5.2-5.5 Integrals Classwork Problems

5.2a: Natural Log Integrals

$\int \frac{1}{u}du = \ln|u| + C$ | **Power Rule**: $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u\,du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

1. $\int \frac{5}{x}dx$

2. $\int \frac{10}{x}dx$

3. $\int \frac{1}{x+1}dx$

4. $\int \frac{1}{x-5}dx$

5. $\int \frac{1}{2x+5}dx$

6. $\int \frac{9}{5-4x}dx$

7. $\int \frac{x}{x^2-3}dx$

8. $\int \frac{x^2}{5-x^3}dx$

9. $\int \frac{4x^3+3}{x^4+3x}dx$

10. $\int \frac{x^2-2x}{x^3-3x^2}dx$

(2)

5.2a: Natural Log Integrals

$\int \frac{1}{u}du = \ln|u| + C$ | **Power Rule**: $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u\,du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

11. $\int \frac{x^2 - 4}{x}\,dx$

12. $\int \frac{x^3 - 8x}{x^2}\,dx$

13. $\int \frac{x^2 + 2x + 3}{x^3 + 3x^2 + 9x}\,dx$

14. $\int \frac{x^2 + 4x}{x^3 + 6x^2 + 5}\,dx$

15. $\int \frac{x^2 - 3x + 2}{x + 1}\,dx$

16. $\int \frac{2x^2 + 7x - 3}{x - 2}\,dx$

5.2a: Natural Log Integrals

$\int \frac{1}{u}du = \ln|u| + C$ | **Power Rule:** $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u\,du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

17. $\int \frac{x^3 - 3x^2 + 5}{x - 3}\,dx$

18. $\int \frac{x^3 - 6x - 20}{x + 5}\,dx$

19. $\int \frac{x^4 + x - 4}{x^2 + 2}\,dx$

20. $\int \frac{x^3 - 4x^2 - 4x + 20}{x^2 - 5}\,dx$

(4)

5.2a: Natural Log Integrals

$\int \frac{1}{u}du = \ln|u| + C$ | **Power Rule**: $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u\,du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

21. $\int \frac{(\ln x)^2}{x}\,dx$

22. $\int \frac{1}{x \ln x^3}\,dx$

23. $\int \frac{1}{\sqrt{x}(1-3\sqrt{x})}\,dx$

24. $\int \frac{1}{x^{2/3}(1+x^{1/3})}\,dx$

$\int \tan u\,du = -\ln\lvert\cos u\rvert + C$	$\int \cot u\,du = \ln\lvert\sin u\rvert + C$
$\int \sec u\,du = \ln\lvert\sec u + \tan u\rvert + C$	$\int \csc u\,du = -\ln\lvert\csc u + \cot u\rvert + C$

$\int \cos u\,du = \sin u + C$ | $\frac{d}{dx}[\sin u] = (\cos u)u'$ | $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$ | $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$

Finding an Indefinite Integral of a Trigonometric Function **In Exercises 31–40, find the indefinite integral.**

31. $\int \cot\frac{\theta}{3}\,d\theta$

32. $\int \tan 5\theta\,d\theta$

33. $\int \csc 2x\,dx$

34. $\int \sec\frac{x}{2}\,dx$

35. $\int (\cos 3\theta - 1)\,d\theta$

36. $\int \left(2 - \tan\frac{\theta}{4}\right)d\theta$

(6)

$\int \tan u\,du = -\ln\lvert\cos u\rvert + C$	$\int \cot u\,du = \ln\lvert\sin u\rvert + C$
$\int \sec u\,du = \ln\lvert\sec u + \tan u\rvert + C$	$\int \csc u\,du = -\ln\lvert\csc u + \cot u\rvert + C$

$\int \cos u\,du = \sin u + C$ | $\frac{d}{dx}[\sin u] = (\cos u)u'$ | $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$ | $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$

Finding an Indefinite Integral of a Trigonometric Function **In Exercises 31–40, find the indefinite integral.**

37. $\int \frac{\cos t}{1 + \sin t}\,dt$

38. $\int \frac{\csc^2 t}{\cot t}\,dt$

39. $\int \frac{\sec x \tan x}{\sec x - 1}\,dx$

40. $\int (\sec 2x + \tan 2x)\,dx$

Ch. 5.4 Exponential Function e^x Integration:

$\int \frac{1}{u}du = \ln\lvert u\rvert + C$	**Power Rule:** $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$	$\int e^u\,du = e^u + C$	$\frac{d}{dx}[\ln u] = \frac{u'}{u}$	$\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

Finding an Indefinite Integral **In Exercises 91–108, find the indefinite integral.**

