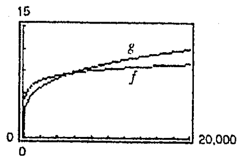
(b) $f(x) = \ln x,\ g(x) = \sqrt[4]{x}$



$f'(x) = \dfrac{1}{x},\ g'(x) = \dfrac{1}{4\sqrt[4]{x^3}}$

For $x > 256$, $g'(x) > f'(x)$. g is increasing at a faster rate than f for "large" values of x. $f(x) = \ln x$ increases very slowly for "large" values of x.

Section 5.2 The Natural Logarithmic Function: Integration

1. $\displaystyle\int \frac{5}{x}\,dx = 5\int \frac{1}{x}\,dx = 5\ln|x| + C$

2. $\displaystyle\int \frac{10}{x}\,dx = 10\int \frac{1}{x}\,dx = 10\ln|x| + C$

3. $u = x + 1,\ du = dx$

$$\int \frac{1}{x+1}\,dx = \ln|x+1| + C$$

4. $u = x - 5,\ du = dx$

$$\int \frac{1}{x-5}\,dx = \ln|x-5| + C$$

5. $u = 2x + 5,\ du = 2\,dx$

$$\int \frac{1}{2x+5}\,dx = \frac{1}{2}\int \frac{1}{2x+5}(2)\,dx = \frac{1}{2}\ln|2x+5| + C$$

6. $u = 5 - 4x,\ du = -4\,dx$

$$\int \frac{9}{5-4x}\,dx = -\frac{9}{4}\int \frac{1}{5-4x}(-4dx) = -\frac{9}{4}\ln|5-4x| + C$$

7. $u = x^2 - 3,\ du = 2x\,dx$

$$\int \frac{x}{x^2-3}\,dx = \frac{1}{2}\int \frac{1}{x^2-3}(2x)\,dx = \frac{1}{2}\ln|x^2-3| + C$$

8. $u = 5 - x^3,\ du = -3x^2\,dx$

$$\int \frac{x^2}{5-x^3}\,dx = -\frac{1}{3}\int \frac{1}{5-x^3}(-3x^2)\,dx = -\frac{1}{3}\ln|5-x^3| + C$$

9. $u = x^4 + 3x,\ du = (4x^3 + 3)\,dx$

$$\int \frac{4x^3+3}{x^4+3x}\,dx = \int \frac{1}{x^4+3x}(4x^3+3)\,dx = \ln|x^4+3x| + C$$

10. $u = x^3 - 3x^2,\ du = (3x^2 - 6x)\,dx = 3(x^2 - 2x)\,dx$

$$\int \frac{x^2-2x}{x^3-3x^2}\,dx = \frac{1}{3}\int \frac{1}{x^3-3x^2}(3x^2-6x)\,dx = \frac{1}{3}\ln|x^3-3x^2| + C$$

11. $\displaystyle\int \frac{x^2-4}{x}\,dx = \int\left(x - \frac{4}{x}\right)dx = \frac{x^2}{2} - 4\ln|x| + C = \frac{x^2}{2} - \ln(x^4) + C$

12. $\displaystyle\int \frac{x^3-8x}{x^2}\,dx = \int\left(x - \frac{8}{x}\right)dx = \frac{x^2}{2} - 8\ln|x| + C$

13. $u = x^3 + 3x^2 + 9x,\ du = 3(x^2 + 2x + 3)\,dx$

$$\int \frac{x^2+2x+3}{x^3+3x^2+9x}\,dx = \frac{1}{3}\int \frac{3(x^2+2x+3)}{x^3+3x^2+9x}\,dx = \frac{1}{3}\ln|x^3+3x^2+9x| + C$$

14. $u = x^3 + 6x^2 + 5,\ du = \left(3x^2 + 12x\right)dx = 3\left(x^2 + 4x\right)dx$

$$\int \frac{x^2 + 4x}{x^3 + 6x^2 + 5}\,dx = \frac{1}{3}\int \frac{1}{x^3 + 6x^2 + 5}\,3\left(x^2 + 4x\right)dx$$
$$= \frac{1}{3}\ln\left|x^3 + 6x^2 + 5\right| + C$$

15. $$\int \frac{x^2 - 3x + 2}{x + 1}\,dx = \int\left(x - 4 + \frac{6}{x + 1}\right)dx$$
$$= \frac{x^2}{2} - 4x + 6\ln|x + 1| + C$$

16. $$\int \frac{2x^2 + 7x - 3}{x - 2}\,dx = \int\left(2x + 11 + \frac{19}{x - 2}\right)dx$$
$$= x^2 + 11x + 19\ln|x - 2| + C$$

17. $$\int \frac{x^3 - 3x^2 + 5}{x - 3}\,dx = \int\left(x^2 + \frac{5}{x - 3}\right)dx$$
$$= \frac{x^3}{3} + 5\ln|x - 3| + C$$

18. $$\int \frac{x^3 - 6x - 20}{x + 5}\,dx = \int\left(x^2 - 5x + 19 - \frac{115}{x + 5}\right)dx$$
$$= \frac{x^3}{3} - \frac{5x^2}{2} + 19x - 115\ln|x + 5| + C$$

