107. Let $f(t) = \ln t$ on $[x, y]$, $0 < x < y$.

By the Mean Value Theorem,

$$\frac{f(y) - f(x)}{y - x} = f'(c), \quad x < c < y,$$

$$\frac{\ln y - \ln x}{y - x} = \frac{1}{c}.$$

Because $0 < x < c < y, \frac{1}{x} > \frac{1}{c} > \frac{1}{y}$. So,

$$\frac{1}{y} < \frac{\ln y - \ln x}{y - x} < \frac{1}{x}.$$

108. $F(x) = \int_x^{2x} \frac{1}{t}\,dt, \quad x > 0$

$F'(x) = \frac{1}{2x}(2) - \frac{1}{x} = 0 \Rightarrow F$ is constant on $(0, \infty)$.

Alternate Solution:

$$F(x) = [\ln t]_x^{2x} = \ln(2x) - \ln x$$
$$= \ln 2 + \ln x - \ln x$$
$$= \ln 2$$

Section 5.3 Inverse Functions

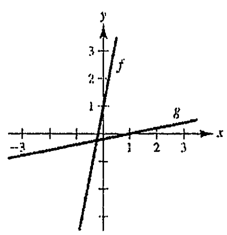
1. (a) $f(x) = 5x + 1$

$g(x) = \frac{x - 1}{5}$

$f(g(x)) = f\left(\frac{x - 1}{5}\right) = 5\left(\frac{x - 1}{5}\right) + 1 = x$

$g(f(x)) = g(5x + 1) = \frac{(5x - 1) - 1}{5} = x$

(b)



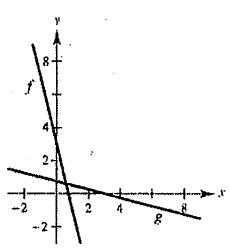
2. (a) $f(x) = 3 - 4x$

$g(x) = \frac{3 - x}{4}$

$f(g(x)) = f\left(\frac{3 - x}{4}\right) = 3 - 4\left(\frac{3 - x}{4}\right) = x$

$g(f(x)) = g(3 - 4x) = \frac{3 - (3 - 4x)}{4} = x$

(b)



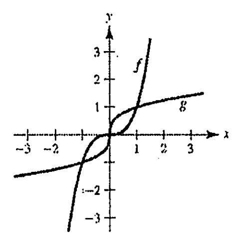
3. (a) $f(x) = x^3$

$g(x) = \sqrt[3]{x}$

$f(g(x)) = f\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 = x$

$g(f(x)) = g(x^3) = \sqrt[3]{x^3} = x$

(b)



4. (a) $f(x) = 1 - x^3$

$g(x) = \sqrt[3]{1 - x}$

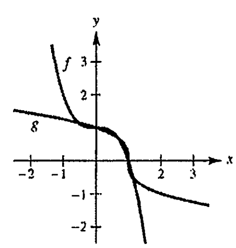
$f(g(x)) = f\left(\sqrt[3]{1 - x}\right) = 1 - \left(\sqrt[3]{1 - x}\right)^3$

$= 1 - (1 - x) = x$

$g(f(x)) = g(1 - x^3)$

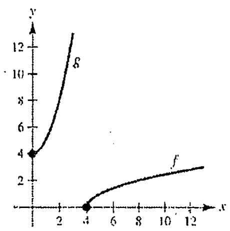
$= \sqrt[3]{1 - (1 - x^3)} = \sqrt[3]{x^3} = x$

(b)



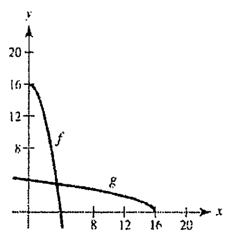
5. (a)
$$f(x) = \sqrt{x-4}$$
$$g(x) = x^2 + 4, \quad x \ge 0$$
$$f(g(x)) = f(x^2+4) = \sqrt{(x^2+4)-4} = \sqrt{x^2} = x$$
$$g(f(x)) = g(\sqrt{x-4}) = (\sqrt{x-4})^2 + 4 = x - 4 + 4 = x$$

(b)



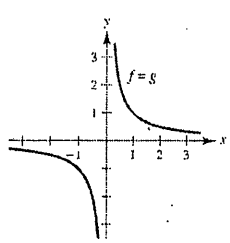
6. (a)
$$f(x) = 16 - x^2, \quad x \ge 0$$
$$g(x) = \sqrt{16-x}$$
$$f(g(x)) = f(\sqrt{16-x}) = 16 - (\sqrt{16-x})^2 = 16 - (16-x) = x$$
$$g(f(x)) = g(16-x^2) = \sqrt{16-(16-x^2)} = \sqrt{x^2} = x$$

(b)