91. $\int e^{5x}(5)\,dx$

92. $\int e^{-x^4}(-4x^3)\,dx$

93. $\int e^{2x-1}\,dx$

94. $\int e^{1-3x}\,dx$

95. $\int x^2 e^{x^3}\,dx$

96. $\int e^x(e^x + 1)^2\,dx$

97. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

98. $\int \frac{e^{1/x^2}}{x^3}\,dx$

(8) **Ch. 5.4 Exponential Function e^x Integration:**

$\int \frac{1}{u}du = \ln|u| + C$ | **Power Rule:** $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u\,du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

Finding an Indefinite Integral **In Exercises 91–108, find the indefinite integral.**

99. $\int \frac{e^{-x}}{1 + e^{-x}}\,dx$

100. $\int \frac{e^{2x}}{1 + e^{2x}}\,dx$

101. $\int e^x\sqrt{1 - e^x}\,dx$

102. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

103. $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx$

104. $\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2}\,dx$

$\int \frac{1}{u}du = \ln|u| + C$ | **Power Rule:** $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u\,du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

Finding an Indefinite Integral **In Exercises 91–108, find the indefinite integral.**

105. $\int \frac{5 - e^x}{e^{2x}}\,dx$

106. $\int \frac{e^{2x} + 2e^x + 1}{e^x}\,dx$

107. $\int e^{-x}\tan(e^{-x})\,dx$

108. $\int e^{2x}\csc(e^{2x})\,dx$

Ch. 5.5 Exponential Function a^u Integration:

$$\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$$

Power Rule:
$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C$$

$$\int e^u\,du = e^u + C$$

$$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$

$$\frac{d}{dx}[e^u] = e^u u'$$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

71. $\int 3^x\,dx$

72. $\int 8^{-x}\,dx$

73. $\int (x^2 + 2^{-x})\,dx$

74. $\int (x^4 + 5^x)\,dx$

75. $\int x(5^{-x^2})\,dx$

76. $\int (x + 4)6^{(x+4)^2}\,dx$

77. $\int \frac{3^{2x}}{1 + 3^{2x}}\,dx$

78. $\int 2^{\sin x}\cos x\,dx$

5.2-5.5 Integrals Classwork Problems

Key

5.2a: Natural Log Integrals

$\int \frac{1}{u}du = \ln|u| + C$ | **Power Rule:** $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u\,du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

1. $\int \frac{5}{x}dx$

$5\int \frac{1}{x}dx$

$= \boxed{5\ln|x|+C}$

2. $\int \frac{10}{x}dx$

$10\int \frac{1}{x}dx = \boxed{10\ln|x|+C}$

3. $\int \frac{1}{x+1}dx$

$u = x+1$ | $dx = du$ | $\ln|u|+C$

$\frac{du}{dx} = 1$ | $\int \frac{1}{u}du$ | $= \boxed{\ln|x+1|+C}$

4. $\int \frac{1}{x-5}dx$

$u = x-5$ | $\int \frac{1}{u}du = \ln|u|+C$

$\frac{du}{dx} = 1$ | $\boxed{\ln|x-5|+C}$

$dx = du$

5. $\int \frac{1}{2x+5}dx$

$u = 2x+5$ | $dx = \frac{du}{2}$ | $\frac{1}{2}\int \frac{1}{u}du$

$\frac{du}{dx} = 2$ | $\int \frac{1}{u}\cdot\frac{du}{2}$ | $\frac{1}{2}\ln|u|+C$

$2dx = du$ | $\boxed{\frac{1}{2}\ln|2x+5|+C}$

6. $\int \frac{9}{5-4x}dx$

$u = 5-4x$ | $dx = \frac{du}{-4}$ | $-\frac{9}{4}\int \frac{1}{u}du$

$\frac{du}{dx} = -4$ | $9\int \frac{1}{u}\cdot\frac{du}{-4}$ | $-\frac{9}{4}\ln|u|+C$

$-4dx = du$ | $\boxed{-\frac{9}{4}\ln|5-4x|+C}$

7. $\int \frac{x}{x^2-3}dx$

$u = x^2-3$ | $dx = \frac{du}{2x}$ | $\frac{1}{2}\int \frac{1}{u}du$

$\frac{du}{dx} = 2x$ | $\int \frac{x}{u}\cdot\frac{du}{2x}$ | $\frac{1}{2}\ln|u|+C$

$2xdx = du$ | $\boxed{\frac{1}{2}\ln|x^2-3|+C}$

8. $\int \frac{x^2}{5-x^3}dx$

$u = 5-x^3$ | $dx = \frac{du}{-3x^2}$ | $-\frac{1}{3}\int \frac{1}{u}du$

$\frac{du}{dx} = -3x^2$ | $\int \frac{x^2}{u}\cdot\frac{du}{-3x^2}$ | $-\frac{1}{3}\ln|u|+C$

