Interval	Sign of $5x^2 - 2$	Sign of $\left(x^2+1\right)$	Sign of $f'(x)$	Conclusion
$\left(-\infty, -\frac{\sqrt{10}}{5}\right)$	+	+	+	Increasing
$\left(\frac{-\sqrt{10}}{5}, \frac{\sqrt{10}}{5}\right)$	−	+	−	Decreasing
$\left(\frac{\sqrt{10}}{5}, \infty\right)$	+	+	+	Increasing

By the First Derivative Test, g has a local minimum at $\boxed{x = \frac{\sqrt{10}}{5}}$

CHOICE B

4. The candidates for local minimum(s) on f are where $f'(x) = 0$, which on the graph of f' is at the points where y is 0, namely (0,0), (4,0) and (5,0).

   Here, using the graph of f', we look for where the y-coordinates change from negative to positive at the critical numbers 0, 4, and 5. By the First Derivative Test, f has a local minimum at $\boxed{\text{5 only}}$

   CHOICE C

5. $f(2) = 0$. $f'(2)$ is the slope of the line tangent to f at $(2, f(2))$, which appears to be negative. $f''(2)$ is the concavity of f at $(2, f(2))$, which appears to be positive, as the graph is concave up for the entire domain. Therefore $\boxed{f'(2) < f(2) < f''(2)}$.

   CHOICE B

6. $f'(c)$ is the slope of the line tangent to f at $(c, f(c))$. $f''(c)$ indicates the concavity of f at $(c, f(c))$, where $f''(x) > 0$ indicates that the graph is concave up and $f''(x) < 0$ indicates that the graph is concave down. The following table summarizes these observations as applied to the graph of f.

Point	Sign of $f'(x)$	Sign of $f''(x)$
A	−	+
B	+	−
C	−	−
D	−	+

   Therefore f' and f'' have the same sign at point $\boxed{\text{C}}$.

   CHOICE C

7. $v(t) = s'(t)$. A value of $v(t)$ in the chart represents the $s'(t)$ value, which is the slope of the line tangent to the graph of $s(t)$ at the specified value of t. Numerous values in the chart could be examined, but note that $v(3) = 0$ and $v(5) = 0$, indicating a horizontal tangent line to $s(t)$ at $(3, s(3))$ and $(5, s(5))$. Reviewing the given graphs shows that only Choice $\boxed{\text{B}}$ satisfies these conditions.

   CHOICE B

8. $f(x) = 3x^4 - 2x^2$. The function f is concave up in the interval(s) where $f''(x) > 0$ and concave down in the interval(s) where $f''(x) < 0$. The point(s) of inflection are those point(s) at which the concavity of the graph of f changes.

   $f'(x) = 12x^3 - 2x^2$

   $f''(x) = 36x^2 - 4$

Set $f''(x) = 0$ and solve for x.

$$36x^2 - 4 = 0$$
$$4(3x-1)(3x+1) = 0$$
$$x = \frac{1}{3} \quad \text{or} \quad x = -\frac{1}{3}$$

The sign of $f''(x)$ with the determination of whether the graph is concave up or concave down in the specified interval is shown in the following table.

Interval	Sign of $3x-1$	Sign of $3x+1$	Sign of $f''(x)$	Conclusion
$(-\infty, -\frac{1}{3})$	$-$	$-$	$+$	Concave Up
$(\frac{-1}{3}, \frac{1}{3})$	$-$	$+$	$-$	Concave Down
$(\frac{1}{3}, \infty)$	$+$	$+$	$+$	Concave Up

From the chart, since the Point of Inflection is at the point where the concavity changes, the Inflection Points are at $\boxed{\left(-\frac{1}{3}, f\left(-\frac{1}{3}\right)\right) = \left(-\frac{1}{3}, -\frac{5}{27}\right)}$ and $\boxed{\left(\frac{1}{3}, f\left(\frac{1}{3}\right)\right) = \left(\frac{1}{3}, -\frac{5}{27}\right)}$.