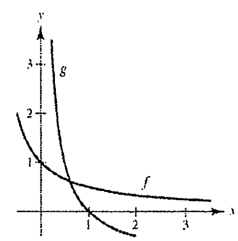
7. (a)
$$f(x) = \frac{1}{x}$$
$$g(x) = \frac{1}{x}$$
$$f(g(x)) = \frac{1}{1/x} = x$$
$$g(f(x)) = \frac{1}{1/x} = x$$

(b)



8. (a)
$$f(x) = \frac{1}{1+x}, \qquad x \ge 0$$
$$g(x) = \frac{1-x}{x}, \qquad 0 < x \le 1$$
$$f(g(x)) = f\left(\frac{1-x}{x}\right) = \frac{1}{1+\frac{1-x}{x}} = \frac{1}{\frac{1}{x}} = x$$
$$g(f(x)) = g\left(\frac{1}{1+x}\right) = \frac{1-\frac{1}{1+x}}{\frac{1}{1+x}} = \frac{x}{1+x} \cdot \frac{1+x}{1} = x$$

(b)



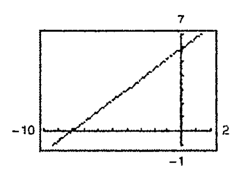
9. Matches (c)

10. Matches (b)

11. Matches (a)

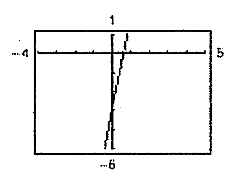
12. Matches (d)

13. $f(x) = \frac{3}{4}x + 6$



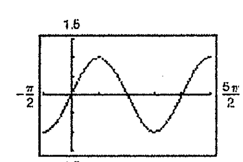
One-to-one; has an inverse

14. $f(x) = 5x - 3$



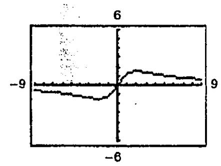
One-to-one; has an inverse

15. $f(\theta) = \sin\theta$



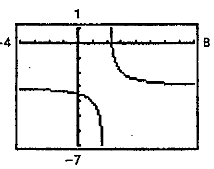
Not one-to-one; does not have an inverse

16. $f(x) = \dfrac{6x}{x^2 + 4}$



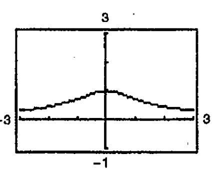
Not one-to-one; does not have an inverse

17. $h(s) = \dfrac{1}{s - 2} - 3$



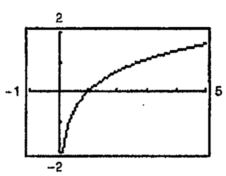
One-to-one; has an inverse

18. $g(t) = \dfrac{1}{\sqrt{t^2 + 1}}$



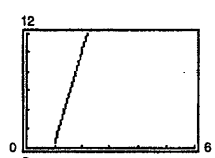
Not one-to-one; does not have an inverse

19. $f(x) = \ln x$



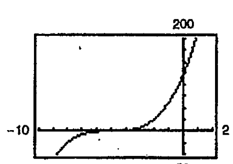
One-to-one; has an inverse

20. $f(x) = 5x\sqrt{x - 1}$



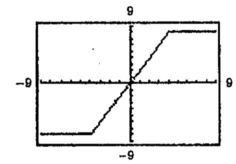
One-to-one; has an inverse

21. $g(x) = (x + 5)^3$



One-to-one; has an inverse

22. $h(x) = |x + 4| - |x - 4|$



Not one-to-one; does not have an inverse

23. $f(x) = 2 - x - x^3$

$f'(x) = -1 - 3x^2 < 0$ for all x

f is decreasing on $(-\infty, \infty)$. Therefore, f is strictly monotonic and has an inverse.

24. $f(x) = x^3 - 6x^2 + 12x$

$f'(x) = 3x^2 - 12x + 12 = 3(x - 2)^2 \geq 0$ for all x

f is increasing on $(-\infty, \infty)$. Therefore, f is strictly monotonic and has an inverse.

25. $f(x) = \dfrac{x^4}{4} - 2x^2$

$f'(x) = x^3 - 4x = 0$ when $x = 0, 2, -2$

f is not strictly monotonic on $(-\infty, \infty)$. Therefore, f does not have an inverse.

26. $f(x) = x^5 + 2x^3$

$f'(x) = 5x^4 + 6x^2 \geq 0$ for all x

f is increasing on $(-\infty, \infty)$. Therefore, f is strictly monotonic and has an inverse.

27. $f(x) = \ln(x - 3), \quad x > 3$

$f'(x) = \dfrac{1}{x - 3} > 0$ for $x > 3$

f is increasing on $(3, \infty)$. Therefore, f is strictly monotonic and has an inverse.

