5.4 Exponential Functions e^x Classwork Worksheet

exponent properties: $e^a e^b = e^{a+b}$ $\quad \frac{e^a}{e^b} = e^{a-b}$ $\quad \frac{d}{dx}e^u = e^u * u'$

Additional $y = \ln x$ and $y = e^x$ Properties: $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln 1 = 0$ | $\ln e = 1$

Solving an Exponential or Logarithmic Equation **In Exercises 1–16, solve for x accurate to three decimal places.**

1. $e^{\ln x} = 4$	**2.** $e^{\ln 3x} = 24$
3. $e^x = 12$	**4.** $5e^x = 36$
5. $9 - 2e^x = 7$	**8.** $100e^{-2x} = 35$
11. $\ln x = 2$	**12.** $\ln x^2 = 10$
13. $\ln(x - 3) = 2$	**14.** $\ln 4x = 1$
15. $\ln\sqrt{x + 2} = 1$	**16.** $\ln(x - 2)^2 = 12$

exponent properties: $e^a e^b = e^{a+b}$ $\quad \frac{e^a}{e^b} = e^{a-b}$ $\quad \frac{d}{dx}e^u = e^u * u'$

Additional $y = \ln x$ *and* $y = e^x$ Properties: $e^{\ln x} = x \;|\; \ln e^x = x \;|\; \ln 1 = 0 \;|\; \ln e = 1$

Finding a Derivative **In Exercises 33–54, find the derivative.**

33. $f(x) = e^{2x}$	**34.** $y = e^{-8x}$
35. $y = e^{\sqrt{x}}$	**36.** $y = e^{-2x^3}$
39. $y = e^x \ln x$	**40.** $y = xe^{4x}$
41. $y = x^3 e^x$	**42.** $y = x^2 e^{-x}$
43. $g(t) = (e^{-t} + e^t)^3$	**44.** $g(t) = e^{-3/t^2}$
45. $y = \ln(1 + e^{2x})$	**46.** $y = \ln\left(\frac{1 + e^x}{1 - e^x}\right)$

Finding a Derivative **In Exercises 33–54, find the derivative.**

$\frac{d}{dx}[\sin u] = (\cos u)u'$

14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$

15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$

$\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$

17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$

18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$

47. $y = \dfrac{2}{e^x + e^{-x}}$

48. $y = \dfrac{e^x - e^{-x}}{2}$

49. $y = \dfrac{e^x + 1}{e^x - 1}$

50. $y = \dfrac{e^{2x}}{e^{2x} + 1}$

51. $y = e^x(\sin x + \cos x)$

52. $y = e^{2x} \tan 2x$

Finding an Equation of a Tangent Line In Exercises 55–62, find an equation of the tangent line to the graph of the function at the given point.

Steps for finding Tangent Line Equation:

1) Identify Ordered Pair: Point: __________
2) Find Slope: Find f '(x) and evaluate the slope at x-value: Slope: m = __________
3) Put equation into point-slope form: $y - y_1 = m(x - x_1)$

55. $f(x) = e^{3x}$, $(0, 1)$	**56.** $f(x) = e^{-2x}$, $(0, 1)$
57. $f(x) = e^{1-x}$, $(1, 1)$	**58.** $y = e^{-2x+x^2}$, $(2, 1)$
59. $f(x) = e^{-x}\ln x$, $(1, 0)$	**62.** $y = xe^x - e^x$, $(1, 0)$

Implicit Differentiation In Exercises 63 and 64, use implicit differentiation to find dy/dx.

63. $xe^y - 10x + 3y = 0$

64. $e^{xy} + x^2 - y^2 = 10$

Finding the Equation of a Tangent Line In Exercises 65 and 66, find an equation of the tangent line to the graph of the function at the given point.

