# 5.3 Exercises

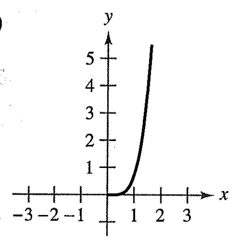
See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

**Verifying Inverse Functions** In Exercises 1–8, show that $f$ and $g$ are inverse functions (a) analytically and (b) graphically.

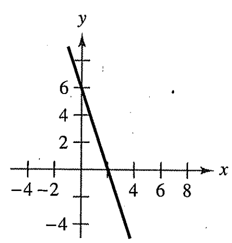
1. $f(x) = 5x + 1, \quad g(x) = \dfrac{x-1}{5}$
2. $f(x) = 3 - 4x, \quad g(x) = \dfrac{3-x}{4}$
3. $f(x) = x^3, \quad g(x) = \sqrt[3]{x}$
4. $f(x) = 1 - x^3, \quad g(x) = \sqrt[3]{1-x}$
5. $f(x) = \sqrt{x-4}, \quad g(x) = x^2 + 4, \quad x \ge 0$
6. $f(x) = 16 - x^2, \quad x \ge 0, \quad g(x) = \sqrt{16-x}$
7. $f(x) = \dfrac{1}{x}, \quad g(x) = \dfrac{1}{x}$
8. $f(x) = \dfrac{1}{1+x}, \quad x \ge 0, \quad g(x) = \dfrac{1-x}{x}, \quad 0 < x \le 1$

**Matching** In Exercises 9–12, match the graph of the function with the graph of its inverse function. [The graphs of the inverse functions are labeled (a), (b), (c), and (d).]

(a)




(b)

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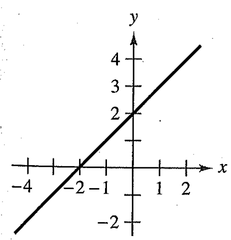
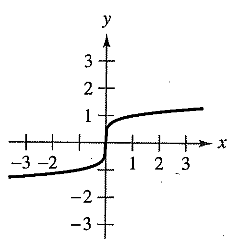



(d)

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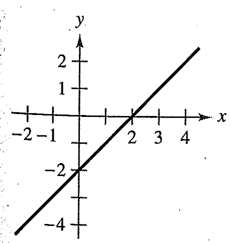


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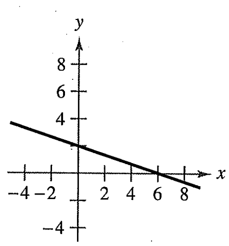


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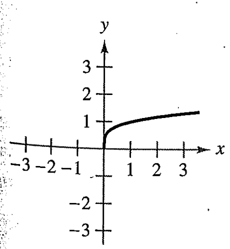


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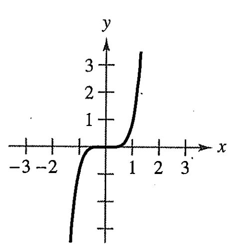



**Using the Horizontal Line Test** In Exercises 13–22, use a graphing utility to graph the function. Then use the Horizontal Line Test to determine whether the function is one-to-one on its entire domain and therefore has an inverse function.

13. $f(x) = \frac{3}{4}x + 6$
14. $f(x) = 5x - 3$
15. $f(\theta) = \sin \theta$
16. $f(x) = \dfrac{6x}{x^2+4}$
17. $h(s) = \dfrac{1}{s-2} - 3$
18. $g(t) = \dfrac{1}{\sqrt{t^2+1}}$
19. $f(x) = \ln x$
20. $f(x) = 5x\sqrt{x-1}$
21. $g(x) = (x+5)^3$
22. $h(x) = |x+4| - |x-4|$

**Determining Whether a Function Has an Inverse Function** In Exercises 23–28, use the derivative to determine whether the function is strictly monotonic on its entire domain and therefore has an inverse function.

23. $f(x) = 2 - x - x^3$
24. $f(x) = x^3 - 6x^2 + 12x$
25. $f(x) = \dfrac{x^4}{4} - 2x^2$
26. $f(x) = x^5 + 2x^3$
27. $f(x) = \ln(x-3)$
28. $f(x) = \cos \dfrac{3x}{2}$

**Verifying a Function Has an Inverse Function** In Exercises 29–34, show that $f$ is strictly monotonic on the given interval and therefore has an inverse function on that interval.