28. $f(x) = \cos \dfrac{3x}{2}$

$f'(x) = -\dfrac{3}{2} \sin \dfrac{3x}{2} = 0$ when $x = 0, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dots$

f is not strictly monotonic on $(-\infty, \infty)$. Therefore, f does not have an inverse.

29. $f(x) = (x-4)^2$ on $[4, \infty)$

$f'(x) = 2(x-4) > 0$ on $[4, \infty)$

f is increasing on $[4, \infty)$. Therefore, f is strictly monotonic and has an inverse.

30. $f(x) = |x+2|$ on $[-2, \infty)$

$f'(x) = \frac{|x+2|}{x+2}(1) = 1 > 0$ on $[-2, \infty)$

f is increasing on $[-2, \infty)$. Therefore, f is strictly monotonic and has an inverse.

31. $f(x) = \frac{4}{x^2}$ on $(0, \infty)$

$f'(x) = -\frac{8}{x^3} < 0$ on $(0, \infty)$

f is decreasing on $(0, \infty)$. Therefore, f is strictly monotonic and has an inverse.

32. $f(x) = \cot x$ on $(0, \pi)$

$f'(x) = -\csc^2 x < 0$ on $(0, \pi)$

f is decreasing on $(0, \pi)$. Therefore, f is strictly monotonic and has an inverse.

33. $f(x) = \cos x$ on $[0, \pi]$

$f'(x) = -\sin x < 0$ on $(0, \pi)$

f is decreasing on $[0, \pi]$. Therefore, f is strictly monotonic and has an inverse.

34. $f(x) = \sec x$ on $\left[0, \frac{\pi}{2}\right)$

$f'(x) = \sec x \tan x > 0$ on $\left(0, \frac{\pi}{2}\right)$

f is increasing on $[0, \pi/2)$. Therefore, f is strictly monotonic and has an inverse.

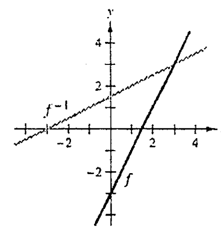
35. (a) $f(x) = 2x - 3 = y$

$$x = \frac{y+3}{2}$$

$$y = \frac{x+3}{2}$$

$$f^{-1}(x) = \frac{x+3}{2}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other across the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

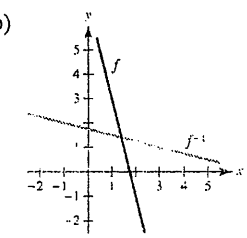
36. (a) $f(x) = 7 - 4x = y$

$$x = \frac{7-y}{4}$$

$$y = \frac{7-x}{4}$$

$$f^{-1}(x) = \frac{7-x}{4}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other across the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

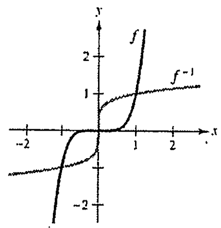
37. (a) $f(x) = x^5 = y$

$x = \sqrt[5]{y}$

$y = \sqrt[5]{x}$

$f^{-1}(x) = \sqrt[5]{x} = x^{1/5}$

(b)



(c) The graphs of f and f^{-1} are reflections of each other across the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

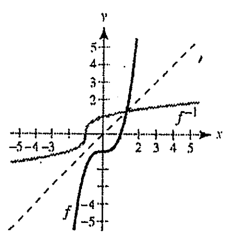
38. (a) $f(x) = x^3 - 1 = y$

$x = \sqrt[3]{y + 1}$

$y = \sqrt[3]{x + 1}$

$f^{-1}(x) = \sqrt[3]{x + 1} = (x + 1)^{1/3}$

(b)



(c) The graphs of f and f^{-1} are reflections of each other across the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

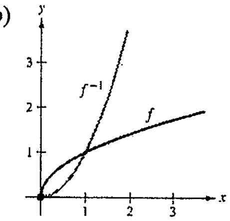
39. (a) $f(x) = \sqrt{x} = y$

$x = y^2$

$y = x^2$

$f^{-1}(x) = x^2, \quad x \geq 0$

(b)



(c) The graphs of f and f^{-1} are reflections of each other across the line $y = x$.

(d) Domain of f: $x \geq 0$
Range of f: $y \geq 0$
Domain of f^{-1}: $x \geq 0$
Range of f^{-1}: $y \geq 0$

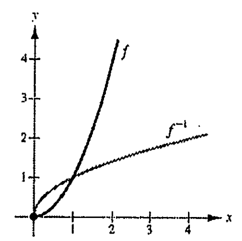
40. (a) $f(x) = x^2 = y, \quad x \geq 0$

$x = \sqrt{y}$

$y = \sqrt{x}$

$f^{-1}(x) = \sqrt{x}$

(b)



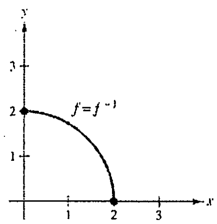
(c) The graphs of f and f^{-1} are reflections of each other across the line $y = x$.

