Derivative of Exponential Function e^x

$y = \ln x$ and $y = e^x$ are inverse functions (meaning $f(g(x)) = x$ and $g(f(x)) = x$)

Example 1: Solve $7 = e^{x+1}$

Example 2: solve $\ln(2x - 3) = 5$

Reminder: exponent properties: $e^a e^b = e^{a+b}$ $\quad \frac{e^a}{e^b} = e^{a-b}$ Reminder: e is a NUMBER: if $y = e^2$, then $y' = 0$

Exponential Function e^x Derivative rule $\quad \frac{d}{dx} e^u = e^u * u'$

Example 3: find y' for $y = \ln(2x - e^{-2x})$

Example 4: Find y' for $y = xe^{(x^2+2x+3)^3}$

Example 5: Find the equation of the tangent line to the graph at the given point:

$y = e^{-x} \ln x \quad (1, 0)$

Example 6: Find dy/dx

$xe^y - 10x + 3y = 0$

Example 7: Find the equation of the tangent line to the graph of the function at the given point.

$xe^y + ye^x = 1$ at $(0, 1)$

Example 8: Find the 2nd derivative of the function

$f(x) = (3 + 2x)e^{-3x}$

Ex. 9: find the extrema and points of inflection for $g(t) = 1 + (2 + t)e^{-t}$

Ex. 10: find the extrema and points of inflection for $f(x) = \frac{e^x - e^{-x}}{2}$ (use common denominators)

Change of Base: $log_a x = \frac{lnx}{lna}$

Ex. 1 solve for x: $3^x = 1/81$

Ex. 2 solve: $\log_2 x = -4$

Derivative Rule for logs of other bases : $\frac{d}{dx}\log_a u = \frac{u'}{(\ln a)\, u'}$

Ex. 3 Find f '(x) for $f(x) = log_5 \sqrt[3]{(2x^2+7)}$

Ex. 4 Find f '(x) for $f(x) = log \frac{5x^3}{(x^2-3x)^3}$

Derivative Rule for Exponential functions of base a^x : $\frac{d}{dx}a^u = (\ln a)a^u * u'$

Ex. 5 Find f '(x) for $f(x) = 5^{x^2-2x}$

Ex. 6 Find f '(x) for $f(x) = x(4^{-x})$

Ex. 7:

1994 #4: A particle moves along the *x*-axis so that at any time $t > 0$ its velocity is given by $v(t) = t\ln t - t$.

a) Write an expression for the acceleration of the particle.

b) For what values of *t* is the particle moving to the right?

c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

Ch.5.4 Notes **Derivative of Exponential Function e^x**

$y = \ln x$ and $y = e^x$ are inverse functions (meaning $f(g(x)) = x$ and $g(f(x)) = x$)

Example 1: Solve $7 = e^{x+1}$

$\ln 7 = \ln e^{x+1}$

$\ln 7 = (x+1)\cancel{\ln e}$

$\boxed{\ln 7 - 1 = x}$

Example 2: solve $\ln(2x - 3) = 5$

$\log_e(2x-3) = 5$

$e^5 = 2x - 3$

$\boxed{\dfrac{e^5+3}{2} = x}$

Reminder: exponent properties: $e^a e^b = e^{a+b}$ $\quad \dfrac{e^a}{e^b} = e^{a-b}$ Reminder: e is a NUMBER: if $y = e^2$, then $y' = 0$

Exponential Function e^x Derivative rule $\dfrac{d}{dx}e^u = e^u * u'$

$\dfrac{d}{dx}e^x = e^x \cdot (1) = e^x$

Example 3: find y' for $y = \ln(2x - e^{-2x})$

$y' = \dfrac{2 - e^{-2x}(-2)}{2x - e^{-2x}} = \dfrac{2 + \frac{2}{e^{2x}}}{2x - \frac{1}{e^{2x}}}$

$\boxed{y' = \dfrac{2e^{2x}+2}{2xe^{2x}-1}}$

Example 4: Find y' for $y = xe^{(x^2+2x+3)^3}$

$y' = 1e^{(x^2+2x+3)^3} + x \cdot e^{(x^2+2x+3)^3} \cdot 3(x^2+2x+3)^2 \cdot (2x+2)$

$y' = e^{(x^2+2x+3)^3} + (6x^2+6x)(x^2+2x+3)^2 e^{(x^2+2x+3)^3}$

$y' = e^{(x^2+2x+3)^3}\left[1 + (6x^2+6x)(x^2+2x+3)^2\right]$

Example 5: Find the equation of the tangent line to the graph at the given point:
$y = e^{-x}\ln x$ (1, 0)