19. $$\int \frac{x^4 + x - 4}{x^2 + 2}\,dx = \int\left(x^2 - 2 + \frac{x}{x^2 + 2}\right)dx$$
$$= \frac{x^3}{3} - 2x + \frac{1}{2}\ln\left(x^2 + 2\right) + C$$
$$= \frac{x^3}{3} - 2x + \ln\sqrt{x^2 + 2} + C$$

20. $$\int \frac{x^3 - 4x^2 - 4x + 20}{x^2 - 5}\,dx = \int\left(x - 4 + \frac{x}{x^2 - 5}\right)dx$$
$$= \frac{x^2}{2} - 4x + \frac{1}{2}\ln\left|x^2 - 5\right| + C$$

21. $u = \ln x,\ du = \frac{1}{x}dx$

$$\int \frac{(\ln x)^2}{x}\,dx = \frac{1}{3}(\ln x)^3 + C$$

22. $$\int \frac{1}{x\ln\left(x^3\right)}\,dx = \frac{1}{3}\int \frac{1}{\ln x}\cdot\frac{1}{x}\,dx$$
$$= \frac{1}{3}\ln\left|\ln|x|\right| + C$$

23. $u = 1 - 3\sqrt{x},\ du = \frac{-3}{2\sqrt{x}}$

$$\int \frac{1}{\sqrt{x}\left(1 - 3\sqrt{x}\right)}\,dx = -\frac{2}{3}\int \frac{1}{1 - 3\sqrt{x}}\left(\frac{-3}{2\sqrt{x}}\right)dx$$
$$= -\frac{2}{3}\ln\left|1 - 3\sqrt{x}\right| + C$$

24. $u = 1 + x^{1/3},\ du = \frac{1}{3x^{2/3}}\,dx$

$$\int \frac{1}{x^{2/3}\left(1 + x^{1/3}\right)}\,dx = 3\int \frac{1}{1 + x^{1/3}}\left(\frac{1}{3x^{2/3}}\right)dx$$
$$= 3\ln\left|1 + x^{1/3}\right| + C$$

25. $$\int \frac{2x}{(x - 1)^2}\,dx = \int \frac{2x - 2 + 2}{(x - 1)^2}\,dx$$
$$= \int \frac{2(x - 1)}{(x - 1)^2}\,dx + 2\int \frac{1}{(x - 1)^2}\,dx$$
$$= 2\int \frac{1}{x - 1}\,dx + 2\int \frac{1}{(x - 1)^2}\,dx$$
$$= 2\ln|x - 1| - \frac{2}{(x - 1)} + C$$

26. $$\int \frac{x(x - 2)}{(x - 1)^3}\,dx = \int \frac{x^2 - 2x + 1 - 1}{(x - 1)^3}\,dx$$
$$= \int \frac{(x - 1)^2}{(x - 1)^3}\,dx - \int \frac{1}{(x - 1)^3}\,dx$$
$$= \int \frac{1}{x - 1}\,dx - \int \frac{1}{(x - 1)^3}\,dx$$
$$= \ln|x - 1| + \frac{1}{2(x - 1)^2} + C$$

27. $u = 1 + \sqrt{2x},\ du = \frac{1}{\sqrt{2x}}\,dx \Rightarrow (u - 1)\,du = dx$

$$\int \frac{1}{1 + \sqrt{2x}}\,dx = \int \frac{(u - 1)}{u}\,du = \int\left(1 - \frac{1}{u}\right)du$$
$$= u - \ln|u| + C_1$$
$$= \left(1 + \sqrt{2x}\right) - \ln\left|1 + \sqrt{2x}\right| + C_1$$
$$= \sqrt{2x} - \ln\left(1 + \sqrt{2x}\right) + C$$

where $C = C_1 + 1$.

28. $u = 1 + \sqrt{3x},\ du = \dfrac{3}{2\sqrt{3x}}\,dx \Rightarrow dx = \dfrac{2}{3}(u - 1)\,du$

$$\begin{aligned}\int \frac{1}{1+\sqrt{3x}}\,dx &= \int \frac{1}{u}\frac{2}{3}(u-1)\,du \\ &= \frac{2}{3}\int\left(1 - \frac{1}{u}\right)du \\ &= \frac{2}{3}\left[u - \ln|u|\right] + C \\ &= \frac{2}{3}\left[1 + \sqrt{3x} - \ln\left(1+\sqrt{3x}\right)\right] + C \\ &= \frac{2}{3}\sqrt{3x} - \frac{2}{3}\ln\left(1+\sqrt{3x}\right) + C_1\end{aligned}$$

29. $u = \sqrt{x} - 3,\ du = \dfrac{1}{2\sqrt{x}}\,dx \Rightarrow 2(u+3)du = dx$

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{x}-3}\,dx &= 2\int\frac{(u+3)^2}{u}\,du \\ &= 2\int\frac{u^2+6u+9}{u}\,du = 2\int\left(u + 6 + \frac{9}{u}\right)du \\ &= 2\left[\frac{u^2}{2} + 6u + 9\ln|u|\right] + C_1 \\ &= u^2 + 12u + 18\ln|u| + C_1 \\ &= \left(\sqrt{x}-3\right)^2 + 12\left(\sqrt{x}-3\right) + 18\ln\left|\sqrt{x}-3\right| + C_1 \\ &= x + 6\sqrt{x} + 18\ln\left|\sqrt{x}-3\right| + C\end{aligned}$$

where $C = C_1 - 27$.