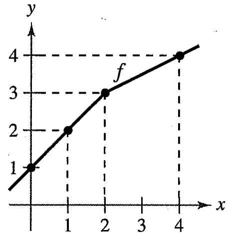
29. $f(x) = (x-4)^2, \quad [4, \infty)$
30. $f(x) = |x+2|, \quad [-2, \infty)$
31. $f(x) = \dfrac{4}{x^2}, \quad (0, \infty)$
32. $f(x) = \cot x, \quad (0, \pi)$
33. $f(x) = \cos x, \quad [0, \pi]$
34. $f(x) = \sec x, \quad \left[0, \dfrac{\pi}{2}\right)$

**Finding an Inverse Function** In Exercises 35–46, (a) find the inverse function of $f$, (b) graph $f$ and $f^{-1}$ on the same set of coordinate axes, (c) describe the relationship between the graphs, and (d) state the domain and range of $f$ and $f^{-1}$.

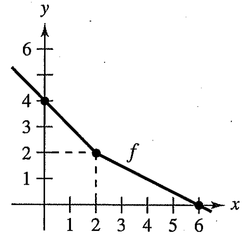
35. $f(x) = 2x - 3$
36. $f(x) = 7 - 4x$
37. $f(x) = x^5$
38. $f(x) = x^3 - 1$
39. $f(x) = \sqrt{x}$
40. $f(x) = x^2, \quad x \ge 0$
41. $f(x) = \sqrt{4-x^2}, \quad 0 \le x \le 2$
42. $f(x) = \sqrt{x^2-4}, \quad x \ge 2$
43. $f(x) = \sqrt[3]{x-1}$
44. $f(x) = x^{2/3}, \quad x \ge 0$
45. $f(x) = \dfrac{x}{\sqrt{x^2+7}}$
46. $f(x) = \dfrac{x+2}{x}$

**Finding an Inverse Function** **In Exercises 47 and 48, use the graph of the function $f$ to make a table of values for the given points. Then make a second table that can be used to find $f^{-1}$, and sketch the graph of $f^{-1}$. To print an enlarged copy of the graph, go to *MathGraphs.com*.**

**47.**



**48.**



**49. Cost** You need 50 pounds of two commodities costing \$1.25 and \$1.60 per pound.

(a) Verify that the total cost is $y = 1.25x + 1.60(50 - x)$, where $x$ is the number of pounds of the less expensive commodity.

(b) Find the inverse function of the cost function. What does each variable represent in the inverse function?

(c) What is the domain of the inverse function? Validate or explain your answer using the context of the problem.

(d) Determine the number of pounds of the less expensive commodity purchased when the total cost is \$73.

**50. Temperature** The formula $C = \frac{5}{9}(F - 32)$, where $F \geq -459.6$, represents Celsius temperature $C$ as a function of Fahrenheit temperature $F$.

(a) Find the inverse function of $C$.

(b) What does the inverse function represent?

(c) What is the domain of the inverse function? Validate or explain your answer using the context of the problem.

(d) The temperature is 22°C. What is the corresponding temperature in degrees Fahrenheit?

**Testing Whether a Function Is One-to-One** **In Exercises 51–54, determine whether the function is one-to-one. If it is, find its inverse function.**

**51.** $f(x) = \sqrt{x - 2}$

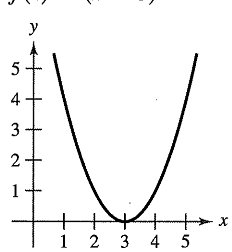
**52.** $f(x) = -3$

**53.** $f(x) = |x - 2|, \quad x \leq 2$

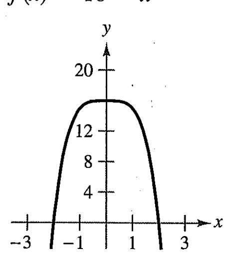
**54.** $f(x) = ax + b, \quad a \neq 0$

**Making a Function One-to-One** **In Exercises 55–58, delete part of the domain so that the function that remains is one-to-one. Find the inverse function of the remaining function and give the domain of the inverse function. (*Note:* There is more than one correct answer.)**

