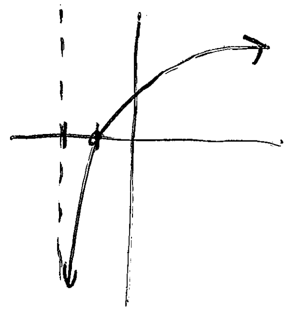
2017

5.1a Derivative of Natural Logs

p. 325-327
#7-15 odd, 19, 21, 31, 33, 47, 49, 53, 55, 59, 71, 73, 75, 83

15) $h(x) = \ln(x+2)$



21) Expand log expression: $\ln \frac{xy}{z} = \boxed{\ln x + \ln y - \ln z}$

31) Condense log expression: $\frac{1}{3}\left[2\ln(x+3) + \ln x - \ln(x^2-1)\right]$

$= \frac{1}{3}\left[\ln(x+3)^2 + \ln x - \ln(x^2-1)\right] = \frac{1}{3}\ln\left[\frac{(x+3)^2 x}{x^2-1}\right] = \boxed{\ln\left[\frac{x(x+3)^2}{x^2-1}\right]^{1/3}}$

33) Condense log expression: $2\ln 3 - \frac{1}{2}\ln(x^2+1)$

$= \ln 3^2 - \ln(x^2+1)^{1/2} = \ln 9 - \ln\sqrt{x^2+1} = \boxed{\ln\left(\frac{9}{\sqrt{x^2+1}}\right)}$

Find derivative

* Recall $\frac{d}{dx}\ln u = \frac{u'}{u}$

47) $y = \ln(t+1)^2 \quad y = 2\ln(t+1) \quad y' = 2 \cdot \frac{1}{t+1} = \boxed{\frac{2}{t+1}}$

49) $y = \ln(x\sqrt{x^2-1})$ → *expand first: $y = \ln x + \ln\sqrt{x^2-1}$

$y = \ln x + \ln(x^2-1)^{1/2}$

$y = \ln x + \frac{1}{2}\ln(x^2-1)$

$y' = \frac{1}{x} + \cancel{\frac{1}{2}}\left(\frac{\cancel{2}x}{x^2-1}\right)$

$y' = \frac{1}{x} + \frac{x}{x^2-1} = \frac{x^2-1+x^2}{x(x^2-1)} = \boxed{\frac{2x^2-1}{x(x^2-1)}}$

53) $g(t) = \frac{\ln t}{t^2}$ *apply quotient rule

$$g'(t) = \frac{\left(\frac{1}{t}\right)(t^2) - (\ln t)(2t)}{(t^2)^2} = \frac{t^2 - 2t\ln t}{t^4} = \frac{t(1-2\ln t)}{t^4} = \boxed{\frac{1-2\ln t}{t^3}}$$

55) $y = \ln(\ln x^2)$ $\quad y' = \frac{u'}{u}$

u-value

$y = \ln[2\ln x]$

$y' = \frac{\cancel{2}\left(\frac{1}{x}\right)}{\cancel{2}\ln x}$

$y' = \frac{\frac{1}{x}}{\ln x}$

$\boxed{y' = \frac{1}{x\ln x}}$

57) $y = \ln\sqrt{\frac{x+1}{x-1}} = \ln\left[\frac{x+1}{x-1}\right]^{1/2} = \frac{1}{2}\ln\left[\frac{x+1}{x-1}\right]$

$y = \frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x-1)$ $\quad y' = \frac{1}{2}\left(\frac{1}{x+1}\right) - \frac{1}{2}\left(\frac{1}{x-1}\right) = \frac{x-1-(x+1)}{2(x+1)(x-1)}$

$x-1-x-1$

$y' = \frac{-\cancel{2}}{\cancel{2}(x+1)(x-1)} = \boxed{\frac{-1}{x^2-1}}$

59) $f(x) = \ln\left(\frac{\sqrt{4+x^2}}{x}\right) = \ln\sqrt{4+x^2} - \ln x$

$f(x) = \ln(4+x^2)^{1/2} - \ln x$

$f(x) = \frac{1}{2}\ln(4+x^2) - \ln x$

$f'(x) = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}x}{4+x^2}\right) - \frac{1}{x}$

$f'(x) = \frac{x}{4+x^2} - \frac{1}{x}$

$f'(x) = \frac{x^2 - 1(4+x^2)}{x(4+x^2)}$

$f'(x) = \frac{x^2 - 4 - x^2}{x(4+x^2)} = \boxed{\frac{-4}{x(x^2+4)}}$

Find equation of tangent line

71) $f(x) = x^3 \ln x \quad (1,0)$

Apply product rule

$$f'(x) = 3x^2 \ln x + x^3\left(\frac{1}{x}\right)$$

$$f'(1) = 3(1)^2 \ln(1) + 1^3\left(\frac{1}{1}\right) = 0 + 1 = 1$$

$$f'(1) = 1$$

point: $(1,0)$

slope: $m = 1$

$y - 0 = 1(x-1)$

$\boxed{y = x - 1}$

73) Find $\frac{dy}{dx}$: $x^2 - 3\ln y + y^2 = 10$ (implicit differentiation)

$$2x - 3\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$$

$$2y\left(\frac{dy}{dx}\right) - \frac{3}{y}\left(\frac{dy}{dx}\right) = -2x$$

$$\frac{dy}{dx}\left(2y - \frac{3}{y}\right) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y - \frac{3}{y}} \cdot \frac{y}{y}$$

$$\boxed{\frac{dy}{dx} = \frac{-2xy}{2y^2 - 3} = \frac{2xy}{3 - 2y^2}}$$

74) Find $\frac{dy}{dx}$ implicitly: $\ln(xy) + 5x = 30$ expand first!