$-3x^2dx = du$ | $\boxed{-\frac{1}{3}\ln|5-x^3|+C}$

9. $\int \frac{4x^3+3}{x^4+3x}dx$

$u = x^4+3x$ | $\int \frac{4x^3+3}{u}\cdot\frac{du}{4x^3+3}$

$\frac{du}{dx} = 4x^3+3$ | $\int \frac{1}{u}du = \ln|u|+C$

$(4x^3+3)dx = du$ | $\boxed{\ln|x^4+3x|+C}$

$dx = \frac{du}{4x^3+3}$

10. $\int \frac{x^2-2x}{x^3-3x^2}dx$

$u = x^3-3x^2$ | $\int \frac{x^2-2x}{u}\cdot\frac{du}{3x^2-6x}$

$\frac{du}{dx} = 3x^2-6x$ | $\int \frac{x^2-2x}{u}\cdot\frac{du}{3(x^2-2x)}$

$(3x^2-6x)dx = du$ | $\frac{1}{3}\int \frac{1}{u}du \rightarrow \frac{1}{3}\ln|u|+C$

$dx = \frac{du}{3x^2-6x}$ | $\rightarrow \boxed{\frac{1}{3}\ln|x^3-3x^2|+C}$

(2)

5.2a: Natural Log Integrals

$\int \frac{1}{u}\,du = \ln|u| + C$ | **Power Rule:** $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u\,du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

11. $\int \frac{x^2-4}{x}\,dx$

$\int (x^2-4)x^{-1}\,dx$

$\int x - 4x^{-1}\,dx$

$\int x - \frac{4}{x}\,dx$

$\boxed{\frac{x^2}{2} - 4\ln|x| + C}$

12. $\int \frac{x^3-8x}{x^2}\,dx$

$\int \frac{x^3}{x^2} - \frac{8x}{x^2}\,dx$

$\int x - \frac{8}{x}\,dx$

$\boxed{\frac{x^2}{2} - 8\ln|x| + C}$

13. $\int \frac{x^2+2x+3}{x^3+3x^2+9x}\,dx$

$u = x^3+3x^2+9x$

$\frac{du}{dx} = 3x^2+6x+9$

$dx(3x^2+6x+9) = du$

$dx = \frac{du}{3x^2+6x+9}$

$\int \frac{x^2+2x+3}{u} \cdot \frac{du}{3(x^2+2x+3)}$

$\frac{1}{3}\int \frac{1}{u}\,du = \boxed{\frac{1}{3}\ln|x^3+3x^2+9x| + C}$

14. $\int \frac{x^2+4x}{x^3+6x^2+5}\,dx$

$u = x^3+6x^2+5$

$\frac{du}{dx} = 3x^2+12x$

$dx = \frac{du}{3(x^2+4x)}$

$\int \frac{x+4}{u} \cdot \frac{du}{3(x^2+4x)}$

$\frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C$

$= \boxed{\frac{1}{3}\ln|x^3+6x^2+5| + C}$

15. $\int \frac{x^2-3x+2}{x+1}\,dx$

* Long Division:

$$\begin{array}{r} x-4+\frac{6}{x+1} \\ x+1 \overline{)\,x^2-3x+2} \\ \ominus x^2 \oplus 1x \\ \hline -4x+2 \\ \oplus 4x \oplus 4 \\ \hline 6 \end{array}$$

$\int x - 4 + \int \frac{6}{x+1}\,dx$

$u = x+1 \quad \frac{du}{dx} = 1$

$6\int \frac{1}{u}\,du = 6\ln|u|$

$\boxed{\frac{x^2}{2} - 4x + 6\ln|x+1| + C}$

* OR synthetic division

$$\begin{array}{r|rrr} -1 & 1 & -3 & 2 \\ & \downarrow & -1 & 4 \\ \hline & 1 & -4 & \boxed{6} \end{array}$$

$\int x - 4 + \frac{6}{x+1}\,dx$

16. $\int \frac{2x^2+7x-3}{x-2}\,dx$

* Long Division

$$\begin{array}{r} 2x+11+\frac{19}{x-2} \\ x-2 \overline{)\,2x^2+7x-3} \\ \ominus 2x^2 \oplus 4x \\ \hline 11x-3 \\ \ominus 11x \oplus 22 \\ \hline 19 \end{array}$$

$\int 2x+11+\frac{19}{x-2}\,dx = \frac{2x^2}{2} + 11x + 19\ln|x-2| + C$

$= \boxed{x^2+11x+19\ln|x-2| + C}$

* OR synthetic division

$$\begin{array}{r|rrr} 2 & 2 & 7 & -3 \\ & \downarrow & 4 & 22 \\ \hline & 2 & 11 & 19 \end{array}$$

$\int 2x+11+\frac{19}{x-2}\,dx$

5.2a: Natural Log Integrals

$\int \frac{1}{u}du = \ln|u| + C$ | **Power Rule:** $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u\,du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