30. $u = x^{1/3} - 1,\ du = \dfrac{1}{3x^{2/3}}\,dx \Rightarrow dx = 3(u+1)^2\,du$

$$\begin{aligned}\int \frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\,dx &= \int\frac{u+1}{u}3(u+1)^2du \\ &= 3\int\frac{u+1}{u}\left(u^2+2u+1\right)du \\ &= 3\int\left(u^2 + 3u + 3 + \frac{1}{u}\right)du \\ &= 3\left[\frac{u^3}{3} + \frac{3u^2}{2} + 3u + \ln|u|\right] + C \\ &= 3\left[\frac{\left(x^{1/3}-1\right)^3}{3} + \frac{3\left(x^{1/3}-1\right)^2}{2} + 3\left(x^{1/3}-1\right) + \ln\left|x^{1/3}-1\right|\right] + C \\ &= 3\ln\left|x^{1/3}-1\right| + \frac{3x^{2/3}}{2} + 3x^{1/3} + x + C_1\end{aligned}$$

31. $$\begin{aligned}\int\cot\left(\frac{\theta}{3}\right)d\theta &= 3\int\cot\left(\frac{\theta}{3}\right)\left(\frac{1}{3}\right)d\theta \\ &= 3\ln\left|\sin\frac{\theta}{3}\right| + C\end{aligned}$$

32. $$\begin{aligned}\int\tan 5\theta\,d\theta &= \frac{1}{5}\int\frac{5\sin 5\theta}{\cos 5\theta}\,d\theta \\ &= -\frac{1}{5}\ln|\cos 5\theta| + C\end{aligned}$$

33. $\int \csc 2x\,dx = \frac{1}{2}\int(\csc 2x)(2)\,dx$

$= -\frac{1}{2}\ln|\csc 2x + \cot 2x| + C$

34. $\int \sec\frac{x}{2}\,dx = 2\int \sec\frac{x}{2}\left(\frac{1}{2}\right)dx = 2\ln\left|\sec\frac{x}{2} + \tan\frac{x}{2}\right| + C$

35. $\int(\cos 3\theta - 1)\,d\theta = \frac{1}{3}\int\cos 3\theta(3)\,d\theta - \int d\theta$

$= \frac{1}{3}\sin 3\theta - \theta + C$

36. $\int\left(2 - \tan\frac{\theta}{4}\right)d\theta = \int 2\,d\theta - 4\int\tan\frac{\theta}{4}\left(\frac{1}{4}\right)d\theta$

$= 2\theta + 4\ln\left|\cos\frac{\theta}{4}\right| + C$

37. $u = 1 + \sin t,\ du = \cos t\,dt$

$\int\frac{\cos t}{1 + \sin t}\,dt = \ln|1 + \sin t| + C$

38. $u = \cot t,\ du = -\csc^2 t\,dt$

$\int\frac{\csc^2 t}{\cot t}\,dt = -\ln|\cot t| + C$

39. $u = \sec x - 1,\ du = \sec x\tan x\,dx$

$\int\frac{\sec x\tan x}{\sec x - 1}\,dx = \ln|\sec x - 1| + C$

40. $\int(\sec 2x + \tan 2x)\,dx = \frac{1}{2}\int(\sec 2x + \tan 2x)(2)\,dx = \frac{1}{2}\ln|\sec 2x + \tan 2x| - \ln|\cos 2x| + C$

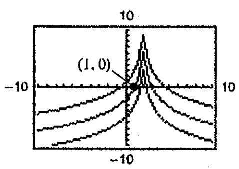
41. $y = \int\frac{3}{2 - x}\,dx$

$= -3\int\frac{1}{x - 2}\,dx$

$= -3\ln|x - 2| + C$

$(1, 0)$: $0 = -3\ln|1 - 2| + C \Rightarrow C = 0$

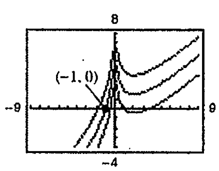
$y = -3\ln|x - 2|$



42. $y = \int\frac{x - 2}{x}\,dx = \int\left(1 - \frac{2}{x}\right)dx = x - 2\ln|x| + C$