$$\ln x + \ln y + 5x = 30$$

$$\frac{1}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right) + 5 = 0$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = -5 - \frac{1}{x}$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{-5x - 1}{x}$$

$$\boxed{\frac{dy}{dx} = \frac{y(-5x-1)}{x} = \frac{-5xy - y}{x}}$$

75) Find $\frac{dy}{dx}$: $4x^3 + \ln y^2 + 2y = 2x$

$$4x^3 + 2\ln y + 2y = 2x$$

$$12x^2 + 2\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) + 2\left(\frac{dy}{dx}\right) = 2$$

$$\frac{2}{y}\left(\frac{dy}{dx}\right) + 2\left(\frac{dy}{dx}\right) = 2 - 12x^2$$

$$\frac{dy}{dx}\left(\frac{2}{y} + 2\right) = 2 - 12x^2$$

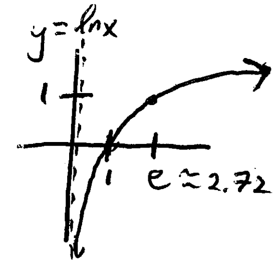
$$\frac{dy}{dx} = \frac{2 - 12x^2}{\frac{2}{y} + 2} \cdot \frac{y}{y}$$

$$\frac{dy}{dx} = \frac{2y - 12x^2y}{2 + 2y}$$

$$\frac{dy}{dx} = \frac{\cancel{2}(y - 6x^2y)}{\cancel{2}(1+y)}$$

$$\boxed{\frac{dy}{dx} = \frac{y - 6x^2y}{1+y}}$$

83) Find relative extrema, points of inflection



$y = \dfrac{x}{\ln x}$ Domain: $(0,1)\cup(1,\infty)$

$y' = \dfrac{(1)\ln x - x\left(\frac{1}{x}\right)}{(\ln x)^2} = \dfrac{\ln x - 1}{(\ln x)^2}$

$\ln x - 1 = 0$

$\ln x = 1$

$e^1 = x$

$\searrow\ \searrow\ \nearrow$; $-$ on $(0,1)$, $-$ on $(1,e)$, $+$ on (e,∞)

$\boxed{\text{Rel. min at } x = e \quad y(e) = e \quad \text{point: } (e,e)}$

$y'' = \dfrac{\left(\frac{1}{x}\right)(\ln x)^2 - (\ln x - 1)\cdot 2(\ln x)\left(\frac{1}{x}\right)}{(\ln x)^2}$

$y''(x) = \dfrac{\frac{1}{x}\cancel{\ln x}\left[\ln x - 2(\ln x - 1)\right]}{(\ln x)^{\cancel{2}}} = \dfrac{\ln x - 2\ln x + 2}{x\ln x}$

$y'' = \dfrac{-\ln x + 2}{x \ln x}$

$-\ln x + 2 = 0$

$\ln x = 2$

$e^2 = x$

$\cap\ \cup\ \cap$; $-$ on $(0,1)$, $+$ on $(1,e^2)$, $-$ on (e^2,∞)

POI at $x = e^2$

$y(e^2) = \frac{e^2}{2}$ $\boxed{\text{POI at point } \left(e^2, \frac{e^2}{2}\right)}$

82) $y = \dfrac{\ln x}{x}$ Domain: $x > 0$

$y' = \dfrac{\left(\frac{1}{x}\right)(x) - \ln x(1)}{x^2} = \dfrac{1 - \ln x}{x^2}$

$1 - \ln x = 0$

$\ln x = 1$

$e^1 = x$

$\nearrow\ \searrow$; $+$ on $(0,e)$, $-$ on (e,∞)

$y(e) = \frac{1}{e}$ Rel. max at $\left(e, \frac{1}{e}\right)$

$y'' = \dfrac{\left(\frac{-1}{x}\right)(x^2) - (1 - \ln x)(2x)}{(x^2)^2}$

$y'' = \dfrac{-x - 2x + 2x\ln x}{x^4} = \dfrac{-3x + 2x\ln x}{x^4}$

$y'' = \dfrac{\cancel{x}(2\ln x - 3)}{x^4} = \dfrac{2\ln x - 3}{x^3}$

$2\ln x - 3 = 0$

$\ln x = 3/2$

$e^{3/2} = x$

$\cap\ \cup$; $-$ on $(0, e^{3/2})$, $+$ on $(e^{3/2},\infty)$

POI at $x = e^{3/2}$

$y(e^{3/2}) = \frac{3}{2}e^{-3/2}$ POI at $\left(e^{3/2}, \frac{3}{2}e^{-3/2}\right)$