65. $xe^y + ye^x = 1, \quad (0, 1)$

66. $1 + \ln xy = e^{x-y}, \quad (1, 1)$

Ch. 5.5 Log and Exponential Derivatives for base a

11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$

12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$

Find the Derivative of the below functions: (Consider Expanding Log Expressions before Deriving if applicable)

39. $y = 5^{-4x}$	**40.** $y = 6^{3x-4}$
41. $f(x) = x\,9^x$	**42.** $y = x(6^{-2x})$
49. $h(t) = \log_5(4 - t)^2$	**48.** $y = \log_3(x^2 - 3x)$
51. $y = \log_5 \sqrt{x^2 - 1}$	**50.** $g(t) = \log_2(t^2 + 7)^3$
53. $f(x) = \log_2 \frac{x^2}{x - 1}$	**52.** $f(x) = \log_2 \sqrt[3]{2x + 1}$
55. $h(x) = \log_3 \frac{x\sqrt{x - 1}}{2}$	**56.** $g(x) = \log_5 \frac{4}{x^2\sqrt{1 - x}}$

5.4 Exponential Functions e^x Classwork Worksheet

Key

exponent properties: $e^a e^b = e^{a+b}$ $\quad \frac{e^a}{e^b} = e^{a-b}$ $\quad \frac{d}{dx} e^u = e^u * u'$ $\quad \frac{d}{dx}\ln u = \frac{u'}{u}$

Additional $y = \ln x$ and $y = e^x$ Properties: $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln 1 = 0$ | $\ln e = 1$

Solving an Exponential or Logarithmic Equation In **Exercises 1–16, solve for x accurate to three decimal places.**

$* \ln a^n = n \cdot \ln a$

1. $e^{\ln x} = 4$

$\boxed{x=4}$

2. $e^{\ln 3x} = 24$

$3x = 24$

$\boxed{x=8}$

3. $e^x = 12$

$\ln e^x = \ln 12$

$x \cancel{\ln e} = \ln 12$

$\boxed{x = \ln 12}$

4. $5e^x = 36$

$\frac{5e^x}{5} = \frac{36}{5}$

$e^x = \frac{36}{5}$

$\ln e^x = \ln\left(\frac{36}{5}\right)$

$x \cancel{\ln e} = \ln\left(\frac{36}{5}\right)$

$\boxed{x = \ln\left(\frac{36}{5}\right)}$

5. $9 - 2e^x = 7$

$-9 \qquad -9$

$-2e^x = -2$

$\frac{-2e^x}{-2} = \frac{-2}{-2}$

$e^x = 1$

$\ln e^x = \ln 1$

$x \cancel{\ln e} = 0$

$\boxed{x = 0}$

8. $100e^{-2x} = 35$

$\frac{100e^{-2x}}{100} = \frac{35}{100}$

$e^{-2x} = \frac{7}{20}$

$\ln e^{-2x} = \ln\left(\frac{7}{20}\right)$

$-2x \cdot \cancel{\ln e} = \ln\left(\frac{7}{20}\right)$

$-2x = \ln\left(\frac{7}{20}\right)$

$\boxed{x = -\frac{1}{2}\ln\left(\frac{7}{20}\right)}$

11. $\ln(x) = 2$

$e^{\ln x} = e^2$

$\boxed{x = e^2}$

12. $\ln x^2 = 10$

$2\ln x = 10$

$\ln x = \frac{10}{2}$

$\ln x = 5$

$e^{\ln x} = e^5$

$\boxed{x = e^5}$

13. $\ln(x - 3) = 2$

$e^{\ln(x-3)} = e^2$

$x - 3 = e^2$

$\boxed{x = e^2 + 3}$

14. $\ln(4x) = 1$

$e^{\ln 4x} = e^1$

$4x = e$

$\boxed{x = \frac{e}{4} = \frac{1}{4}e}$

15. $\ln\sqrt{x + 2} = 1$

$\ln(x+2)^{1/2} = 1$

$2\left(\frac{1}{2}\ln(x+2) = 1\right)$

$\ln(x+2) = 2$

$e^{\ln(x+2)} = e^2$

$x + 2 = e^2$

$\boxed{x = e^2 - 2}$

16. $\ln(x - 2)^2 = 12$

$\frac{2\ln(x-2)}{2} = \frac{12}{2}$

$\ln(x-2) = 6$

$e^{\ln(x-2)} = e^6$

$x - 2 = e^6$

$\boxed{x = e^6 + 2}$

exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ $\frac{d}{dx}e^u = e^u * u'$