$(-1, 0)$: $0 = -1 - 2\ln|-1| + C = -1 + C \Rightarrow C = 1$

$y = x - 2\ln|x| + 1$

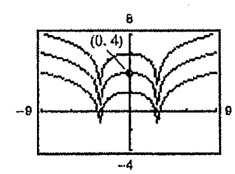


43. $y = \int\frac{2x}{x^2 - 9}\,dx$

$= \ln|x^2 - 9| + C$

$(0, 4)$: $4 = \ln|0 - 9| + C \Rightarrow C = 4 - \ln 9$

$y = \ln|x^2 - 9| + 4 - \ln 9$

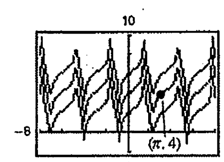


44. $r = \int\frac{\sec^2 t}{\tan t + 1}\,dt$

$= \ln|\tan t + 1| + C$

$(\pi, 4)$: $4 = \ln|0 + 1| + C \Rightarrow C = 4$

$r = \ln|\tan t + 1| + 4$



45. $f''(x) = \dfrac{2}{x^2} = 2x^{-2}, \quad x > 0$

$f'(x) = \dfrac{-2}{x} + C$

$f'(1) = 1 = -2 + C \Rightarrow C = 3$

$f'(x) = \dfrac{-2}{x} + 3$

$f(x) = -2 \ln x + 3x + C_1$

$f(1) = 1 = -2(0) + 3 + C_1 \Rightarrow C_1 = -2$

$f(x) = -2 \ln x + 3x - 2$

46. $f''(x) = \dfrac{-4}{(x-1)^2} - 2 = -4(x-1)^{-2} - 2, \quad x > 1$

$f'(x) = \dfrac{4}{(x-1)} - 2x + C$

$f'(2) = 0 = 4 - 4 + C \Rightarrow C = 0$

$f'(x) = \dfrac{4}{x-1} - 2x$

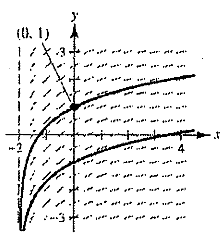
$f(x) = 4\ln(x-1) - x^2 + C_1$

$f(2) = 3 = 4(0) - 4 + C_1 \Rightarrow C_1 = 7$

$f(x) = 4\ln(x-1) - x^2 + 7$

47. $\dfrac{dy}{dx} = \dfrac{1}{x+2}, (0, 1)$

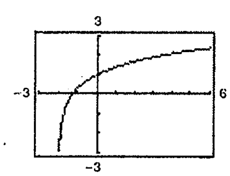
(a)



(b) $y = \displaystyle\int \frac{1}{x+2}\,dx = \ln|x+2| + C$

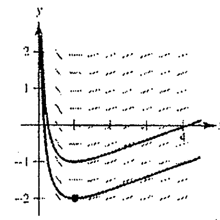
$y(0) = 1 \Rightarrow 1 = \ln 2 + C \Rightarrow C = 1 - \ln 2$

So, $y = \ln|x+2| + 1 - \ln 2 = \ln\left(\dfrac{x+2}{2}\right) + 1.$



48. $\dfrac{dy}{dx} = \dfrac{\ln x}{x}, (1, -2)$

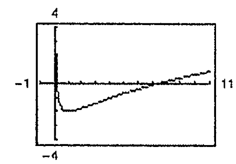
(a)



(b) $y = \displaystyle\int \frac{\ln x}{x}\,dx = \frac{(\ln x)^2}{2} + C$

$y(1) = -2 \Rightarrow -2 = \dfrac{(\ln 1)^2}{2} + C \Rightarrow C = -2$

So, $y = \dfrac{(\ln x)^2}{2} - 2.$



49. $\displaystyle\int_0^4 \frac{5}{3x+1}\,dx = \left[\frac{5}{3}\ln|3x+1|\right]_0^4 = \frac{5}{3}\ln 13 \approx 4.275$

50. $\displaystyle\int_{-1}^{1} \frac{1}{2x+3}\,dx = \frac{1}{2}\Big[\ln|2x+3|\Big]_{-1}^{1}$

$= \dfrac{1}{2}[\ln 5 - \ln 1] = \dfrac{1}{2}\ln 5 \approx 0.805$

51. $u = 1 + \ln x, du = \dfrac{1}{x}\,dx$

$\displaystyle\int_1^e \frac{(1+\ln x)^2}{x}\,dx = \left[\frac{1}{3}(1+\ln x)^3\right]_1^e = \frac{7}{3}$

52. $u = \ln x, du = \dfrac{1}{x}dx$

$\displaystyle\int_e^{e^2} \frac{1}{x\ln x}\,dx = \int_e^{e^2}\left(\frac{1}{\ln x}\right)\frac{1}{x}\,dx = \Big[\ln|\ln|x||\Big]_e^{e^2} = \ln 2$

≈ 0.693

53. $\displaystyle\int_0^2 \frac{x^2-2}{x+1}\,dx = \int_0^2\left(x - 1 - \frac{1}{x+1}\right)dx$

$= \left[\dfrac{1}{2}x^2 - x - \ln|x+1|\right]_0^2 = -\ln 3$

≈ -1.099

54. $\displaystyle\int_0^1 \frac{x-1}{x+1}\,dx = \int_0^1 1\,dx + \int_0^1 \frac{-2}{x+1}\,dx$

$= \Big[x - 2\ln|x+1|\Big]_0^1 = 1 - 2\ln 2$

≈ -0.386

55. $\int_1^2 \frac{1 - \cos\theta}{\theta - \sin\theta}\,d\theta = \left[\ln|\theta - \sin\theta|\right]_1^2$