$y' = e^{-x}(-1)\ln x + e^{-x}\left(\frac{1}{x}\right)$

$= \dfrac{-\ln x}{e^x} + \dfrac{1}{xe^x}$

$y' = \dfrac{-x\ln x + 1}{xe^x}$

$y'(1) = \dfrac{-\ln(1)+1}{(1)e^1} = \dfrac{1}{e}$

$y - 0 = \frac{1}{e}(x-1)$

$\boxed{y = \frac{1}{e}(x-1)}$

Example 6: Find dy/dx

$xe^y - 10x + 3y = 0$

$1e^y + xe^y\left(\frac{dy}{dx}\right) - 10 + 3\frac{dy}{dx} = 0$

$\frac{dy}{dx}(xe^y + 3) = 10 - e^y$

$\boxed{\dfrac{dy}{dx} = \dfrac{10 - e^y}{xe^y + 3}}$

Example 7: Find the equation of the tangent line to the graph of the function at the given point.

$xe^y + ye^x = 1$ at (0, 1)

$1e^y + xe^y\left(\frac{dy}{dx}\right) + \frac{dy}{dx}e^x + ye^x = 0$

$1e^1 + 0e^1\left(\frac{dy}{dx}\right) + \frac{dy}{dx}e^0 + 1e^0 = 0$

$e + \frac{dy}{dx} + 1 = 0$

$\left.\dfrac{dy}{dx}\right|_{(0,1)} = -e - 1$

$\boxed{y - 1 = (-e-1)[x - 0]}$

Example 8: Find the 2nd derivative of the function

$f(x) = (3 + 2x)e^{-3x}$

$f' = (2)e^{-3x} + (3+2x)e^{-3x}(-3) = \dfrac{2 - 9 - 6x}{e^{3x}}$

$= \dfrac{-7-6x}{e^{3x}}$

$f'' = \dfrac{(-6)e^{3x} - (-7-6x)e^{3x}(3)}{(e^{3x})^2}$

$\dfrac{-6e^{3x} + 21e^{3x} + 18xe^{3x}}{(e^{3x})^2} = \boxed{\dfrac{15+18x}{e^{3x}}}$

$g(t) = 1 + \frac{2+t}{e^t}$

Ex. 9: find the extrema and points of inflection for $g(t) = 1 + (2+t)e^{-t}$

$g' = 0 + 1(e^{-t}) + (2+t)e^{-t}(-1) = \frac{1-2-t}{e^t} = \frac{-1-t}{e^t}$ $\boxed{t=-1}$

$+$ | $-$ at -1

Rel. max at $(-1, 1+e)$ b/c $f'(x)$ changes from + to −.

$g(-1) = 1 + (2-1)e^{1}$

$= 1 + e$

$g''(t) = \frac{(-1)e^t - (-1-t)e^t}{e^{2t}}$

$g''(t) = \frac{-e^t + e^t + te^t}{e^{2t}} = \frac{t}{e^t}$

$0 = \frac{t}{e^t}$

$t = 0$

$-$ | $+$ at 0

$f(0) = 1 + (2+0)e^0$

$= 1 + 2 = 3$

POI at $(0,3)$ b/c $f''(x)$ changes signs

Ex. 10: find the extrema and points of inflection for $f(x) = \frac{e^x - e^{-x}}{2}$ (use common denominators)

$f(x) = \frac{e^x - \frac{1}{e^x}}{2} = \frac{\frac{e^{2x}-1}{e^x}}{2} = \frac{e^{2x}-1}{2e^x}$

$f'(x) = \frac{e^{2x}(2)[2e^x] - (e^{2x}-1)(2e^x)}{4e^{2x}}$

$= \frac{4e^{3x} - 2e^{3x} + 2e^x}{4e^{2x}} = \frac{2e^{3x} + 2e^x}{4e^{2x}} = \frac{2e^x(e^{2x}+1)}{4e^{2x}} = \frac{e^{2x}+1}{2e^x} > 0$

$e^{2x} + 1 = 0$

$e^{2x} = -1$

$\boxed{\text{f(x) always increasing, no relative extrema}}$

$f''(x) = \frac{e^{2x}(2)[2e^x] - [e^{2x}+1]2e^x}{(2e^x)^2}$

$= \frac{4e^{3x} - 2e^{3x} - 2e^x}{4e^{2x}}$

$= \frac{2e^{3x} - 2e^x}{4e^{2x}}$

$= \frac{2e^x(e^{2x}-1)}{4e^{2x}} = \frac{e^{2x}-1}{2e^x} = 0$

$e^{2x} - 1 = 0$

$e^{2x} = 1$

$\ln e^{2x} = \ln 1$

$2x = 0$

$x = 0$

$f(0) = \frac{e^0 - e^{-0}}{2} = \frac{1-1}{2} = 0$

$-$ | $+$ at 0

$\boxed{\text{POI at (0,0) b/c } f''(x) \text{ changes sign}}$

Ch. 5.5

~~Ch.5.4~~ Notes **Derivative of Logs of other bases**

Change of Base: $log_a x = \frac{lnx}{lna}$

Ex. 1 solve for x: $3^x = 1/81$

$$\log_3 3^x = \log_3(1/81)$$

$$x = \log_3(3)^{-4}$$

$$x = -4$$

Ex. 2 solve: $\log_2 x = -4$

$$2^{\log_2 x} = 2^{-4}$$

$$x = 2^{-4} = \boxed{\frac{1}{16}}$$

Derivative Rule for logs of other bases : $\frac{d}{dx}\log_a u = \frac{u'}{(\ln a)u'}$ $= \left(\frac{1}{\ln a}\right)\left(\frac{u'}{u}\right)$