(d) Domain of f: $x \geq 0$
Range of f: $y \geq 0$
Domain of f^{-1}: $x \geq 0$
Range of f^{-1}: $y \geq 0$

41. (a) $f(x) = \sqrt{4 - x^2} = y, \quad 0 \le x \le 2$

$$4 - x^2 = y^2$$
$$x^2 = 4 - y^2$$
$$x = \sqrt{4 - y^2}$$
$$y = \sqrt{4 - x^2}$$
$$f^{-1}(x) = \sqrt{4 - x^2}, \quad 0 \le x \le 2$$

(b)



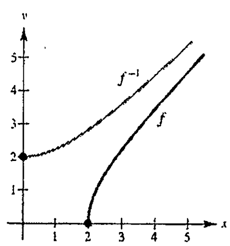
(c) The graphs of f and f^{-1} are reflections of each other across the line $y = x$. In fact, the graphs are identical.

(d) Domain of f: $0 \le x \le 2$
Range of f: $0 \le y \le 2$
Domain of f^{-1}: $0 \le x \le 2$
Range of f^{-1}: $0 \le y \le 2$

42. (a) $f(x) = \sqrt{x^2 - 4} = y, \quad x \ge 2$

$$x^2 = y^2 + 4$$
$$x = \sqrt{y^2 + 4}$$
$$y = \sqrt{x^2 + 4}$$
$$f^{-1}(x) = \sqrt{x^2 + 4}, \quad x \ge 0$$

(b)



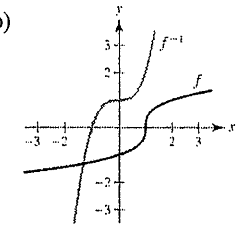
(c) The graphs of f and f^{-1} are reflections of each other across the line $y = x$

(d) Domain of f: $x \ge 2$
Range of f: $y \ge 0$
Domain of f^{-1}: $x \ge 0$
Range of f^{-1}: $y \ge 2$

43. (a) $f(x) = \sqrt[3]{x - 1} = y$

$$x - 1 = y^3$$
$$x = y^3 + 1$$
$$y = x^3 + 1$$
$$f^{-1}(x) = x^3 + 1$$

(b)



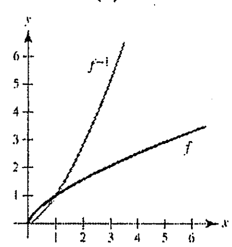
(c) The graphs of f and f^{-1} are reflections of each other across the line $y = x$

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

44. (a) $f(x) = x^{2/3} = y, \quad x \ge 0$

$$x = y^{3/2}$$
$$y = x^{3/2}$$
$$f^{-1}(x) = x^{3/2}, \quad x \ge 0$$

(b)



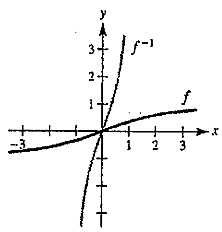
(c) The graphs of f and f^{-1} are reflections of each other across the line $y = x$

(d) Domain of f: $x \ge 0$
Range of f: $y \ge 0$
Domain of f^{-1}: $x \ge 0$
Range of f^{-1}: $y \ge 0$

45. (a) $f(x) = \dfrac{x}{\sqrt{x^2 + 7}} = y$

$$x = y\sqrt{x^2 + 7}$$
$$x^2 = y^2(x^2 + 7) = y^2x^2 + 7y^2$$
$$x^2(1 - y^2) = 7y^2$$
$$x = \frac{\sqrt{7}y}{\sqrt{1 - y^2}}$$
$$y = \frac{\sqrt{7}x}{\sqrt{1 - x^2}}$$
$$f^{-1}(x) = \frac{\sqrt{7}x}{\sqrt{1 - x^2}}, \quad -1 < x < 1$$

(b)



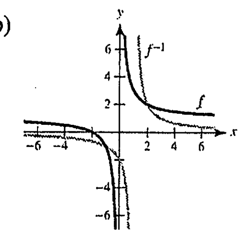
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$

(d) Domain of f: all real numbers
Range of f: $-1 < y < 1$
Domain of f^{-1}: $-1 < x < 1$
Range of f^{-1}: all real numbers

46. (a) $f(x) = \dfrac{x + 2}{x} = y, \quad x \neq 0$

$$x + 2 = yx$$
$$x(1 - y) = -2$$
$$x = \frac{2}{y - 1}$$
$$y = \frac{2}{x - 1}$$
$$f^{-1}(x) = \frac{2}{x - 1}, \quad x \neq 1$$

(b)



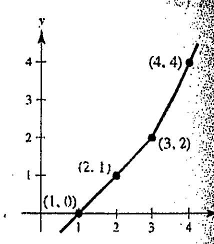
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$