17. $\int \frac{x^3 - 3x^2 + 5}{x - 3}\,dx$

$x-3 \overline{)\, x^3 - 3x^2 + 5}$ $\quad x^2 + \frac{5}{x-3}$

$\ominus x^3 \oplus 3x^2$

5

$\int x^2 + \frac{5}{x-3}\,dx$

$\frac{x^3}{3} + 5\ln|x-3| + C$

$3 \mid 1 \;\; -3 \;\; 0 \;\; 5$

$\downarrow \;\; 3 \;\; 0 \;\; 0$

$1 \;\; 0 \;\; 0 \;\; 5$

$\int x^2 + 0x + 0 + \frac{5}{x-3}\,dx$

18. $\int \frac{x^3 - 6x - 20}{x + 5}\,dx$

$x+5 \overline{)\, x^3 - 6x - 20}$ $\quad x^2 - 5x + 19 + \frac{-115}{x+5}$

$\ominus x^3 \oplus 5x^2$

$-5x^2 - 6x - 20$

$\oplus 5x^2 \oplus 25x$

$19x - 20$

$\ominus 19x \oplus 95$

-115

$\int x^2 - 5x + 19 - \frac{115}{x+5}\,dx$

$\frac{x^3}{3} - \frac{5x^2}{2} + 19x - 115\ln|x+5| + C$

*synthetic division

$-5 \mid 1 \;\; 0 \;\; -6 \;\; -20$

$\downarrow \;\; -5 \;\; 25 \;\; -95$

$1 \;\; -5 \;\; 19 \;\; -115$

$\int x^2 - 5x + 19 - \frac{115}{x+5}\,dx$

19. $\int \frac{x^4 + x - 4}{x^2 + 2}\,dx$

$x^2+2 \overline{)\, x^4 + x - 4}$ $\quad x^2 - 2 + \frac{x}{x^2+2}$

$\ominus x^4 \oplus 2x^2$

$-2x^2 + x - 4$

$\oplus 2x^2 \oplus 4$

x

$\int x^2 - 2\,dx + \int \frac{x}{x^2+2}\,dx$

$u = x^2 + 2 \quad dx = \frac{du}{2x}$

$\frac{du}{dx} = 2x$

$\int \frac{x}{u} \cdot \frac{du}{2x} = \frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u|$

$\frac{x^3}{3} - 2x + \frac{1}{2}\ln|x^2+2| + C$

20. $\int \frac{x^3 - 4x^2 - 4x + 20}{x^2 - 5}\,dx$

$x^2-5 \overline{)\, x^3 - 4x^2 - 4x + 20}$ $\quad x - 4 + \frac{x}{x^2-5}$

$\ominus x^3 \quad \oplus 5x$

$-4x^2 + x + 20$

$\oplus 4x^2 \quad \ominus 20$

x

$\int x - 4\,dx + \int \frac{x}{x^2-5}\,dx$

$u = x^2 - 5 \quad dx = \frac{du}{2x}$

$\frac{du}{dx} = 2x$

$\int \frac{x}{u} \cdot \frac{du}{2x} = \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u|$

$\frac{x^2}{2} - 4x + \frac{1}{2}\ln|x^2 - 5| + C$

(4) **5.2a: Natural Log Integrals**

$\int \frac{1}{u}du = \ln|u| + C$ | **Power Rule:** $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u\,du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

21. $\int \frac{(\ln x)^2}{x}\,dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$\int \frac{u^2}{x} \cdot x\,du = \int u^2\,du$