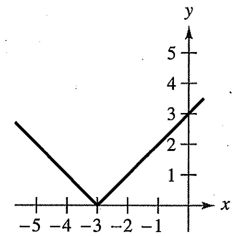
**55.** $f(x) = (x - 3)^2$



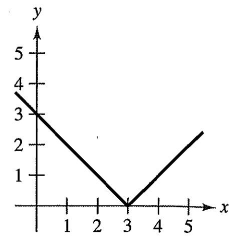
**56.** $f(x) = 16 - x^4$



**57.** $f(x) = |x + 3|$



**58.** $f(x) = |x - 3|$



**Think About It** **In Exercises 59–62, decide whether the function has an inverse function. If so, what is the inverse function?**

**59.** $g(t)$ is the volume of water that has passed through a water line $t$ minutes after a control valve is opened.

**60.** $h(t)$ is the height of the tide $t$ hours after midnight, where $0 \leq t < 24$.

**61.** $C(t)$ is the cost of a long distance call lasting $t$ minutes.

**62.** $A(r)$ is the area of a circle of radius $r$.

**Evaluating the Derivative of an Inverse Function** **In Exercises 63–70, verify that $f$ has an inverse. Then use the function $f$ and the given real number $a$ to find $(f^{-1})'(a)$. (*Hint:* See Example 5.)**

**63.** $f(x) = 5 - 2x^3, \quad a = 7$

**64.** $f(x) = x^3 + 2x - 1, \quad a = 2$

**65.** $f(x) = \frac{1}{27}(x^5 + 2x^3), \quad a = -11$

**66.** $f(x) = \sqrt{x - 4}, \quad a = 2$

**67.** $f(x) = \sin x, \quad -\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}, \quad a = \dfrac{1}{2}$

**68.** $f(x) = \cos 2x, \quad 0 \leq x \leq \dfrac{\pi}{2}, \quad a = 1$

**69.** $f(x) = \dfrac{x + 6}{x - 2}, \quad x > 2, \quad a = 3$

**70.** $f(x) = \dfrac{x + 3}{x + 1}, \quad x > -1, \quad a = 2$

**Using Inverse Functions** **In Exercises 71–74, (a) find the domains of $f$ and $f^{-1}$, (b) find the ranges of $f$ and $f^{-1}$, (c) graph $f$ and $f^{-1}$, and (d) show that the slopes of the graphs of $f$ and $f^{-1}$ are reciprocals at the given points.**

| Functions | Point |
|---|---|
| **71.** $f(x) = x^3$ | $\left(\frac{1}{2}, \frac{1}{8}\right)$ |
| $f^{-1}(x) = \sqrt[3]{x}$ | $\left(\frac{1}{8}, \frac{1}{2}\right)$ |
| **72.** $f(x) = 3 - 4x$ | $(1, -1)$ |
| $f^{-1}(x) = \dfrac{3 - x}{4}$ | $(-1, 1)$ |
| **73.** $f(x) = \sqrt{x - 4}$ | $(5, 1)$ |
| $f^{-1}(x) = x^2 + 4, \quad x \geq 0$ | $(1, 5)$ |
| **74.** $f(x) = \dfrac{4}{1 + x^2}, \quad x \geq 0$ | $(1, 2)$ |
| $f^{-1}(x) = \sqrt{\dfrac{4 - x}{x}}$ | $(2, 1)$ |

**Using Composite and Inverse Functions** In Exercises 75–78, use the functions $f(x) = \frac{1}{8}x - 3$ and $g(x) = x^3$ to find the given value.

75. $(f^{-1} \circ g^{-1})(1)$
76. $(g^{-1} \circ f^{-1})(-3)$
77. $(f^{-1} \circ f^{-1})(6)$
78. $(g^{-1} \circ g^{-1})(-4)$

**Using Composite and Inverse Functions** In Exercises 79–82, use the functions $f(x) = x + 4$ and $g(x) = 2x - 5$ to find the given function.