(d) Domain of f: all $x \neq 0$
Range of f: all $y \neq 1$
Domain of f^{-1}: all $x \neq 1$
Range of f^{-1}: all $y \neq 0$

47.

x	0	1	2	3
$f(x)$	1	2	3	4

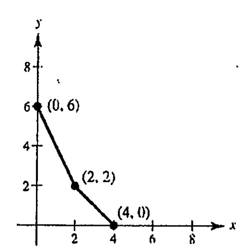
x	1	2	3	4
$f^{-1}(x)$	0	1	2	4



48.

x	0	2	6
$f(x)$	4	2	0

x	0	2	4
$f^{-1}(x)$	6	2	0



49. (a) Let x be the number of pounds of the commodity costing 1.25 per pound. Because there are 50 pounds total, the amount of the second commodity is $50 - x$. The total cost is

$$y = 1.25x + 1.60(50 - x)$$
$$= -0.35x + 80, \quad 0 \le x \le 50.$$

(b) Find the inverse of the original function.

$$y = -0.35x + 80$$
$$0.35x = 80 - y$$
$$x = \tfrac{100}{35}(80 - y)$$

Inverse: $y = \frac{100}{35}(80 - x) = \frac{20}{7}(80 - x)$

x represents cost and y represents pounds.

(c) Domain of inverse is $62.5 \le x \le 80$.

(d) If $x = 73$ in the inverse function,

$y = \frac{100}{35}(80 - 73) = \frac{100}{5} = 20$ pounds.

50. $C = \frac{5}{9}(F - 32), \quad F \ge -459.6$

(a) $\frac{9}{5}C = F - 32$

$F = 32 + \frac{9}{5}C$

(b) The inverse function gives the temperature F corresponding to the Celsius temperature C.

(c) For $F \ge -459.6$, $C = \frac{5}{9}(F - 32) \ge -273.1\overline{1}$.

Therefore, domain is $C \ge -273.\overline{1} = -273\frac{1}{9}$.

(d) If $C = 22°$, then $F = 32 + \frac{9}{5}(22) = 71.6°\text{F}$.

51. $f(x) = \sqrt{x - 2}$, Domain: $x \ge 2$

$$f'(x) = \frac{1}{2\sqrt{x - 2}} > 0 \text{ for } x > 2$$

f is one-to-one; has an inverse

$$\sqrt{x - 2} = y$$
$$x - 2 = y^2$$
$$x = y^2 + 2$$
$$y = x^2 + 2$$
$$f^{-1}(x) = x^2 + 2, \quad x \ge 0$$

52. $f(x) = -3$

Not one-to-one; does not have an inverse

53. $f(x) = |x - 2|, \quad x \le 2$

$$= -(x - 2)$$
$$= 2 - x$$

f is one-to-one; has an inverse

$$2 - x = y$$
$$2 - y = x$$
$$f^{-1}(x) = 2 - x, \quad x \ge 0$$

54. $f(x) = ax + b$

f is one-to-one; has an inverse

$$ax + b = y$$
$$x = \frac{y - b}{a}$$
$$y = \frac{x - b}{a}$$
$$f^{-1}(x) = \frac{x - b}{a}, \quad a \ne 0$$

55. $f(x) = (x - 3)^2$ is one-to-one for $x \ge 3$.

$$(x - 3)^2 = y$$
$$x - 3 = \sqrt{y}$$
$$x = \sqrt{y} + 3$$
$$y = \sqrt{x} + 3$$
$$f^{-1}(x) = \sqrt{x} + 3, \quad x \ge 0$$

(Answer is not unique.)

56. $f(x) = 16 - x^4$ is one-to-one for $x \ge 0$.

$$16 - x^4 = y$$
$$16 - y = x^4$$
$$\sqrt[4]{16 - y} = x$$
$$\sqrt[4]{16 - x} = y$$
$$f^{-1}(x) = \sqrt[4]{16 - x}, \quad x \le 16$$

(Answer is not unique.)

57. $f(x) = |x + 3|$ is one-to-one for $x \ge -3$.

$$x + 3 = y$$
$$x = y - 3$$
$$y = x - 3$$
$$f^{-1}(x) = x - 3, \quad x \ge 0$$

(Answer is not unique.)

58. $f(x) = |x - 3|$ is one-to-one for $x \geq 3$.

$$x - 3 = y$$
$$x = y + 3$$
$$y = x + 3$$
$$f^{-1}(x) = x + 3, \quad x \geq 0$$

(Answer is not unique.)

59. Yes, the volume is an increasing function, and therefore one-to-one. The inverse function gives the time t corresponding to the volume V.

60. No, there could be two times $t_1 \neq t_2$ for which $h(t_1) = h(t_2)$.

61. No, $C(t)$ is not one-to-one because long distance costs are step functions. A call lasting 2.1 minutes costs the same as one lasting 2.2 minutes.