$\frac{u^3}{3} + C$

$= \boxed{\frac{(\ln x)^3}{3} + C}$

22. $\int \frac{1}{x \ln x^3}\,dx$

$\int \frac{1}{x \ln x \cdot 3}\,dx$

$\int \frac{1}{3x \cdot \ln x}\,dx$

$u = \ln x \quad | \quad dx = x\,du$

$\frac{du}{dx} = \frac{1}{x}$

$\int \frac{1}{3x \cdot u} \cdot x\,du$

$\frac{1}{3}\int \frac{1}{u}du = \frac{1}{3}\ln|u| + C$

$\boxed{\frac{1}{3}\ln|\ln|x|| + C}$

23. $\int \frac{1}{\sqrt{x}(1 - 3\sqrt{x})}\,dx$

$u = 1 - 3\sqrt{x}$

$u = 1 - 3x^{1/2}$

$\frac{du}{dx} = -\frac{3}{2}x^{-1/2}$

$\frac{du}{dx} = \frac{-3}{2\sqrt{x}}$

$-3dx = 2\sqrt{x}\,du$

$dx = -\frac{2\sqrt{x}\,du}{3}$

$\int \frac{1}{\sqrt{x} \cdot u} \cdot -\frac{2\sqrt{x}\,du}{3}$

$-\frac{2}{3}\int \frac{1}{u}du$

$-\frac{2}{3}\ln|u| + C$

$\boxed{-\frac{2}{3}\ln|1 - 3\sqrt{x}| + C}$

24. $\int \frac{1}{x^{2/3}(1 + x^{1/3})}\,dx$

$u = 1 + x^{1/3}$

$\frac{du}{dx} = \frac{1}{3}x^{-2/3}$

$\frac{du}{dx} = \frac{1}{3x^{2/3}}$

$dx = 3x^{2/3}\,du$

$\int \frac{1}{x^{2/3} \cdot u} \cdot 3x^{2/3}\,du$

$3\int \frac{1}{u}du$

$3\ln|u| + C$

$\boxed{3\ln|1 + x^{1/3}| + C}$

$\int \tan u\,du = -\ln\lvert\cos u\rvert + C$	$\int \cot u\,du = \ln\lvert\sin u\rvert + C$
$\int \sec u\,du = \ln\lvert\sec u + \tan u\rvert + C$	$\int \csc u\,du = -\ln\lvert\csc u + \cot u\rvert + C$

$\int \cos u\,du = \sin u + C$ | $\frac{d}{dx}[\sin u] = (\cos u)u'$ | $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$ | $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$

Finding an Indefinite Integral of a Trigonometric Function **In Exercises 31–40, find the indefinite integral.**

31. $\int \cot\frac{\theta}{3}\,d\theta$

$u = \frac{1}{3}\theta$

$\frac{du}{d\theta} = \frac{1}{3}$

$d\theta = 3du$

$\int \cot u \cdot 3du = 3\int \cot u\,du$

$3\ln\lvert\sin u\rvert + C$

$\boxed{3\ln\left\lvert\sin\left(\frac{\theta}{3}\right)\right\rvert + C}$

32. $\int \tan 5\theta\,d\theta$

$u = 5\theta$

$\frac{du}{d\theta} = 5$

$d\theta = \frac{du}{5}$

$\int \tan u \cdot \frac{du}{5}$

$\frac{1}{5}\int \tan u\,du$

$\frac{1}{5}\left(-\ln\lvert\cos u\rvert\right) + C$

$\boxed{-\frac{1}{5}\ln\lvert\cos 5\theta\rvert + C}$

33. $\int \csc 2x\,dx$

$u = 2x$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$\int \csc u \cdot \frac{du}{2} = \frac{1}{2}\int \csc u\,du$

$\frac{1}{2}\cdot -\ln\lvert\csc u + \cot u\rvert + C$

$\boxed{-\frac{1}{2}\ln\lvert\csc 2x + \cot 2x\rvert + C}$

34. $\int \sec\frac{x}{2}\,dx$

$u = \frac{x}{2} = \frac{1}{2}x$

$\frac{du}{dx} = \frac{1}{2}$

$dx = 2du$

$\int \sec u \cdot 2du = 2\int \sec u\,du$

$2\ln\lvert\sec u + \tan u\rvert + C$

$\boxed{2\ln\left\lvert\sec\left(\frac{x}{2}\right) + \tan\left(\frac{x}{2}\right)\right\rvert + C}$

35. $\int (\cos 3\theta - 1)\,d\theta$

$\int \cos 3\theta\,d\theta - \int 1\,d\theta$

$u = 3\theta$

$\frac{du}{d\theta} = 3$

$d\theta = \frac{du}{3}$

$\int \cos u \cdot \frac{du}{3}$

$\frac{1}{3}\int \cos u\,du$

$\frac{1}{3}\sin u - \int 1\,d\theta$

$\boxed{\frac{1}{3}\sin(3\theta) - \theta + C}$

36. $\int \left(2 - \tan\frac{\theta}{4}\right)d\theta$

$\int 2\,d\theta - \int \tan\left(\frac{\theta}{4}\right)d\theta$

$\boxed{2\theta + 4\ln\left\lvert\cos\left(\frac{\theta}{4}\right)\right\rvert + C}$

$u = \frac{\theta}{4} = \frac{1}{4}\theta$

$\frac{du}{d\theta} = \frac{1}{4}$ $d\theta = 4du$

$\int \tan u \cdot 4du$

$4\int \tan u\,du = 4\left(-\ln\lvert\cos u\rvert\right)$

(6)

$\int \tan u\,du = -\ln\lvert\cos u\rvert + C$	$\int \cot u\,du = \ln\lvert\sin u\rvert + C$
$\int \sec u\,du = \ln\lvert\sec u + \tan u\rvert + C$	$\int \csc u\,du = -\ln\lvert\csc u + \cot u\rvert + C$

$\int \cos u\,du = \sin u + C$ | $\frac{d}{dx}[\sin u] = (\cos u)u'$ | $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$ | $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$

Finding an Indefinite Integral of a Trigonometric Function **In Exercises 31–40, find the indefinite integral.**