79. $g^{-1} \circ f^{-1}$
80. $f^{-1} \circ g^{-1}$
81. $(f \circ g)^{-1}$
82. $(g \circ f)^{-1}$

### WRITING ABOUT CONCEPTS

83. **In Your Own Words** Describe how to find the inverse function of a one-to-one function given by an equation in $x$ and $y$. Give an example.

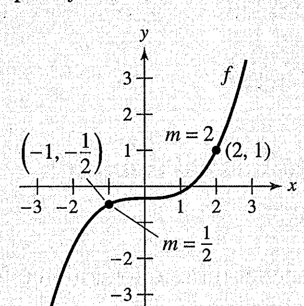
84. **A Function and Its Inverse** Describe the relationship between the graph of a function and the graph of its inverse function.

**Explaining Why a Function Is Not One-to-One** In Exercises 85 and 86, the derivative of the function has the same sign for all $x$ in its domain, but the function is not one-to-one. Explain.

85. $f(x) = \tan x$
86. $f(x) = \dfrac{x}{x^2 - 4}$

87. **Think About It** The function $f(x) = k(2 - x - x^3)$ is one-to-one and $f^{-1}(3) = -2$. Find $k$.

88. **HOW DO YOU SEE IT?** Use the information in the graph of $f$ below.




(a) What is the slope of the tangent line to the graph of $f^{-1}$ at the point $\left(-\frac{1}{2}, -1\right)$? Explain.

(b) What is the slope of the tangent line to the graph of $f^{-1}$ at the point $(1, 2)$? Explain.

**True or False?** In Exercises 89–92, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

89. If $f$ is an even function, then $f^{-1}$ exists.

90. If the inverse function of $f$ exists, then the $y$-intercept of $f$ is an $x$-intercept of $f^{-1}$.

91. If $f(x) = x^n$, where $n$ is odd, then $f^{-1}$ exists.

92. There exists no function $f$ such that $f = f^{-1}$.

93. **Making a Function One-to-One**

(a) Show that $f(x) = 2x^3 + 3x^2 - 36x$ is not one-to-one on $(-\infty, \infty)$.

(b) Determine the greatest value $c$ such that $f$ is one-to-one on $(-c, c)$.

94. **Proof** Let $f$ and $g$ be one-to-one functions. Prove that

(a) $f \circ g$ is one-to-one.

(b) $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$.

95. **Proof** Prove that if $f$ has an inverse function, then $(f^{-1})^{-1} = f$.

96. **Proof** Prove that if a function has an inverse function, then the inverse function is unique.

97. **Proof** Prove that a function has an inverse function if and only if it is one-to-one.

98. **Using Theorem 5.7** Is the converse of the second part of Theorem 5.7 true? That is, if a function is one-to-one (and therefore has an inverse function), then must the function be strictly monotonic? If so, prove it. If not, give a counterexample.

99. **Concavity** Let $f$ be twice-differentiable and one-to-one on an open interval $I$. Show that its inverse function $g$ satisfies

$$g''(x) = -\frac{f''(g(x))}{[f'(g(x))]^3}.$$

When $f$ is increasing and concave downward, what is the concavity of $f^{-1} = g$?

100. **Derivative of an Inverse Function** Let

$$f(x) = \int_2^x \frac{dt}{\sqrt{1 + t^4}}.$$

Find $(f^{-1})'(0)$.

101. **Derivative of an Inverse Function** Show that

$$f(x) = \int_2^x \sqrt{1 + t^2}\, dt$$

is one-to-one and find

$(f^{-1})'(0)$.

102. **Inverse Function** Let

$$y = \frac{x - 2}{x - 1}.$$

Show that $y$ is its own inverse function. What can you conclude about the graph of $f$? Explain.

103. **Using a Function** Let $f(x) = \dfrac{ax + b}{cx + d}$.

(a) Show that $f$ is one-to-one if and only if $bc - ad \neq 0$.

(b) Given $bc - ad \neq 0$, find $f^{-1}$.

(c) Determine the values of $a$, $b$, $c$, and $d$ such that $f = f^{-1}$.