Additional $y = \ln x$ and $y = e^x$ Properties: $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln 1 = 0$ | $\ln e = 1$

Finding a Derivative **In Exercises 33–54, find the derivative.**

33. $f(x) = e^{2x}$

$f'(x) = e^{2x} \cdot 2 = \boxed{2e^{2x}}$

34. $y = e^{-8x}$

$y' = e^{-8x} \cdot -8$ $\boxed{y' = -8e^{-8x}}$

35. $y = e^{\sqrt{x}}$

$y = e^{x^{1/2}}$

$y' = e^{x^{1/2}} \cdot \frac{1}{2}x^{-1/2}$

$\boxed{y' = \frac{e^{\sqrt{x}}}{2\sqrt{x}}}$

36. $y = e^{-2x^3}$

$y' = e^{-2x^3} \cdot -6x^2$

$y' = -6x^2 e^{-2x^3}$

39. $y = \underbrace{e^x}_{f}\underbrace{\ln x}_{g}$

$y' = \underbrace{e^x}_{f'} \cdot \underbrace{\ln x}_{g} + \underbrace{e^x}_{f} \cdot \underbrace{\frac{1}{x}}_{g'}$

*product rule

$\boxed{y' = e^x \ln x + \frac{e^x}{x}}$

40. $y = \underbrace{x}_{f}\underbrace{e^{4x}}_{g}$

*product rule

$y' = \underbrace{1}_{f'} \cdot \underbrace{e^{4x}}_{g} + \underbrace{x}_{f} \cdot \underbrace{e^{4x} \cdot 4}_{g'}$

$\boxed{y' = e^{4x} + 4xe^{4x}}$

41. $y = x^3e^x$

*product rule

$y' = \underbrace{3x^2}_{f'} \cdot \underbrace{e^x}_{g} + \underbrace{x^3}_{f} \cdot \underbrace{e^x}_{g'}$

$\boxed{y' = 3x^2e^x + x^3e^x}$

42. $y = x^2e^{-x}$

*product rule

$y' = \underbrace{2x}_{f'} \cdot \underbrace{e^{-x}}_{g} + \underbrace{x^2}_{f} \cdot \underbrace{e^{-x}(-1)}_{g'}$

$\boxed{y' = 2xe^{-x} - x^2e^{-x}}$

43. $g(t) = (e^{-t} + e^t)^3$

*chain rule:

outside: $(\ )^3$

inside: $e^{-t} + e^t$

$g'(t) = 3(\ )^2 \cdot (e^{-t}(-1) + e^t)$

$\boxed{g'(t) = 3(e^{-t} + e^t)^2(-e^{-t} + e^t)}$

44. $g(t) = e^{-3/t^2}$

$g'(t) = e^{-3t^{-2}} \cdot 6t^{-3}$

$\boxed{g'(t) = \frac{6e^{-3/t^2}}{t^3}}$

45. $y = \ln(1 + e^{2x})$

$y' = \frac{u'}{u}$

$y' = \frac{e^{2x} \cdot 2}{1 + e^{2x}}$

$\boxed{y' = \frac{2e^{2x}}{1 + e^{2x}}}$

46. $y = \ln\left(\frac{1 + e^x}{1 - e^x}\right)$

*expand first:

$y = \ln(1 + e^x) - \ln(1 - e^x)$

$y' = \frac{e^x}{1 + e^x} - \frac{-e^x}{1 - e^x}$

$\boxed{y' = \frac{e^x}{1 + e^x} + \frac{e^x}{1 - e^x}}$

Finding a Derivative In Exercises 33–54, find the derivative.