37. $\int \frac{\cos t}{1 + \sin t}\,dt$

$u = 1 + \sin t$

$\frac{du}{dt} = \cos(t)$

$dt = \frac{du}{\cos(t)}$

$\int \frac{\cos(t)}{u} \cdot \frac{du}{\cos(t)}$

$\int \frac{1}{u}\,du = \ln\lvert u\rvert + C$

$= \boxed{\ln\lvert 1 + \sin t\rvert + C}$

38. $\int \frac{\csc^2 t}{\cot t}\,dt$

$u = \cot t$

$\frac{du}{dt} = -\csc^2 t$

$dt = \frac{du}{-\csc^2 t}$

$\int \frac{\csc^2 t}{u} \cdot \frac{du}{-\csc^2 t}$

$-\int \frac{1}{u}\,du = -\ln\lvert u\rvert + C$

$= \boxed{-\ln\lvert \cot(t)\rvert + C}$

39. $\int \frac{\sec x \tan x}{\sec x - 1}\,dx$

$u = \sec x - 1$

$\frac{du}{dx} = \sec x \tan x$

$dx = \frac{du}{\sec x \tan x}$

$\int \frac{\sec x \tan x}{u} \cdot \frac{du}{\sec x \tan x}$

$\int \frac{1}{u}\,du$

$\ln\lvert u\rvert + C$

$\boxed{\ln\lvert \sec x - 1\rvert + C}$

40. $\int (\sec 2x + \tan 2x)\,dx$

$\int \sec 2x\,dx + \int \tan 2x\,dx$

$u = 2x$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$\int \sec u \cdot \frac{du}{2} + \int \tan u \cdot \frac{du}{2}$

$\frac{1}{2}\int \sec u\,du + \frac{1}{2}\int \tan u\,du$

$\frac{1}{2}\ln\lvert \sec u + \tan u\rvert + \frac{1}{2}\left(-\ln\lvert \cos u\rvert\right) + C$

$\boxed{\frac{1}{2}\ln\lvert \sec 2x + \tan 2x\rvert - \frac{1}{2}\ln\lvert \cos 2x\rvert + C}$

Ch. 5.4 Exponential Function e^x Integration:

$\int \frac{1}{u}\,du = \ln|u| + C$ | **Power Rule:** $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u\,du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

Finding an Indefinite Integral **In Exercises 91–108, find the indefinite integral.**

91. $\int e^{5x}(5)\,dx$

$u = 5x$

$\frac{du}{dx} = 5$

$dx = \frac{du}{5}$

$\int e^u \cdot 5 \cdot \frac{du}{5}$

$e^u + C$

$\boxed{e^{5x} + C}$

92. $\int e^{-x^4}(-4x^3)\,dx$

$u = -x^4$

$\frac{du}{dx} = -4x^3$

$dx = \frac{du}{-4x^3}$

$\int e^u \cdot -4x^3 \cdot \frac{du}{-4x^3}$

$e^u + C$

$\boxed{e^{-x^4} + C}$

93. $\int e^{2x-1}\,dx$

$u = 2x-1$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$\int e^u \cdot \frac{du}{2}$

$\frac{1}{2}\int e^u\,du$

$\frac{1}{2}e^u + C$

$\boxed{\frac{1}{2}e^{2x-1} + C}$

94. $\int e^{1-3x}\,dx$

$u = 1-3x$

$\frac{du}{dx} = -3$

$dx = \frac{du}{-3}$

$\int e^u \cdot \frac{du}{-3}$

$-\frac{1}{3}\int e^u\,du$

$-\frac{1}{3}e^u + C$

$\boxed{-\frac{1}{3}e^{1-3x} + C}$

95. $\int x^2 e^{x^3}\,dx$

$u = x^3$

$\frac{du}{dx} = 3x^2$

$dx = \frac{du}{3x^2}$

$\int x^2 \cdot e^u \cdot \frac{du}{3x^2}$

$\frac{1}{3}\int e^u\,du$

$\frac{1}{3}e^u + C$

$\boxed{\frac{1}{3}e^{x^3} + C}$

96. $\int e^x(e^x + 1)^2\,dx$

$u = e^x + 1$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

$\int e^x \cdot u^2 \cdot \frac{du}{e^x}$

$\int u^2\,du$

$\frac{u^3}{3} + C$

$\boxed{\frac{1}{3}(e^x+1)^3 + C}$

97. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$u = \sqrt{x} = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$