$= \ln\left|\frac{2 - \sin 2}{1 - \sin 1}\right| \approx 1.929$

56. $u = 2\theta, du = 2\,d\theta, \theta = \frac{\pi}{8} \Rightarrow u = \frac{\pi}{4}, \theta = \frac{\pi}{4} \Rightarrow u = \frac{\pi}{2}$

$$\int_{\pi/8}^{\pi/4} (\csc 2\theta - \cot 2\theta)\,d\theta = \frac{1}{2}\int_{\pi/4}^{\pi/2} (\csc u - \cot u)\,du$$

$$= \frac{1}{2}\left[-\ln|\csc u + \cot u| - \ln|\sin u|\right]_{\pi/4}^{\pi/2}$$

$$= \frac{1}{2}\left[-\ln(1 + 0) - \ln(1) + \ln\left(\sqrt{2} + 1\right) + \ln\frac{\sqrt{2}}{2}\right]$$

$$= \frac{1}{2}\left[\ln\left(\sqrt{2} + 1\right) + \ln\frac{\sqrt{2}}{2}\right]$$

$$= \frac{1}{2}\ln\left(1 + \frac{\sqrt{2}}{2}\right)$$

57. $\int \frac{1}{1 + \sqrt{x}}\,dx = 2\sqrt{x} - 2\ln\left(1 + \sqrt{x}\right) + C$

58. $\int \frac{1 - \sqrt{x}}{1 + \sqrt{x}}\,dx = 4\sqrt{x} - x - 4\ln\left(1 + \sqrt{x}\right) + C$

59. $\int \frac{\sqrt{x}}{x - 1}\,dx = \ln\left(\frac{\sqrt{x} - 1}{\sqrt{x} + 1}\right) + 2\sqrt{x} + C$

60. $\int \frac{x^2}{x - 1}\,dx = \ln|x - 1| + \frac{x^2}{2} + x + C$

61. $\int_{\pi/4}^{\pi/2} (\csc x - \sin x)\,dx = \ln\left(\sqrt{2} + 1\right) - \frac{\sqrt{2}}{2} \approx 0.174$

62. $\int_{-\pi/4}^{\pi/4} \frac{\sin^2 x - \cos^2 x}{\cos x}\,dx = \ln\left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1}\right) - 2\sqrt{2}$

≈ -1.066

Note: In Exercises 63–66, you can use the Second Fundamental Theorem of Calculus or integrate the function.

63. $F(x) = \int_1^x \frac{1}{t}\,dt$

$F'(x) = \frac{1}{x}$

64. $F(x) = \int_0^x \tan t\,dt$

$F'(x) = \tan x$

65. $F(x) = \int_1^{3x} \frac{1}{t}\,dt$

$F'(x) = \frac{1}{3x}(3) = \frac{1}{x}$

(by Second Fundamental Theorem of Calculus)

Alternate Solution:

$F(x) = \int_1^{3x} \frac{1}{t}\,dt = \left[\ln|t|\right]_1^{3x} = \ln|3x|$

$F'(x) = \frac{1}{3x}(3) = \frac{1}{x}$

66. $F(x) = \int_1^{x^2} \frac{1}{t}\,dt$

$F'(x) = \frac{2x}{x^2} = \frac{2}{x}$

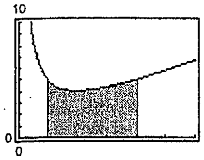
67. $A = \int_1^3 \frac{6}{x}\,dx = \left[6\ln|x|\right]_1^3 = 6\ln 3$

68. $A = \int_2^4 \frac{2}{x\ln x}\,dx = 2\int_2^4 \frac{1}{\ln x}\frac{1}{x}\,dx = 2\ln|\ln x|\Big]_2^4 = 2\left[\ln(\ln 4) - \ln(\ln 2)\right] = 2\ln\left(\frac{2\ln 2}{\ln 2}\right) = 2\ln 2$

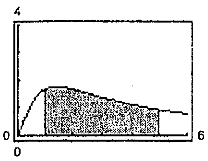
69. $A = \int_0^{\pi/4} \tan x\,dx = -\ln|\cos x|\Big]_0^{\pi/4} = -\ln\frac{\sqrt{2}}{2} + 0 = \ln\sqrt{2} = \frac{\ln 2}{2}$

70. $A = \int_{\pi/4}^{3\pi/4} \frac{\sin x}{1 + \cos x}\, dx = -\ln|1 + \cos x|\Big]_{\pi/4}^{3\pi/4} = -\ln\left(1 - \frac{\sqrt{2}}{2}\right) + \ln\left(1 + \frac{\sqrt{2}}{2}\right) = \ln\left(\frac{2 + \sqrt{2}}{2 - \sqrt{2}}\right) = \ln\left(3 + 2\sqrt{2}\right)$

71. $A = \int_1^4 \frac{x^2 + 4}{x}\, dx = \int_1^4 \left(x + \frac{4}{x}\right) dx = \left[\frac{x^2}{2} + 4 \ln x\right]_1^4 = (8 + 4 \ln 4) - \frac{1}{2} = \frac{15}{2} + 8 \ln 2 \approx 13.045$