62. Yes, the area function is increasing and therefore one-to-one. The inverse function gives the radius r corresponding to the area A.

63. $f(x) = 5 - 2x^3, \quad a = 7$

$$f'(x) = -6x^2$$

f is monotonic (decreasing) on $(-\infty, \infty)$ therefore f has an inverse.

$$f(-1) = 7 \Rightarrow f^{-1}(7) = -1$$

$$\left(f^{-1}\right)'(7) = \frac{1}{f'\left(f^{-1}(7)\right)} = \frac{1}{f'(-1)} = \frac{1}{-6(-1)^2} = \frac{-1}{6}$$

64. $f(x) = x^3 + 2x - 1, \quad a = 2$

$$f'(x) = 3x^2 + 2 > 0$$

f is monotonic (increasing) on $(-\infty, \infty)$ therefore f has an inverse.

$$f(1) = 2 \Rightarrow f^{-1}(2) = 1$$

$$\left(f^{-1}\right)'(2) = \frac{1}{f'\left(f^{-1}(2)\right)} = \frac{1}{f'(1)} = \frac{1}{3(1^2) + 2} = \frac{1}{5}$$

65. $f(x) = \frac{1}{27}\left(x^5 + 2x^3\right), \quad a = -11$

$$f'(x) = \frac{1}{27}\left(5x^4 + 6x^2\right)$$

f is monotonic (increasing) on $(-\infty, \infty)$ therefore f has an inverse.

$$f(-3) = \frac{1}{27}(-243 - 54) = -11 \Rightarrow f^{-1}(-11) = -3$$

$$\left(f^{-1}\right)'(-11) = \frac{1}{f'\left(f^{-1}(-11)\right)} = \frac{1}{f'(-3)}$$

$$= \frac{1}{\frac{1}{27}\left(5(-3)^4 + 6(-3)^2\right)} = \frac{1}{\frac{1}{27}(459)} = \frac{1}{17}$$

66. $f(x) = \sqrt{x - 4}, \quad a = 2, \quad x \geq 4$

$$f'(x) = \frac{1}{2\sqrt{x - 4}} > 0 \text{ on } (4, \infty)$$

f is monotonic (increasing) on $[4, \infty)$ therefore f has an inverse.

$$f(8) = 2 \Rightarrow f^{-1}(2) = 8$$

$$f'(8) = \frac{1}{2\sqrt{8 - 4}} = \frac{1}{4}$$

$$\left(f^{-1}\right)'(2) = \frac{1}{f'\left(f^{-1}(2)\right)} = \frac{1}{f'(8)} = \frac{1}{1/4} = 4$$

67. $f(x) = \sin x, \quad a = 1/2, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$$f'(x) = \cos x > 0 \text{ on } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

f is monotonic (increasing) on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ therefore f has an inverse.

$$f\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2} \Rightarrow f^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\left(f^{-1}\right)'\left(\frac{1}{2}\right) = \frac{1}{f'\left(f^{-1}\left(\frac{1}{2}\right)\right)}$$

$$= \frac{1}{f'\left(\frac{\pi}{6}\right)} = \frac{1}{\cos\left(\frac{\pi}{6}\right)} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

68. $f(x) = \cos 2x, \quad a = 1, 0 \le x \le \pi/2$

$f'(x) = -2\sin 2x < 0$ on $(0, \pi/2)$

f is monotonic (decreasing) on $[0, \pi/2]$ therefore f has an inverse.

$f(0) = 1 \Rightarrow f^{-1}(1) = 0$

$$\left(f^{-1}\right)'(1) = \frac{1}{f'\left(f^{-1}(1)\right)} = \frac{1}{f'(0)} = \frac{1}{-2\sin 0} = \frac{1}{0}$$

So, $\left(f^{-1}\right)'(1)$ is undefined.

69. $f(x) = \dfrac{x+6}{x-2}, \quad x > 0, a = 3$

$$f'(x) = \frac{(x-2)(1) - (x+6)(1)}{(x-2)^2}$$

$$= \frac{-8}{(x-2)^2} < 0 \text{ on } (2, \infty)$$

f is monotonic (decreasing) on $(2, \infty)$ therefore f has an inverse.

$f(6) = 3 \Rightarrow f^{-1}(3) = 6$

$$\left(f^{-1}\right)'(3) = \frac{1}{f'\left(f^{-1}(3)\right)} = \frac{1}{f'(6)} = \frac{1}{-8/(6-2)^2} = -2$$

70. $f(x) = \dfrac{x+3}{x+1}, \quad x > -1, \quad a = 2$

$$f'(x) = \frac{(x+1)(1) - (x+3)(1)}{(x+1)^2}$$

$$= \frac{-2}{(x+1)^2} < 0 \text{ on } (-1, \infty)$$

f is monotonic (decreasing) on $(-1, \infty)$ therefore f has an inverse.