$\int \frac{e^u}{\sqrt{x}} \cdot 2\sqrt{x}\,du$

$2\int e^u\,du = 2e^u + C$

$\boxed{2e^{\sqrt{x}} + C}$

98. $\int \frac{e^{1/x^2}}{x^3}\,dx$

$u = \frac{1}{x^2} = x^{-2}$

$\frac{du}{dx} = -2x^{-3}$

$\frac{du}{dx} = \frac{-2}{x^3}$

$-2dx = x^3\,du$

$dx = \frac{x^3}{-2}\,du$

$\int \frac{e^u}{x^3} \cdot \frac{x^3}{-2}\,du$

$-\frac{1}{2}\int e^u\,du$

$-\frac{1}{2}e^u + C$

$\boxed{-\frac{1}{2}e^{\frac{1}{x^2}} + C}$

(8) **Ch. 5.4 Exponential Function e^x Integration:**

$\int \frac{1}{u}\,du = \ln|u| + C$ | **Power Rule:** $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u\,du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

Finding an Indefinite Integral **In Exercises 91–108, find the indefinite integral.**

99. $\int \frac{e^{-x}}{1+e^{-x}}\,dx$

$u = 1+e^{-x}$

$\frac{du}{dx} = e^{-x}(-1)$

$dx = \frac{du}{-e^{-x}}$

$\int \frac{e^{-x}}{u} \cdot \frac{du}{-e^{-x}}$

$-1\int \frac{1}{u}\,du = -\ln|u| + C$

$= \boxed{-\ln|1+e^{-x}| + C}$

100. $\int \frac{e^{2x}}{1+e^{2x}}\,dx$

$u = 1+e^{2x}$

$\frac{du}{dx} = e^{2x}\cdot 2$

$dx = \frac{du}{2e^{2x}}$

$\int \frac{e^{2x}}{u} \cdot \frac{du}{2e^{2x}} = \frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u| + C$

$\boxed{\frac{1}{2}\ln|1+e^{2x}| + C}$

101. $\int e^x\sqrt{1-e^x}\,dx$

$u = 1-e^x$

$\frac{du}{dx} = -e^x$

$dx = \frac{du}{-e^x}$

$\int e^x(1-e^x)^{1/2}\,dx$

$\int e^x \cdot u^{1/2} \cdot \frac{du}{-e^x}$

$-1\int u^{1/2}\,du = -\frac{u^{3/2}}{3/2} + C$

$-\frac{2}{3}u^{3/2} + C = \boxed{-\frac{2}{3}(1-e^x)^{3/2} + C}$

102. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

$u = e^x + e^{-x}$

$\frac{du}{dx} = e^x + e^{-x}(-1)$

$dx = \frac{du}{e^x - e^{-x}}$

$\int \frac{e^x - e^{-x}}{u} \cdot \frac{du}{e^x - e^{-x}}$

$\int \frac{1}{u}\,du = \ln|u| + C$

$= \boxed{\ln|e^x + e^{-x}| + C}$

103. $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx$

$u = e^x - e^{-x}$

$\frac{du}{dx} = e^x - e^{-x}(-1)$

$dx = \frac{du}{e^x + e^{-x}}$

$\int \frac{e^x + e^{-x}}{u} \cdot \frac{du}{e^x + e^{-x}}$

$\int \frac{1}{u}\,du = \ln|u| + C$

$= \boxed{\ln|e^x - e^{-x}| + C}$

104. $\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2}\,dx$

$\int 2e^x - 2e^{-x}(e^x + e^{-x})^{-2}\,dx$

$u = e^x + e^{-x}$

$\frac{du}{dx} = e^x + e^{-x}(-1)$

$dx = \frac{du}{e^x - e^{-x}}$

$\int 2(e^x - e^{-x}) \cdot u^{-2} \cdot \frac{du}{e^x - e^{-x}}$

$2\int u^{-2}\,du = 2 \cdot \frac{u^{-1}}{-1} + C$

$-\frac{2}{u} + C = \boxed{\frac{-2}{e^x + e^{-x}} + C}$

$\int \frac{1}{u}du = \ln|u| + C$ | **Power Rule:** $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u\,du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

Finding an Indefinite Integral **In Exercises 91–108, find the indefinite integral.**

