \ln x - \ln 4$
36. $f(x) = \ln \sqrt{x(x^2 + 1)}, \quad g(x) = \frac{1}{2}[\ln x + \ln(x^2 + 1)]$

Finding a Limit **In Exercises 37–40, find the limit.**

37. $\lim\limits_{x \to 3^+} \ln(x - 3)$
38. $\lim\limits_{x \to 6^-} \ln(6 - x)$
39. $\lim\limits_{x \to 2^-} \ln[x^2(3 - x)]$
40. $\lim\limits_{x \to 5^+} \ln \dfrac{x}{\sqrt{x - 4}}$

Finding a Derivative **In Exercises 41–64, find the derivative of the function.**

41. $f(x) = \ln(3x)$
42. $f(x) = \ln(x - 1)$
43. $g(x) = \ln x^2$
44. $h(x) = \ln(2x^2 + 1)$
45. $y = (\ln x)^4$
46. $y = x^2 \ln x$
47. $y = \ln(t + 1)^2$
48. $y = \ln \sqrt{x^2 - 4}$
49. $y = \ln\left(x\sqrt{x^2 - 1}\right)$
50. $y = \ln[t(t^2 + 3)^3]$
51. $f(x) = \ln\left(\dfrac{x}{x^2 + 1}\right)$
52. $f(x) = \ln\left(\dfrac{2x}{x + 3}\right)$
53. $g(t) = \dfrac{\ln t}{t^2}$
54. $h(t) = \dfrac{\ln t}{t}$
55. $y = \ln(\ln x^2)$
56. $y = \ln(\ln x)$
57. $y = \ln \sqrt{\dfrac{x + 1}{x - 1}}$
58. $y = \ln \sqrt[3]{\dfrac{x - 1}{x + 1}}$
59. $f(x) = \ln\left(\dfrac{\sqrt{4 + x^2}}{x}\right)$
60. $f(x) = \ln\left(x + \sqrt{4 + x^2}\right)$
61. $y = \ln|\sin x|$
62. $y = \ln|\csc x|$
63. $y = \ln\left|\dfrac{\cos x}{\cos x - 1}\right|$
64. $y = \ln|\sec x + \tan x|$

Finding an Equation of a Tangent Line **In Exercises 65–72, (a) find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of a graphing utility to confirm your results.**

65. $y = \ln x^4, \quad (1, 0)$

66. $y = \ln x^{3/2}, \quad (1, 0)$

67. $f(x) = 3x^2 - \ln x, \quad (1, 3)$

68. $f(x) = 4 - x^2 - \ln\left(\frac{1}{2}x + 1\right), \quad (0, 4)$

69. $f(x) = \ln\sqrt{1 + \sin^2 x}, \quad \left(\frac{\pi}{4}, \ln\sqrt{\frac{3}{2}}\right)$

70. $f(x) = \sin 2x \ln x^2, \quad (1, 0)$

71. $f(x) = x^3 \ln x, \quad (1, 0)$

72. $f(x) = \frac{1}{2}x \ln x^2, \quad (-1, 0)$

Finding a Derivative Implicitly **In Exercises 73–76, use implicit differentiation to find dy/dx.**

73. $x^2 - 3 \ln y + y^2 = 10$

74. $\ln xy + 5x = 30$

75. $4x^3 + \ln y^2 + 2y = 2x$

76. $4xy + \ln x^2y = 7$

Differential Equation **In Exercises 77 and 78, show that the function is a solution of the differential equation.**

Function	Differential Equation
77. $y = 2 \ln x + 3$	$xy'' + y' = 0$
78. $y = x \ln x - 4x$	$x + y - xy' = 0$

Relative Extrema and Points of Inflection **In Exercises 79–84, locate any relative extrema and points of inflection. Use a graphing utility to confirm your results.**