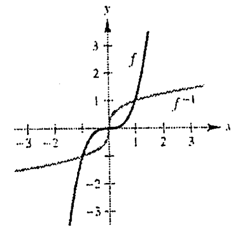
$f(1) = 2 \Rightarrow f^{-1}(2) = 1$

$$\left(f^{-1}\right)'(2) = \frac{1}{f'\left(f^{-1}(2)\right)} = \frac{1}{f'(1)} = \frac{1}{(-2)/(1+1)^2} = -2$$

71. (a) Domain f = Domain $f^{-1} = (-\infty, \infty)$

(b) Range f = Range $f^{-1} = (-\infty, \infty)$

(c)



(d) $f(x) = x^3, \quad \left(\frac{1}{2}, \frac{1}{8}\right)$

$f'(x) = 3x^2$

$f'\left(\frac{1}{2}\right) = \frac{3}{4}$

$f^{-1}(x) = \sqrt[3]{x}, \quad \left(\frac{1}{8}, \frac{1}{2}\right)$

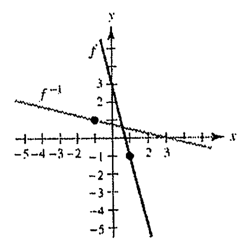
$\left(f^{-1}\right)'(x) = \dfrac{1}{3\sqrt[3]{x^2}}$

$\left(f^{-1}\right)'\left(\frac{1}{8}\right) = \frac{4}{3}$

72. (a) Domain f = Domain $f^{-1} = (-\infty, \infty)$

(b) Range f = Range $f^{-1} = (-\infty, \infty)$

(c)



(d) $f(x) = 3 - 4x, \quad (1, -1)$

$f'(x) = -4$

$f'(1) = -4$

$f^{-1}(x) = \dfrac{3-x}{4}, \quad (-1, 1)$

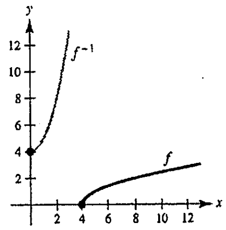
$\left(f^{-1}\right)'(x) = -\frac{1}{4}$

$\left(f^{-1}\right)'(-1) = -\frac{1}{4}$

73. (a) Domain $f = [4, \infty)$, Domain $f^{-1} = [0, \infty)$

(b) Range $f = [0, \infty)$, Range $f^{-1} = [4, \infty)$

(c)



(d) $f(x) = \sqrt{x - 4}$, $(5, 1)$

$$f'(x) = \frac{1}{2\sqrt{x - 4}}$$

$$f'(5) = \frac{1}{2}$$

$$f^{-1}(x) = x^2 + 4, \quad (1, 5)$$

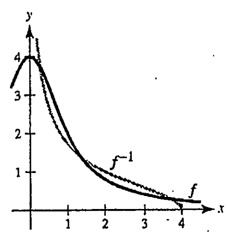
$$\left(f^{-1}\right)'(x) = 2x$$

$$\left(f^{-1}\right)'(1) = 2$$

74. (a) Domain $f = [0, \infty)$, Domain $f^{-1} = (0, 4]$

(b) Range $f = (0, 4]$, Range $f^{-1} = [0, \infty)$

(c)



(d) $f(x) = \dfrac{4}{1 + x^2}$

$$f'(x) = \frac{-8x}{\left(x^2 + 1\right)^2}$$

$$f'(1) = -2$$

$$f^{-1}(x) = \sqrt{\frac{4 - x}{x}}$$

$$\left(f^{-1}\right)'(x) = \frac{-2}{x^2\sqrt{(4 - x)/x}}$$

$$\left(f^{-1}\right)'(2) = -\frac{1}{2}$$

In Exercises 75–78, use the following.

$f(x) = \frac{1}{8}x - 3$ **and** $g(x) = x^3$

$f^{-1}(x) = 8(x + 3)$ **and** $g^{-1}(x) = \sqrt[3]{x}$

75. $\left(f^{-1} \circ g^{-1}\right)(1) = f^{-1}\left(g^{-1}(1)\right) = f^{-1}(1) = 32$

76. $\left(g^{-1} \circ f^{-1}\right)(-3) = g^{-1}\left(f^{-1}(-3)\right) = g^{-1}(0) = 0$

77. $\left(f^{-1} \circ f^{-1}\right)(6) = f^{-1}\left(f^{-1}(6)\right) = f^{-1}(72) = 600$

78. $\left(g^{-1} \circ g^{-1}\right)(-4) = g^{-1}\left(g^{-1}(-4)\right) = g^{-1}\left(\sqrt[3]{-4}\right)$

$$= \sqrt[3]{\sqrt[3]{-4}} = -\sqrt[9]{4}$$

In Exercises 79–82, use the following.

