

5.16 Log Differentiation p. 326 # 82, 89-93 odd

89) $y = x\sqrt{x^2+1}$

$\ln y = \ln[x\sqrt{x^2+1}]$

$\ln y = \ln x + \ln(\sqrt{x^2+1})$

$\ln y = \ln x + \ln(x^2+1)^{1/2}$

$\ln y = \ln x + \frac{1}{2} \ln(x^2+1)$

$\frac{1}{y} \left(\frac{dy}{dx}\right) = \frac{1}{x} + \frac{1}{2} \left(\frac{2x}{x^2+1}\right)$

$\frac{dy}{dx} = y \left[\frac{1}{x} + \frac{x}{x^2+1} \right] = y \left[\frac{x^2+1+x^2}{x(x^2+1)} \right]$

$\frac{dy}{dx} = x\sqrt{x^2+1} \left[\frac{2x^2+1}{x(x^2+1)} \right] = \frac{2x^2+1}{\sqrt{x^2+1}}$

91) $y = \frac{x^2\sqrt{3x-2}}{(x+1)^2}$

$\ln y = \ln \left[\frac{x^2\sqrt{3x-2}}{(x+1)^2} \right]$

$\ln y = \ln x^2 + \ln(3x-2)^{1/2} - \ln(x+1)^2$

$\ln y = 2\ln x + \frac{1}{2} \ln(3x-2) - 2\ln(x+1)$

$\frac{1}{y} \left(\frac{dy}{dx}\right) = 2\left(\frac{1}{x}\right) + \frac{1}{2} \left(\frac{3}{3x-2}\right) - 2\left(\frac{1}{x+1}\right)$

$\frac{dy}{dx} = y \left[\frac{2}{x} + \frac{3}{2(3x-2)} - \frac{2}{x+1} \right]$

$\frac{dy}{dx} = \frac{x^2\sqrt{3x-2}}{(x+1)^2} \left[\frac{2}{x} + \frac{3}{2(3x-2)} - \frac{2}{x+1} \right]$

93) $y = \frac{x(x-1)^{3/2}}{\sqrt{x+1}}$

$\ln y = \ln \left[\frac{x(x-1)^{3/2}}{(x+1)^{1/2}} \right]$

$\ln y = \ln x + \ln(x-1)^{3/2} - \ln(x+1)^{1/2}$

$\ln y = \ln x + \frac{3}{2} \ln(x-1) - \frac{1}{2} \ln(x+1)$

$\frac{1}{y} \left(\frac{dy}{dx}\right) = \frac{1}{x} + \frac{3}{2} \left(\frac{1}{x-1}\right) - \frac{1}{2} \left(\frac{1}{x+1}\right)$

$\frac{dy}{dx} = y \left[\frac{1}{x} + \frac{3}{2(x-1)} - \frac{1}{2(x+1)} \right]$

$\frac{dy}{dx} = \frac{x(x-1)^{3/2}}{\sqrt{x+1}} \left[\frac{1}{x} + \frac{3}{2(x-1)} - \frac{1}{2(x+1)} \right]$