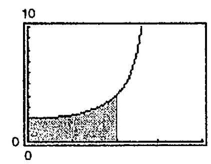


72. $A = \int_1^5 \frac{5x}{x^2 + 2}\, dx = \frac{5}{2}\int_1^5 \frac{1}{x^2 + 2}(2x\, dx) = \left[\frac{5}{2} \ln|x^2 + 2|\right]_1^5 = \frac{5}{2}(\ln 27 - \ln 3) = \frac{5}{2} \ln 9 = 5 \ln 3 \approx 5.4931$

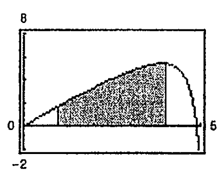


73. $\int_0^2 2 \sec \frac{\pi x}{6}\, dx = \frac{12}{\pi}\int_0^2 \sec\left(\frac{\pi x}{6}\right)\frac{\pi}{6}\, dx = \frac{12}{\pi}\left[\ln\left|\sec \frac{\pi x}{6} + \tan \frac{\pi x}{6}\right|\right]_0^2$

$= \frac{12}{\pi}\left(\ln\left|\sec \frac{\pi}{3} + \tan \frac{\pi}{3}\right| - \ln|1 + 0|\right) = \frac{12}{\pi} \ln\left(2 + \sqrt{3}\right) \approx 5.03041$



74. $\int_1^4 \left(2x - \tan(0.3x)\right) dx = \left[x^2 + \frac{10}{3} \ln|\cos(0.3x)|\right]_1^4 = \left[16 + \frac{10}{3} \ln \cos(1.2)\right] - \left[1 + \frac{10}{3} \ln \cos(0.3)\right] \approx 11.7686$



75. $f(x) = \frac{12}{x}, b - a = 5 - 1 = 4, n = 4$

Trapezoid: $\frac{4}{2(4)}\left[f(1) + 2f(2) + 2f(3) + 2f(4) + f(5)\right] = \frac{1}{2}[12 + 12 + 8 + 6 + 2.4] = 20.2$

Simpson: $\frac{4}{3(4)}\left[f(1) + 4f(2) + 2f(3) + 4f(4) + f(5)\right] = \frac{1}{3}[12 + 24 + 8 + 12 + 2.4] \approx 19.4667$

Calculator: $\int_1^5 \frac{12}{x}\, dx \approx 19.3133$

Exact: $12 \ln 5$

76. $f(x) = \dfrac{8x}{x^2 + 4}, b - a = 4 - 0 = 4, n = 4$

Trapezoid: $\dfrac{4}{2(4)}\left[f(0) + 2f(1) + 2f(2) + 2f(3) + f(4)\right] = \dfrac{1}{2}\left[0 + 3.2 + 4 + 3.6923 + 1.6\right] \approx 6.2462$

Simpson: $\dfrac{4}{3(4)}\left[f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)\right] \approx 6.4615$

Calculator: $\displaystyle\int_0^4 \frac{8x}{x^2 + 4}\,dx \approx 6.438$

Exact: $4 \ln 5$

77. $f(x) = \ln x, b - a = 6 - 2 = 4, n = 4$

Trapezoid: $\dfrac{4}{2(4)}\left[f(2) + 2f(3) + 2f(4) + 2f(5) + f(6)\right] = \dfrac{1}{2}\left[0.6931 + 2.1972 + 2.7726 + 3.2189 + 1.7918\right] \approx 5.3368$

Simpson: $\dfrac{4}{3(4)}\left[f(2) + 4f(3) + 2f(4) + 4f(5) + f(6)\right] \approx 5.3632$

Calculator: $\displaystyle\int_2^6 \ln x\,dx \approx 5.3643$

78. $f(x) = \sec x, b - a = \dfrac{\pi}{3} - \left(-\dfrac{\pi}{3}\right) = \dfrac{2\pi}{3}, n = 4$

Trapezoid: $\dfrac{2\pi/3}{2(4)}\left[f\left(-\dfrac{\pi}{3}\right) + 2f\left(-\dfrac{\pi}{6}\right) + 2f(0) + 2f\left(\dfrac{\pi}{6}\right) + f\left(\dfrac{\pi}{3}\right)\right] \approx \dfrac{\pi}{12}\left[2 + 2.3094 + 2 + 2.3094 + 2\right] \approx 2.780$

Simpson: $\dfrac{2\pi/3}{3(4)}\left[f\left(-\dfrac{\pi}{3}\right) + 4f\left(-\dfrac{\pi}{6}\right) + 2f(0) + 4f\left(\dfrac{\pi}{6}\right) + f\left(\dfrac{\pi}{3}\right)\right] \approx 2.6595$

Calculator: $\displaystyle\int_{-\pi/3}^{\pi/3} \sec x\,dx \approx 2.6339$

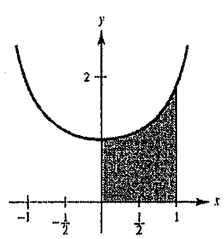
79. Power Rule

80. Substitution: $(u = x^2 + 4)$ and Power Rule

81. Substitution: $(u = x^2 + 4)$ and Log Rule

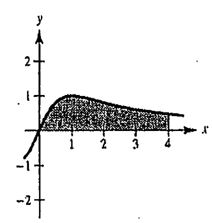
82. Substitution: $(u = \tan x)$ and Log Rule

83.