$f(x) = x + 4$ **and** $g(x) = 2x - 5$

$f^{-1}(x) = x - 4$ **and** $g^{-1}(x) = \dfrac{x + 5}{2}$

79.
$$\begin{aligned}\left(g^{-1} \circ f^{-1}\right)(x) &= g^{-1}\left(f^{-1}(x)\right)\\ &= g^{-1}(x - 4)\\ &= \frac{(x - 4) + 5}{2}\\ &= \frac{x + 1}{2}\end{aligned}$$

80.
$$\begin{aligned}\left(f^{-1} \circ g^{-1}\right)(x) &= f^{-1}\left(g^{-1}(x)\right)\\ &= f^{-1}\left(\frac{x + 5}{2}\right)\\ &= \frac{x + 5}{2} - 4\\ &= \frac{x - 3}{2}\end{aligned}$$

81.
$$\begin{aligned}(f \circ g)(x) &= f(g(x))\\ &= f(2x - 5)\\ &= (2x - 5) + 4\\ &= 2x - 1\end{aligned}$$

So, $(f \circ g)^{-1}(x) = \dfrac{x + 1}{2}$.

Note: $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

82. $(g \circ f)(x) = g(f(x))$
$= g(x + 4)$
$= 2(x + 4) - 5$
$= 2x + 3$

So, $(g \circ f)^{-1}(x) = \dfrac{x - 3}{2}.$

Note: $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

83. Let $y = f(x)$ be one-to-one. Solve for x as a function of y. Interchange x and y to get $y = f^{-1}(x)$. Let the domain of f^{-1} be the range of f. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

Example:
$f(x) = x^3;\ y = x^3;\ x = \sqrt[3]{y};\ y = \sqrt[3]{x};$
$f^{-1}(x) = \sqrt[3]{x}$

84. The graphs of f and f^{-1} are mirror images with respect to the line $y = x$.

85. f is not one-to-one because many different x-values yield the same y-value.

Example: $f(0) = f(\pi) = 0$

Not continuous at $\dfrac{(2n - 1)\pi}{2}$, where n is an integer.

86. f is not one-to-one because different x-values yield the same y-value.

Example: $f(3) = f\left(-\dfrac{4}{3}\right) = \dfrac{3}{5}$

Not continuous at ± 2.

87. $f(x) = k(2 - x - x^3)$ is one-to-one. Because

$f^{-1}(3) = -2,$

$f(-2) = 3 = k(2 - (-2) - (-2)^3) = 12k \Rightarrow k = \frac{1}{4}.$

88. (a) Since the slope of the tangent line to f at $\left(-1, -\frac{1}{2}\right)$ is $\frac{1}{2}$, the slope of the tangent line to f^{-1} at $\left(-\frac{1}{2}, 1\right)$ is

$m = \dfrac{1}{(1/2)} = 2.$

(b) Since the slope of the tangent line to f at $(2, 1)$ is 2, the slope of the tangent line to f^{-1} at $(1, 2)$ is

$m = \dfrac{1}{2}.$

89. False. Let $f(x) = x^2$.

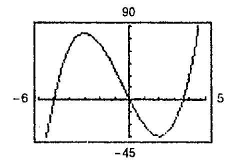
90. True; if f has a y-intercept.

91. True

92. False. Let $f(x) = x$ or $g(x) = 1/x$.

93. (a) $f(x) = 2x^3 + 3x^2 - 36x$

f does not pass the horizontal line test.



(b) $f'(x) = 6x^2 + 6x - 36$
$= 6(x^2 + x - 6) = 6(x + 3)(x - 2)$

$f'(x) = 0$ at $x = 2, -3$

On the interval $(-2, 2)$, f is one-to-one, so, $c = 2$.

94. Let f and g be one-to-one functions.

(a) Let

$(f \circ g)(x_1) = (f \circ g)(x_2)$
$f(g(x_1)) = f(g(x_2))$
$g(x_1) = g(x_2)$ (Because f is one-to-one.)
$x_1 = x_2$ (Because g is one-to-one.)

So, $f \circ g$ is one-to-one.

(b) Let $(f \circ g)(x) = y$, then $x = (f \circ g)^{-1}(y)$. Also:

$(f \circ g)(x) = y$
$f(g(x)) = y$
$g(x) = f^{-1}(y)$
$x = g^{-1}(f^{-1}(y))$
$x = (g^{-1} \circ f^{-1})(y)$

So, $(f \circ g)^{-1}(y) = (g^{-1} \circ f^{-1})(y)$ and $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

95. If f has an inverse, then f and f^{-1} are both one-to-one. Let $(f^{-1})^{-1}(x) = y$ then $x = f^{-1}(y)$ and $f(x) = y$. So, $(f^{-1})^{-1} = f$.