$\frac{d}{dx}[\sin u] = (\cos u)u'$ 14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$ 15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$

$\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$ 17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$ 18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$

47. $y = \dfrac{2}{e^x + e^{-x}}$

$y = 2(e^x + e^{-x})^{-1}$

*chain rule:

outside: $2(\ )^{-1}$

inside: $e^x + e^{-x}$

$y' = -2(\ )^{-2} \cdot (e^x + e^{-x}(-1))$

$y' = -2(e^x + e^{-x})^{-2}(e^x - e^{-x})$

$$\boxed{y' = \frac{-2(e^x - e^{-x})}{(e^x + e^{-x})^2}}$$

48. $y = \dfrac{e^x - e^{-x}}{2}$

$y = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

$y' = \frac{1}{2}e^x - \frac{1}{2}e^{-x}(-1)$

$$\boxed{y' = \frac{1}{2}e^x + \frac{1}{2}e^{-x}}$$

49. $y = \dfrac{e^x + 1}{e^x - 1}$

*quotient rule

$$y' = \frac{\overbrace{e^x}^{f'} \cdot \overbrace{(e^x - 1)}^{g} - \overbrace{(e^x + 1)}^{f} \cdot \overbrace{e^x}^{g'}}{\underbrace{(e^x - 1)^2}_{g^2}}$$

50. $y = \dfrac{e^{2x}}{e^{2x} + 1}$

*quotient rule

$$y' = \frac{\overbrace{e^{2x} \cdot 2}^{f'} \overbrace{(e^{2x} + 1)}^{g} - \overbrace{e^{2x}}^{f} \cdot \overbrace{(e^{2x} \cdot 2)}^{g'}}{\underbrace{(e^{2x} + 1)^2}_{g^2}}$$

51. $y = e^x(\sin x + \cos x)$

$$y' = \overbrace{e^x}^{f'} \cdot \overbrace{(\sin x + \cos x)}^{g} + \overbrace{e^x}^{f} \cdot \overbrace{(\cos x - \sin x)}^{g'}$$

*product rule

52. $y = e^{2x} \tan 2x$

*product rule

$$y' = \overbrace{e^{2x} \cdot 2}^{f'} \ \overbrace{\tan 2x}^{g} + \overbrace{e^{2x}}^{f} \cdot \overbrace{\sec^2(2x) \cdot 2}^{g'}$$

Finding an Equation of a Tangent Line In Exercises 55–62, find an equation of the tangent line to the graph of the function at the given point.

Steps for finding Tangent Line Equation:

1) Identify Ordered Pair: Point: ___________
2) Find Slope: Find f '(x) and evaluate the slope at x-value: Slope: m = ___________
3) Put equation into point-slope form: $y - y_1 = m(x - x_1)$

55. $f(x) = e^{3x}, \quad (0, 1)$

$f'(x) = e^{3x} \cdot 3$

$f'(0) = e^{3(0)} \cdot 3 = 1(3) = 3$

point: $(0,1)$ slope: $m = 3$

$\boxed{y - 1 = 3(x - 0)}$

56. $f(x) = e^{-2x}, \quad (0, 1)$

$f'(x) = e^{-2x}(-2)$

$f'(0) = -2e^{-2(0)} = -2(1) = -2$

point: $(0,1)$

slope: $m = -2$

$\boxed{y - 1 = -2(x - 0)}$

57. $f(x) = e^{1-x}, \quad (1, 1)$

$f'(x) = e^{1-x} \cdot (-1)$

$f'(1) = e^{1-1} \cdot (-1) = e^{0}(-1) = 1(-1) = -1$

point: $(1,1)$

slope: $m = -1$

$\boxed{y - 1 = -1(x - 1)}$

58. $y = e^{-2x+x^2}, \quad (2, 1)$

$y' = e^{-2x+x^2} \cdot (-2 + 2x)$

$y'(2) = e^{-4+4} \cdot (-2 + 4) = e^{0}(2) = 1(2) = 2$

point: $(2,1)$

slope: $m = 2$

$\boxed{y - 1 = 2(x - 2)}$

59. $f(x) = e^{-x}\ln x, \quad (1, 0)$

$f'(x) = \overbrace{e^{-x}(-1)}^{f'}\,\overbrace{\ln x}^{g} + \overbrace{e^{-x}}^{f} \cdot \overbrace{\frac{1}{x}}^{g'}$