$A \approx 1.25$; Matches (d)

84.



$A \approx 3$; Matches (a)

85.
$$\int_1^x \frac{3}{t}\,dt = \int_{1/4}^x \frac{1}{t}\,dt$$
$$\left[3 \ln|t|\right]_1^x = \left[\ln|t|\right]_{1/4}^x$$
$$3 \ln x = \ln x - \ln\left(\frac{1}{4}\right)$$
$$2 \ln x = -\ln\left(\frac{1}{4}\right) = \ln 4$$
$$\ln x = \frac{1}{2} \ln 4 = \ln 2$$
$$x = 2$$

86. $\int_1^x \frac{1}{t}\,dt = \left[\ln|t|\right]_1^x = \ln x$ (assume $x > 0$)

(a) $\ln x = \ln 5 \Rightarrow x = 5$

(b) $\ln x = 1 \Rightarrow x = e$

87. $\int \cot u\,du = \int \frac{\cos u}{\sin u}\,du = \ln|\sin u| + C$

Alternate solution:

$$\frac{d}{du}\left[\ln|\sin u| + C\right] = \frac{1}{\sin u}\cos u + C = \cot u + C$$

88. $\int \csc u\,du = \int \csc u\left(\frac{\csc u + \cot u}{\csc u + \cot u}\right)du = -\int \frac{1}{\csc u + \cot u}\left(-\csc u \cot u - \csc^2 u\right)du = -\ln|\csc u + \cot u| + C$

Alternate solution:

$$\frac{d}{du}\left[-\ln|\csc u + \cot u| + C\right] = -\frac{1}{\csc u + \cot u}\left(-\csc u \cot u - \csc^2 u\right) = \frac{\csc u(\cot u + \csc u)}{\csc u + \cot u} = \csc u$$

89. $-\ln|\cos x| + C = \ln\left|\frac{1}{\cos x}\right| + C = \ln|\sec x| + C$

90. $\ln|\sin x| + C = \ln\left|\frac{1}{\csc x}\right| + C = -\ln|\csc x| + C$

91.
$$\begin{aligned}\ln|\sec x + \tan x| + C &= \ln\left|\frac{(\sec x + \tan x)(\sec x - \tan x)}{(\sec x - \tan x)}\right| + C\\ &= \ln\left|\frac{\sec^2 x - \tan^2 x}{\sec x - \tan x}\right| + C\\ &= \ln\left|\frac{1}{\sec x - \tan x}\right| + C = -\ln|\sec x - \tan x| + C\end{aligned}$$

92.
$$\begin{aligned}-\ln|\csc x + \cot x| + C &= -\ln\left|\frac{(\csc x + \cot x)(\csc x - \cot x)}{(\csc x - \cot x)}\right| + C\\ &= -\ln\left|\frac{\csc^2 x - \cot^2 x}{\csc x - \cot x}\right| + C\\ &= -\ln\left|\frac{1}{\csc x - \cot x}\right| + C = \ln|\csc x - \cot x| + C\end{aligned}$$

93.
$$\begin{aligned}\text{Average value} &= \frac{1}{4-2}\int_2^4 \frac{8}{x^2}\,dx\\ &= 4\int_2^4 x^{-2}\,dx\\ &= \left[-4\frac{1}{x}\right]_2^4\\ &= -4\left(\frac{1}{4} - \frac{1}{2}\right) = 1\end{aligned}$$

94.
$$\begin{aligned}\text{Average value} &= \frac{1}{4-2}\int_2^4 \frac{4(x+1)}{x^2}\,dx\\ &= 2\int_2^4\left(\frac{1}{x} + \frac{1}{x^2}\right)dx\\ &= 2\left[\ln x - \frac{1}{x}\right]_2^4\\ &= 2\left[\ln 4 - \frac{1}{4} - \ln 2 + \frac{1}{2}\right]\\ &= 2\left[\ln 2 + \frac{1}{4}\right] = \ln 4 + \frac{1}{2} \approx 1.8863\end{aligned}$$

95. Average value $= \dfrac{1}{e-1}\displaystyle\int_1^e \frac{2 \ln x}{x}\,dx$

$= \dfrac{2}{e-1}\left[\dfrac{(\ln x)^2}{2}\right]_1^e$

$= \dfrac{1}{e-1}(1-0)$

$= \dfrac{1}{e-1} \approx 0.582$

96. Average value $= \dfrac{1}{2-0}\displaystyle\int_0^2 \sec\frac{\pi x}{6}\,dx$

$= \left[\dfrac{1}{2}\left(\dfrac{6}{\pi}\right)\ln\left|\sec\dfrac{\pi x}{6} + \tan\dfrac{\pi x}{6}\right|\right]_0^2$