79. $y = \frac{x^2}{2} - \ln x$

80. $y = 2x - \ln(2x)$

81. $y = x \ln x$

82. $y = \frac{\ln x}{x}$

83. $y = \frac{x}{\ln x}$

84. $y = x^2 \ln \frac{x}{4}$

Linear and Quadratic Approximation **In Exercises 85 and 86, use a graphing utility to graph the function. Then graph**

$$P_1(x) = f(1) + f'(1)(x - 1)$$

and

$$P_2(x) = f(1) + f'(1)(x - 1) + \tfrac{1}{2}f''(1)(x - 1)^2$$

in the same viewing window. Compare the values of f, P_1, P_2, and their first derivatives at $x = 1$.

85. $f(x) = \ln x$

86. $f(x) = x \ln x$

Using Newton's Method **In Exercises 87 and 88, use Newton's Method to approximate, to three decimal places, the x-coordinate of the point of intersection of the graphs of the two equations. Use a graphing utility to verify your result.**

87. $y = \ln x, \quad y = -x$

88. $y = \ln x, \quad y = 3 - x$

Logarithmic Differentiation **In Exercises 89–94, use logarithmic differentiation to find dy/dx.**

89. $y = x\sqrt{x^2 + 1}, \quad x > 0$

90. $y = \sqrt{x^2(x + 1)(x + 2)}, \quad x > 0$

91. $y = \frac{x^2\sqrt{3x - 2}}{(x + 1)^2}, \quad x > \frac{2}{3}$

92. $y = \sqrt{\frac{x^2 - 1}{x^2 + 1}}, \quad x > 1$

93. $y = \frac{x(x - 1)^{3/2}}{\sqrt{x + 1}}, \quad x > 1$

94. $y = \frac{(x + 1)(x - 2)}{(x - 1)(x + 2)}, \quad x > 2$

WRITING ABOUT CONCEPTS

95. Properties In your own words, state the properties of the natural logarithmic function.

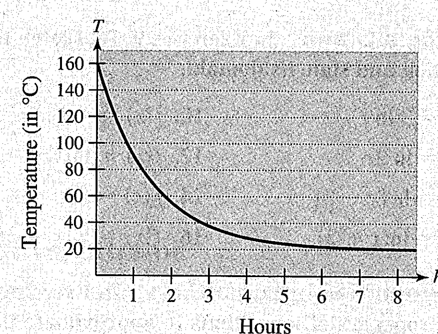
96. Base Define the base for the natural logarithmic function.

97. Comparing Functions Let f be a function that is positive and differentiable on the entire real number line. Let $g(x) = \ln f(x)$.

(a) When g is increasing, must f be increasing? Explain.

(b) When the graph of f is concave upward, must the graph of g be concave upward? Explain.

98. HOW DO YOU SEE IT? The graph shows the temperature T (in °C) of an object h hours after it is removed from a furnace.



(a) Find $\lim_{h\to\infty} T$. What does this limit represent?

(b) When is the temperature changing most rapidly?

True or False? **In Exercises 99–102, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

99. $\ln(x + 25) = \ln x + \ln 25$

100. $\ln xy = \ln x \ln y$

101. If $y = \ln \pi$, then $y' = 1/\pi$.

102. If $y = \ln e$, then $y' = 1$.

103. Home Mortgage The term t (in years) of a \$200,000 home mortgage at 7.5% interest can be approximated by

$$t = 13.375 \ln\left(\frac{x}{x - 1250}\right), \quad x > 1250$$

where x is the monthly payment in dollars.

(a) Use a graphing utility to graph the model.

(b) Use the model to approximate the term of a home mortgage for which the monthly payment is \$1398.43. What is the total amount paid?

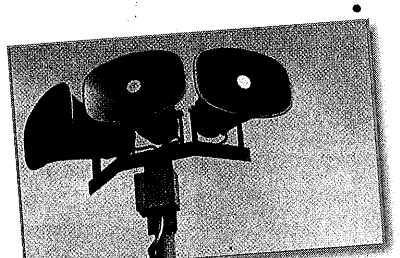
(c) Use the model to approximate the term of a home mortgage for which the monthly payment is \$1611.19. What is the total amount paid?