$f'(1) = -e^{-1}(\ln 1) + e^{-1}\left(\frac{1}{1}\right)$

$= 0 - \frac{1}{e} = \frac{1}{e}$

point: $(1,0)$

slope: $m = \frac{1}{e}$

$\boxed{y - 0 = \frac{1}{e}(x - 1)}$

62. $y = xe^x - e^x, \quad (1, 0)$

$y' = \overbrace{1}^{f'}\,\overbrace{e^x}^{g} + \overbrace{x}^{f} \cdot \overbrace{e^x}^{g'} - e^x$

$y'(1) = e + 1e - e = e$

point: $(1,0)$

slope: $m = e$

$\boxed{y - 0 = e(x - 1)}$

Implicit Differentiation In Exercises 63 and 64, use implicit differentiation to find dy/dx.

63. $\underbrace{x}_{f}\underbrace{e^y}_{g} - 10x + 3y = 0$

$\underbrace{1}_{f'}\cdot\underbrace{e^y}_{g} + \underbrace{x}_{f}\cdot\underbrace{e^y\left(\frac{dy}{dx}\right)}_{g'} - 10 + 3\left(\frac{dy}{dx}\right) = 0$

$xe^y\left(\frac{dy}{dx}\right) + 3\left(\frac{dy}{dx}\right) = 10 - e^y$

$\frac{dy}{dx}\left(xe^y + 3\right) = 10 - e^y$

$\boxed{\frac{dy}{dx} = \frac{10 - e^y}{xe^y + 3}}$

64. $e^{xy} + x^2 - y^2 = 10$

$e^{xy}\left[1y + x\left(\frac{dy}{dx}\right)\right] + 2x - 2y\left(\frac{dy}{dx}\right) = 0$

$xe^{xy}\left(\frac{dy}{dx}\right) + ye^{xy} + 2x - 2y\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}\left(xe^{xy} - 2y\right) = -2x - ye^{xy}$

$\boxed{\frac{dy}{dx} = \frac{-2x - ye^{xy}}{xe^{xy} - 2y}}$

Finding the Equation of a Tangent Line In Exercises 65 and 66, find an equation of the tangent line to the graph of the function at the given point.

65. $xe^y + ye^x = 1, \quad (0, 1)$

$1\cdot e^y + x\cdot e^y\left(\frac{dy}{dx}\right) + \frac{dy}{dx}e^x + ye^x = 0$

$1e^1 + 0e^1\left(\frac{dy}{dx}\right) + \frac{dy}{dx}e^0 + 1e^0 = 0$

$e + \frac{dy}{dx} + 1 = 0 \qquad \frac{dy}{dx} = -e - 1$

$\boxed{y - 1 = (-1 - e)(x - 0)}$

66. $1 + \ln xy = e^{x-y}, \quad (1, 1)$

$1 + \ln x + \ln y = e^{x-y}$

$0 + \frac{1}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right) = e^{x-y}\left[1 - \frac{dy}{dx}\right]$

$1 + 1\left(\frac{dy}{dx}\right) = e^0\left[1 - \frac{dy}{dx}\right]$

$1 + \frac{dy}{dx} = 1 - \frac{dy}{dx}$

$2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 0$

point $(1, 1)$

slope: $m = 0$

$y - 1 = 0(x - 1)$

$\boxed{y = 1}$

Ch. 5.5 Log and Exponential Derivatives for base a

11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$ **12.** $\frac{d}{dx}[a^u] = (\ln a)a^u u'$