CHOICE D

9.

$$f(x) = \frac{x^4}{2} - 2x^3 - 9x^2 - 12x + 5$$
$$f'(x) = 2x^3 - 6x^2 - 18x - 12$$
$$f''(x) = 6x^2 - 12x - 18$$

Set $f''(x) = 0$ and solve for x.

$$6x^2 - 12x - 18 = 0$$
$$6(x-3)(x+1) = 0$$
$$x = 3 \quad \text{or} \quad x = -1$$

The function f is concave up where $f''(x) > 0$ and concave down where $f''(x) < 0$.

To determine where $f''(x) > 0$ and $f''(x) < 0$ use the numbers $x = -1$ and $x = 3$ to determine the intervals to test for concavity. The sign of $f''(x)$ with the determination of whether the graph is concave up or concave down in the specified interval is shown in the following table.

Interval	Sign of $x-3$	Sign of $x+1$	Sign of $f''(x)$	Conclusion
$(-\infty, -1)$	$-$	$-$	$+$	Concave Up
$(-1, 3)$	$-$	$+$	$-$	Concave Down
$(3, \infty)$	$+$	$+$	$+$	Concave Up

f is concave down on the interval $\boxed{(-1, 3)}$

CHOICE A

10. Set $f'(x) = x^2(x-1)(x+2)(x+3) = 0$ and solve for x.

$x = 0, x = 1, x = -2,$ or $x = -3$

Interval	Sign of x^2	Sign of $x-1$	Sign of $x+2$	Sign of $x+3$	Sign of $f'(x)$	Conclusion
$(-\infty,-3)$	+	−	−	−	−	Decreasing
$(-3,-2)$	+	−	−	+	+	Increasing
$(-2,0)$	+	−	+	+	−	Decreasing
$(0,1)$	+	−	+	+	−	Decreasing
$(1,\infty)$	+	+	+	+	+	Increasing

By the First Derivative Test, f has a relative minimum at $\boxed{-3 \text{ and } 1}$

CHOICE D

11. $f(x) = (x-1)^{4/5} - 2$

$$f'(x) = \frac{4}{5}(x-1)^{-1/5} = \frac{4}{5(x-1)^{1/5}}$$

$$f''(x) = \frac{4}{5}\left(-\frac{1}{5}\right)(x-1)^{-6/5} = -\frac{4}{25(x-1)^{6/5}}$$

f has a critical number at $x = 1$, since $f'(x)$ does not exist at $x = 1$. There are no other critical numbers, since $f'(x)$ never equals 0.

Interval	Sign of $f'(x)$	Conclusion	Sign of $f''(x)$	Conclusion
$(-\infty,1)$	−	Decreasing	−	Concave Down
$(1,\infty)$	+	Increasing	−	Concave Down

By the First Derivative Test, $(1,-2)$ is a local minimum. Choice A is true.

f is concave down on $(-\infty,1)$ and $(1,\infty)$. Choice B is true.

The point $(1,-2)$ is not an inflection point. Choice $\boxed{C}$ is false.

f does have a vertical tangent at $x = 1$, since f is continuous everywhere but $f'(x) \to \pm\infty$ as $x \to \pm\infty$. Choice D is true.

CHOICE C

12. $f(x) = x \ln x$

$f'(x) = 1 + \ln x$

Set $f'(x) = 0$ and solve for x.

$$1 + \ln x = 0$$
$$\ln x = -1$$
$$x = e^{-1} = \frac{1}{e}$$

For the domain $x > 0$ the table for the local maximum and minimum follows.

Interval	Sign of $f'(x)$	Conclusion
$\left(0, \frac{1}{e}\right)$	$-$	Decreasing
$\left(\frac{1}{e}, \infty\right)$	$+$	Increasing

By the First Derivative Test there is a local minimum at $\left(\frac{1}{e}, f\left(\frac{1}{e}\right)\right) = \left(\frac{1}{e}, -\frac{1}{e}\right)$

So the minimum value is $\boxed{-\frac{1}{e}}$. (Be careful—the problem doesn't ask for the x-value at the local minimum, merely the minimum value itself, i.e. the function value at the local minimum.)

