5.3 AP Practice Problems (p. 347-348)

1. If $f''(x) = x(x+1)^2(x-2)$ is the second derivative of the function f, then list the x-coordinate(s) of the inflection point(s) of f.

(A) $-1, 0,$ and 2 (B) 0 only

(C) 2 only $\boxed{\text{(D)}}$ 0 and 2 only

$f''(x)$: $+$ (-2) | -1 | $+$ (-0.5) | 0 | $-$ (1) | 2 | $+$ (3)

POI at $x=0, x=2$ since $f''(x)$ change signs.

* set $f''(x)=0$
* find critical points
* create $f''(x)$ sign line

$0 = x(x+1)^2(x-2)$

$x = 0, -1, 2$

2. If $f(x) = x + \sin x$, find the smallest positive number x at which the function changes concavity.

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ $\boxed{\text{(C)}}$ π (D) $\frac{3\pi}{2}$

* create $f''(x)$ sign line

$f'(x) = 1 + \cos x$

$f''(x) = -\sin x$

$0 = -\sin x$

$x = 0, \pi, 2\pi, 3\pi, \ldots$

$f''(x)$: 0 | $-$ $(\pi/2)$ | π | $+$ $(3\pi/2)$

POI at $x=\pi$ since $f''(x)$ change signs

3. The function $g(x) = x^5 + x^3 - 2x - 1$ has a local minimum at

(A) $x = -\frac{\sqrt{10}}{5}$ $\boxed{\text{(B)}}$ $x = \frac{\sqrt{10}}{5}$

(C) $x = \frac{2}{5}$ (D) $x = 0$

$g'(x) = 5x^4 + 3x^2 - 2$

$0 = (5x^2-2)(x^2+1)$

$5x^2 - 2 = 0$ | $x^2 + 1 = 0$

$x^2 = 2/5$ | $x^2 \neq -1$

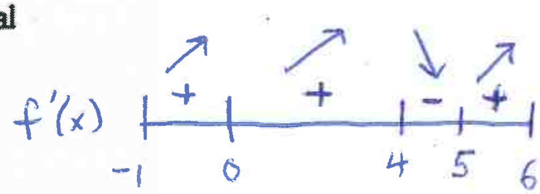
$x = \pm\sqrt{2/5}$

$g'(x)$: $+$ (-2) | $-\sqrt{2/5}$ | $-$ (0) | $\sqrt{2/5}$ | $+$ (2)

Relative min at $x = \sqrt{2/5}$ since $g'(x)$ changes from $-$ to $+$

$\frac{\sqrt{2}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{\sqrt{10}}{5}}$

4. A function f is continuous for all real numbers. The graph of its derivative function f' is shown. The graph has horizontal tangent lines at $(0, 0)$, $(2, 3)$, and $(4.5, -1)$. At which number(s) x does f have a local minimum?

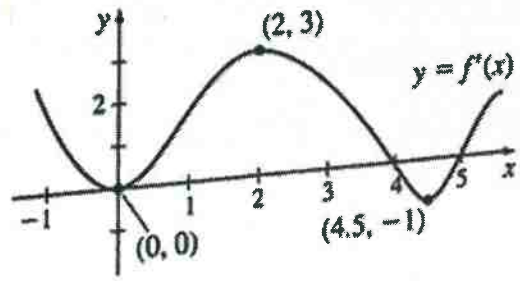


Relative min at $x=5$ since $f'(x)$ changes from $-$ to $+$.

pos. slope

m=0

neg. slope



(A) 0, 4, and 5
(B) 4 only
$\boxed{\text{(C)}}$ 5 only
(D) 0, 2, and 4.5

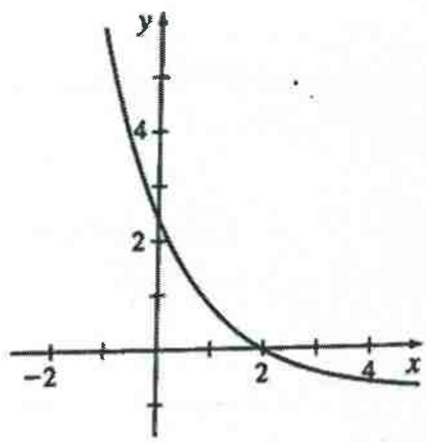
5. For the function f graphed below, both f' and f'' exist for all numbers x. Which of the following is true?

$f(2)=0$

$f'(2)<0$ (negative slope)

$f''(2)>0$ (concave up)

$\boxed{f'(2)<f(2)<f''(2)}$



(A) $f(2) < f'(2) < f''(2)$
$\boxed{\text{(B)}}$ $f'(2) < f(2) < f''(2)$
(C) $f''(2) < f'(2) < f(2)$
(D) $f''(2) < f(2) < f'(2)$