Find the Derivative of the below functions: (Consider Expanding Log Expressions before Deriving if applicable)

39. $y = 5^{-4x}$

$y' = \ln 5 \cdot 5^{-4x} \cdot (-4)$ $\boxed{y' = -4\ln 5 \cdot 5^{-4x}}$

40. $y = 6^{3x-4}$

$y' = \ln 6 \cdot 6^{3x-4} \cdot 3$ $\boxed{y' = 3\ln 6 \cdot 6^{3x-4}}$

41. $f(x) = x\,9^x$

$f'(x) = \underbrace{1}_{f'} \cdot \underbrace{9^x}_{g} + \underbrace{x}_{f} \cdot \underbrace{\ln 9 \cdot 9^x}_{g'}$

42. $y = x(6^{-2x})$

$y' = \underbrace{1}_{f'} \cdot \underbrace{6^{-2x}}_{g} + \underbrace{x}_{f} \cdot \underbrace{\ln 6 \cdot 6^{-2x} \cdot (-2)}_{g'}$

49. $h(t) = \log_5(4-t)^2$

$h(t) = 2\log_5(4-t)$

$h'(t) = 2 \cdot \frac{1}{\ln 5} \cdot \frac{-1}{4-t} = \boxed{\frac{-2}{\ln 5(4-t)}}$

48. $y = \log_3(x^2 - 3x)$

$y' = \frac{1}{\ln 3} \cdot \frac{2x-3}{x^2-3x} = \boxed{\frac{2x-3}{\ln 3(x^2-3x)}}$

51. $y = \log_5 \sqrt{x^2 - 1}$

$y = \log_5(x^2-1)^{1/2}$

$y = \frac{1}{2}\log_5(x^2-1)$

$y' = \frac{1}{2} \cdot \frac{1}{\ln 5} \cdot \frac{2x}{x^2-1}$

$\boxed{y' = \frac{x}{(\ln 5)(x^2-1)}}$

50. $g(t) = \log_2(t^2 + 7)^3$

$g(t) = 3\log_2(t^2+7)$

$g'(t) = 3 \cdot \frac{1}{\ln 2} \cdot \frac{2t}{t^2+7}$

$\boxed{g'(t) = \frac{6t}{(\ln 2)(t^2+7)}}$

53. $f(x) = \log_2 \frac{x^2}{x-1}$

$f(x) = \log_2(x^2) - \log_2(x-1)$

$f'(x) = \frac{1}{\ln 2} \cdot \frac{2x}{x^2} - \frac{1}{\ln 2} \cdot \frac{1}{x-1}$

52. $f(x) = \log_2 \sqrt[3]{2x+1}$

$f(x) = \log_2(2x+1)^{1/3}$

$f(x) = \frac{1}{3}\log_2(2x+1)$

$f'(x) = \frac{1}{3} \cdot \frac{1}{\ln 2} \cdot \frac{2}{2x+1}$

$\boxed{f'(x) = \frac{2}{3\ln 2(2x+1)}}$

55. $h(x) = \log_3 \frac{x\sqrt{x-1}}{2}$

$h(x) = \log_3 x + \log_3(x-1)^{1/2} - \log_3 2$

$h(x) = \log_3(x) + \frac{1}{2}\log_3(x-1) - \log_3 2$

$h'(x) = \frac{1}{\ln 3} \cdot \frac{1}{x} + \frac{1}{2} \frac{1}{\ln 3} \frac{1}{x-1} - 0$

56. $g(x) = \log_5 \frac{4}{x^2\sqrt{1-x}}$

$g(x) = \log_5 4 - \log_5 x^2 - \log_5(1-x)^{1/2}$

$g(x) = \log_5 4 - \log_5 x^2 - \frac{1}{2}\log_5(1-x)$

$g'(x) = 0 - \frac{1}{\ln 5} \cdot \frac{2x}{x^2} - \frac{1}{2} \cdot \frac{1}{\ln 5} \cdot \frac{-1}{1-x}$