CHOICE A

13. All of the extrema are less than 0, so any possible zeros must be to the left of -5 or to the right of 2.

Since $(-5, -3)$ is a local minimum and f is a polynomial, $f(x) \to +\infty$ as $x \to -\infty$, and there is a zero to the left of -5.

Since $(2, -6)$ is a local minimum and f is a polynomial, $f(x) \to +\infty$ as $x \to +\infty$, and there is a zero to the right of 2.

Therefore f has $\boxed{\text{two}}$ zeros.

CHOICE B

14.
$$\begin{aligned} f(x) &= \frac{2}{x+3} \\ &= 2(x+3)^{-1} \\ f'(x) &= -2(x+3)^{-2} \\ &= -\frac{2}{(x+3)^2} \\ f''(x) &= 4(x+3)^{-3} \\ &= \frac{4}{(x+3)^3} \end{aligned}$$

Interval	Sign of $f''(x)$	Conclusion
$(-\infty, -3)$	$-$	Concave Down
$\boxed{(-3, \infty)}$	$+$	Concave Up

CHOICE A

15. To investigate the motion of $x(t)$, we find the velocity v.

$$x(t) = \frac{\ln t}{t}$$

$$v = x'(t) = \frac{\left(\frac{1}{t}\right) t - 1 \ln t}{t^2} = \frac{1 - \ln t}{t^2}$$

15) (Continued)

In order to determine when the object turns, set

$v = 0$
$1 - \ln t = 0$
$\ln t = 1$
$t = e$

We examine the intervals $(1, e)$ and (e, ∞)

Checking a test point in the interval $(1,\ e)$ for v such as $v(2) = \dfrac{1 - \ln 2}{2^2} = 0.306 > 0$ showing that the object is moving to the right in the interval $(1,\ e)$.

Checking a test point in the interval (e, ∞) for v such as $v(4) = \dfrac{1 - \ln 4}{4^2} = -0.024 < 0$ showing that the object is moving to the left in the interval (e, ∞). Therefore the object is farthest from the origin at its one and only turning point $\boxed{t = e \text{ seconds}}$.

CHOICE D

5.4 Use Calculus to Graph Functions

Skill Building

1. Let $f(x) = x^4 - 6x^2 + 10$.

Step 1 The polynomial function f has a domain of $\boxed{\text{all real numbers}}$. $f(0) = 10$, so the $\boxed{y\text{-intercept is } 10}$. To find the x-intercepts, solve the equation $f(x) = 0$. Because

$$x^4 - 6x^2 + 10 = (x^2 - 3)^2 + 1,$$

it follows that there are no real solutions to the equation $f(x) = 0$, so the graph of f has $\boxed{\text{no } x\text{-intercepts}}$.

Step 2 The $\boxed{\text{graphs of polynomial functions do not have asymptotes}}$, but the end behavior of the graph of f will resemble the power function $y = x^4$.

Step 3 Now

$$\begin{aligned} f'(x) &= 4x^3 - 12x = 4x(x^2 - 3); \text{ and} \\ f''(x) &= 12x^2 - 12 = 12(x - 1)(x + 1). \end{aligned}$$

The critical numbers of the polynomial function f occur where $f'(x) = 0$, so $\boxed{0 \text{ and } \pm\sqrt{3}}$ are the critical numbers. At the points $(-\sqrt{3}, 1)$, $(0, 10)$, and $(\sqrt{3}, 1)$, the tangent lines are horizontal.

Step 4 To apply the Increasing/Decreasing Function Test, use the critical numbers 0 and $\pm\sqrt{3}$ to divide the number line into four intervals.

Interval	Sign of f'	Conclusion
$(-\infty, -\sqrt{3})$	$-$	f is decreasing on $(-\infty, -\sqrt{3})$
$(-\sqrt{3}, 0)$	$+$	f is increasing on $(-\sqrt{3}, 0)$
$(0, \sqrt{3})$	$-$	f is decreasing on $(0, \sqrt{3})$
$(\sqrt{3}, \infty)$	$+$	f is increasing on $(\sqrt{3}, \infty)$