$= \dfrac{3}{\pi}\left[\ln\left(2+\sqrt{3}\right) - \ln(1+0)\right]$

$= \dfrac{3}{\pi}\ln\left(2+\sqrt{3}\right)$

97. $P(t) = \displaystyle\int \frac{3000}{1+0.25t}\,dt = (3000)(4)\int\frac{0.25}{1+0.25t}\,dt$

$= 12{,}000 \ln|1+0.25t| + C$

$P(0) = 12{,}000\ln|1+0.25(0)| + C = 1000$

$C = 1000$

$P(t) = 12{,}000\ln|1+0.25t| + 1000$

$= 1000[12\ln|1+0.25t| + 1]$

$P(3) = 1000[12(\ln 1.75) + 1] \approx 7715$

98. $\dfrac{dS}{dt} = \dfrac{k}{t}$

$S(t) = \displaystyle\int\frac{k}{t}\,dt = k\ln|t| + C = k\ln t + C$ because $t > 1$.

$S(2) = k\ln 2 + C = 200$

$S(4) = k\ln 4 + C = 300$

Solving this system yields $k = 100/\ln 2$ and $C = 100$. So,

$S(t) = \dfrac{100\ln t}{\ln 2} + 100 = 100\left[\dfrac{\ln t}{\ln 2} + 1\right].$

99. $t = \dfrac{10}{\ln 2}\displaystyle\int_{250}^{300}\frac{1}{T-100}\,dT$

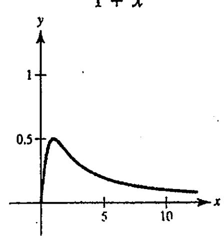
$= \dfrac{10}{\ln 2}\left[\ln(T-100)\right]_{250}^{300} = \dfrac{10}{\ln 2}[\ln 200 - \ln 150]$

$= \dfrac{10}{\ln 2}\left[\ln\left(\dfrac{4}{3}\right)\right] \approx 4.1504$ min

100. $\dfrac{1}{50-40}\displaystyle\int_{40}^{50}\frac{90{,}000}{400+3x}\,dx = \left[3000\ln|400+3x|\right]_{40}^{50}$

$\approx \$168.27$

101. $f(x) = \dfrac{x}{1+x^2}$



(a) $y = \dfrac{1}{2}x$ intersects $f(x) = \dfrac{x}{1+x^2}$:

$\dfrac{1}{2}x = \dfrac{x}{1+x^2}$

$1 + x^2 = 2$

$x = 1$

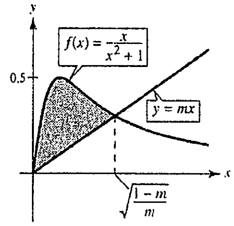
$A = \displaystyle\int_0^1\left(\left[\frac{x}{1+x^2}\right] - \frac{1}{2}x\right)dx = \left[\frac{1}{2}\ln(x^2+1) - \frac{x^2}{4}\right]_0^1 = \frac{1}{2}\ln 2 - \frac{1}{4}$

(b) $f'(x) = \dfrac{(1 + x^2) - x(2x)}{(1 + x^2)^2} = \dfrac{1 - x^2}{(1 + x^2)^2}$

$f'(0) = 1$

So, for $0 < m < 1$, the graphs of f and $y = mx$ enclose a finite region.

(c)



$f(x) = \dfrac{x}{x^2 + 1}$ intersects $y = mx$:

$$\frac{x}{1 + x^2} = mx$$
$$1 = m + mx^2$$
$$x^2 = \frac{1 - m}{m}$$
$$x = \sqrt{\frac{1 - m}{m}}$$

$$A = \int_0^{\sqrt{(1-m)/m}} \left(\frac{x}{1 + x^2} - mx\right) dx, \quad 0 < m < 1$$
$$= \left[\frac{1}{2}\ln(1 + x^2) - \frac{mx^2}{2}\right]_0^{\sqrt{(1-m)/m}}$$
$$= \frac{1}{2}\ln\left(1 + \frac{1 - m}{m}\right) - \frac{1}{2}m\left(\frac{1 - m}{m}\right)$$
$$= \frac{1}{2}\ln\left(\frac{1}{m}\right) - \frac{1}{2}(1 - m)$$
$$= \frac{1}{2}\left[m - \ln(m) - 1\right]$$

102. (a) At $x = -1$, $f'(-1) \approx \frac{1}{2}$.

The slope of f at $x = -1$ is approximately $\frac{1}{2}$.

(b) Since the slope is positive for $x > -2$, f is increasing on $(-2, \infty)$. Similarly, f is decreasing on $(-\infty, -2)$.

103. False

$\frac{1}{2}(\ln x) = \ln(x^{1/2}) \neq (\ln x)^{1/2}$

104. False

$\dfrac{d}{dx}[\ln x] = \dfrac{1}{x}$

105. True

$\int \frac{1}{x}\,dx = \ln|x| + C_1 = \ln|x| + \ln|C| = \ln|Cx|, C \neq 0$

106. False; the integrand has a nonremovable discontinuity at $x = 0$.