105. $\int \frac{5 - e^x}{e^{2x}}\,dx$

$\int (5-e^x)e^{-2x}\,dx$

$\int 5e^{-2x} - e^{-1x}\,dx$

$u=-2x \quad | \quad u=-1x$

$\frac{du}{dx}=-2 \quad | \quad \frac{du}{dx}=-1$

$dx=\frac{du}{-2} \quad | \quad dx=-du$

$5\int e^u \cdot \frac{du}{-2} - \int e^u \cdot (-du)$

$-\frac{5}{2}\int e^u du + \int e^u du$

$-\frac{5}{2}e^u + e^u + C$

$\boxed{-\frac{5}{2}e^{-2x} + e^{-x} + C}$

106. $\int \frac{e^{2x} + 2e^x + 1}{e^x}\,dx$

$\int (e^{2x}+2e^x+1)e^{-x}\,dx$

$u=-x \quad \frac{du}{dx}=-1$

$\int e^x + 2 + 1e^{-x}\,dx$

$\boxed{e^x + 2x - e^{-x} + C}$

$dx=-du$

$\int 1e^u \cdot (-du)$

$= -e^u + C$

$= -e^{-x} + C$

107. $\int e^{-x}\tan(e^{-x})\,dx$

$u=e^{-x}$

$\frac{du}{dx}=e^{-x}(-1)$

$dx=\frac{du}{-e^{-x}}$

$\int e^{-x} \cdot \tan u \cdot \frac{du}{-e^{-x}}$

$-1\int \tan u\,du$

$-1 \cdot -\ln|\cos u| + C$

$= \boxed{\ln|\cos e^{-x}| + C}$

108. $\int e^{2x}\csc(e^{2x})\,dx$

$u=e^{2x}$

$\frac{du}{dx}=e^{2x}\cdot 2$

$dx=\frac{du}{2e^{2x}}$

$\int e^{2x}\csc u \cdot \frac{du}{2e^{2x}}$

$\frac{1}{2}\int \csc u\,du$

$= -\frac{1}{2}\ln|\csc u + \cot u| + C$

$= \boxed{-\frac{1}{2}\ln|\csc(e^{2x}) + \cot(e^{2x})| + C}$

(10) **Ch. 5.5 Exponential Function a^u Integration:**

$\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$ | **Power Rule:** $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u\,du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u u'$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

71. $\int 3^x\,dx$ $= \frac{1}{\ln 3}\cdot 3^x + C$

$= \boxed{\frac{3^x}{\ln 3} + C}$

72. $\int 8^{-x}\,dx$

$u = -x$

$\frac{du}{dx} = -1$

$dx = \frac{du}{-1} = -du$

$\int 8^u \cdot -du$ | $-\frac{1}{\ln 8}\cdot 8^u + C$

$-1\int 8^u\,du$ | $\boxed{\frac{-8^{-x}}{\ln 8} + C}$

73. $\int (x^2 + 2^{-x})\,dx$

$u = -x \quad dx = -du$

$\frac{du}{dx} = -1$

$\int x^2\,dx + \int 2^{-x}\,dx$

$\int 2^u \cdot -du = -1\int 2^u\,du$

$\frac{x^3}{3} - \frac{1}{\ln 2}\cdot 2^{-x} + C$ | $\boxed{\frac{x^3}{3} - \frac{2^{-x}}{\ln 2} + C}$

74. $\int (x^4 + 5^x)\,dx$

$\int x^4\,dx + \int 5^x\,dx$

$\frac{x^5}{5} + \frac{1}{\ln 5}\cdot 5^x + C$ | $\boxed{\frac{x^5}{5} + \frac{5^x}{\ln 5} + C}$

75. $\int x(5^{-x^2})\,dx$

$u = -x^2$

$\frac{du}{dx} = -2x$

$dx = \frac{du}{-2x}$

$\int x\cdot 5^u \cdot \frac{du}{-2x}$ | $-\frac{1}{2}\cdot\frac{1}{\ln 5}\cdot 5^u + C$

$-\frac{1}{2}\int 5^u\,du$ | $\boxed{\frac{-5^{-x^2}}{2\ln 5} + C}$

76. $\int (x+4)6^{(x+4)^2}\,dx$

$u = (x+4)^2$

$\frac{du}{dx} = 2(x+4)(1)$

$dx = \frac{du}{2(x+4)}$

$\int (x+4)\cdot 6^u \cdot \frac{du}{2(x+4)}$ | $\frac{1}{2}\cdot\frac{1}{\ln 6}\cdot 6^u + C = \boxed{\frac{6^{(x+4)^2}}{2\ln 6} + C}$

$\frac{1}{2}\int 6^u\,du$

77. $\int \frac{3^{2x}}{1 + 3^{2x}}\,dx$

$*\frac{d}{dx}a^u = \ln a \cdot a^u \cdot u'$

$u = 1 + 3^{2x}$

$\frac{du}{dx} = \ln 3 \cdot 3^{2x}\cdot 2$

$dx = \frac{du}{2\ln 3\cdot 3^{2x}}$

$\int \frac{3^{2x}}{u}\cdot\frac{du}{2\ln 3\cdot 3^{2x}}$ | $\frac{1}{2\ln 3}\int \frac{1}{u}\,du = \frac{1}{2\ln 3}\cdot\ln|u| + C$

$\boxed{\frac{1}{2\ln 3}\ln|1+3^{2x}| + C}$

78. $\int 2^{\sin x}\cos x\,dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$\int 2^u \cdot \cos x \cdot \frac{du}{\cos x}$ | $\int 2^u\,du = \frac{1}{\ln 2}\cdot 2^u + C$

$= \boxed{\frac{2^{\sin x}}{\ln 2} + C}$