$$\frac{d}{dx}\log_a x = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \left(\frac{1}{\ln a}\right)\cdot\left(\frac{1}{x}\right) = \frac{1}{(\ln a)x}$$

Recall: $\frac{d}{dx}\ln u = \frac{u'}{u}$

$$\frac{d}{dx}\log_a u = \frac{d}{dx}\left(\frac{\ln u}{\ln a}\right) = \left(\frac{1}{\ln a}\right)\cdot\frac{d}{dx}(\ln u) = \left(\frac{1}{\ln a}\right)\left(\frac{u'}{u}\right)$$

Ex. 3 Find f '(x) for $f(x) = log_5 \sqrt[3]{(2x^2+7)}$

$$f(x) = \log_5(2x^2+7)^{1/3}$$

$$= \frac{1}{3}\log_5(2x^2+7)$$

$$f'(x) = \frac{1}{3}\left(\frac{1}{\ln 5}\right)\left(\frac{4x}{2x^2+7}\right) = \boxed{\frac{4x}{3\ln 5(2x^2+7)}}$$

Ex. 4 Find f '(x) for $f(x) = log\,\frac{5x^3}{(x^2-3x)^3}$

$$f(x) = \log 5x^3 - \log(x^2-3x)^3$$

$$= \log 5x^3 - 3\log(x^2-3x)$$

$$f'(x) = \left(\frac{1}{\ln 10}\right)\left(\frac{15x^2}{5x^3}\right) - 3\left(\frac{1}{\ln 10}\right)\left(\frac{2x-3}{x^2-3x}\right)$$

$$= \frac{3}{x\ln 10} - \frac{3(2x-3)}{(\ln 10)(x^2-3x)}$$

$* e^{\ln x} = x$

Derivative Rule for Exponential functions of base a^x : $\frac{d}{dx}a^u = (\ln a)a^u * u'$

$$\frac{d}{dx}a^x = e^{(\ln a)x} \rightarrow \frac{d}{dx}e^{(\ln a)x} = e^{(\ln a)(x)} \cdot \ln a = (\ln a)e^{(\ln a)x} = (\ln a)a^x$$

Recall: $\frac{d}{dx}e^u = e^u \cdot u'$

$$\frac{d}{dx}a^u = \ln a \cdot a^u \cdot u'$$

Ex. 5 Find f '(x) for f(x) = 5^{x^2-2x}

$$f'(x) = (\ln 5)(5^{x^2-2x})(2x-2)$$

Ex. 6 Find f '(x) for f(x) = $x(4^{-x})$

$$f'(x) = (1)(4^{-x}) + x \cdot (\ln 4)(4^{-x})(-1)$$

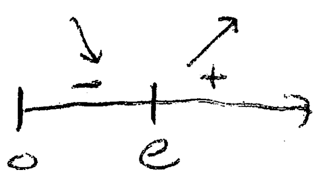
$$= \frac{1}{4^x} - \frac{x\ln 4}{4^x} = \boxed{\frac{1-x\ln 4}{4^x}}$$

Ex 7

$v(t) = t\ln t - t$

a) $a(t) = v'(t) = \ln t + t\left(\frac{1}{t}\right) - 1 = \ln t + 1 - 1 = \ln t$

b) Find critical points : set $v(t) = 0$



particle moving right (e, ∞) b/c $v(t) > 0$

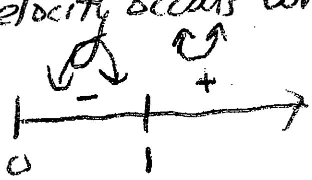
$v(t) = t\ln t - t$ | $\ln t - 1 = 0$

$0 = t(\ln t - 1)$ | $\ln t = 1$

| $e^1 = t$

$t = 0,\ t = e$

c) * minimum velocity occurs where $f''(x)$ changes from − to +



$a(t) = \ln t$

$0 = \ln t$

$e^0 = t$

$t = 1$

$v(1) = 1\ln(1) - 1$

$\boxed{v(1) = -1}$

Minimum velocity is −1 b/c $a(t) < 0$ for t in $(0, 1)$ and $a(t) > 0$ for all $t > 1$