(d) Find the instantaneous rates of change of t with respect to x when $x = \$1398.43$ and $x = \$1611.19$.

(e) Write a short paragraph describing the benefit of the higher monthly payment.

104. Sound Intensity

The relationship between the number of decibels β and the intensity of a sound I in watts per centimeter squared is



$$\beta = \frac{10}{\ln 10} \ln\left(\frac{I}{10^{-16}}\right).$$

(a) Use the properties of logarithms to write the formula in simpler form.

(b) Determine the number of decibels of a sound with an intensity of 10^{-5} watt per square centimeter.

105. Modeling Data The table shows the temperatures T (in °F) at which water boils at selected pressures p (in pounds per square inch). (*Source: Standard Handbook of Mechanical Engineers*)

p	5	10	14.696 (1 atm)	20
T	162.24°	193.21°	212.00°	227.96°

p	30	40	60	80	100
T	250.33°	267.25°	292.71°	312.03°	327.81°

A model that approximates the data is

$$T = 87.97 + 34.96 \ln p + 7.91\sqrt{p}.$$

(a) Use a graphing utility to plot the data and graph the model.

(b) Find the rates of change of T with respect to p when $p = 10$ and $p = 70$.

(c) Use a graphing utility to graph T'. Find $\lim_{p\to\infty} T'(p)$ and interpret the result in the context of the problem.

106. Modeling Data The atmospheric pressure decreases with increasing altitude. At sea level, the average air pressure is one atmosphere (1.033227 kilograms per square centimeter). The table shows the pressures p (in atmospheres) at selected altitudes h (in kilometers).

h	0	5	10	15	20	25
p	1	0.55	0.25	0.12	0.06	0.02

(a) Use a graphing utility to find a model of the form $p = a + b \ln h$ for the data. Explain why the result is an error message.

(b) Use a graphing utility to find the logarithmic model $h = a + b \ln p$ for the data.

(c) Use a graphing utility to plot the data and graph the model.

(d) Use the model to estimate the altitude when $p = 0.75$.

(e) Use the model to estimate the pressure when $h = 13$.

(f) Use the model to find the rates of change of pressure when $h = 5$ and $h = 20$. Interpret the results.

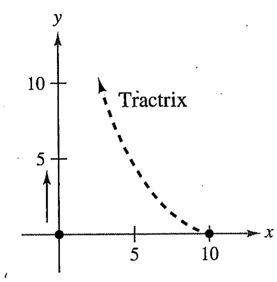
107. Tractrix A person walking along a dock drags a boat by a 10-meter rope. The boat travels along a path known as a *tractrix* (see figure). The equation of this path is

$$y = 10 \ln\left(\frac{10 + \sqrt{100 - x^2}}{x}\right) - \sqrt{100 - x^2}.$$

(a) Use a graphing utility to graph the function.

(b) What are the slopes of this path when $x = 5$ and $x = 9$?

(c) What does the slope of the path approach as $x \to 10$?



108. Prime Number Theorem There are 25 prime numbers less than 100. The **Prime Number Theorem** states that the number of primes less than x approaches

$$p(x) \approx \frac{x}{\ln x}.$$

Use this approximation to estimate the rate (in primes per 100 integers) at which the prime numbers occur when

(a) $x = 1000$.

(b) $x = 1{,}000{,}000$.

(c) $x = 1{,}000{,}000{,}000$.

109. Conjecture Use a graphing utility to graph f and g in the same viewing window and determine which is increasing at the greater rate for large values of x. What can you conclude about the rate of growth of the natural logarithmic function?

(a) $f(x) = \ln x, \quad g(x) = \sqrt{x}$

(b) $f(x) = \ln x, \quad g(x) = \sqrt[4]{x}$