5.4-5.5 Exponential Functions Derivatives Review WS #4 Morning Review

exponent properties: $e^a e^b = e^{a+b}$ $\quad \frac{e^a}{e^b} = e^{a-b}$ $\quad \frac{d}{dx}e^u = e^u * u'$ $\quad \frac{d}{dx}[\ln u] = \frac{u'}{u}$

$\ln(ab) = \ln a + \ln b$ | $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ | $\ln a^n = n\ln a$ | $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln e = 1$ | $\ln 1 = 0$

Solve the below Exponential or Logarithmic equation

1. $2\ln\sqrt{(x^3-6)^3} - 5 = 22$

2. $13 - 4e^{1-6x} = 1$

Find the derivative of the following functions:

3. $y = 2e^{5\sqrt{x^3}}$

4. $y = (\ln \sqrt[3]{(5-2x)^5})e^{5x}$

5. $f(x) = \frac{-3}{\sqrt{(\ln x^2 - e^{4x})^3}}$

6. $f(x) = \ln \sqrt[5]{\left(\frac{\sqrt{e^{4x}-x}}{3x-e^{2x}}\right)}$

exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ $\frac{d}{dx}e^u = e^u * u'$ $\frac{d}{dx}[\ln u] = \frac{u'}{u}$

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$\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$ $\frac{d}{dx}[a^u] = (\ln a)a^u u'$

Find the equation of a tangent line: $y - y_1 = m(x - x_1)$

7) $f(x) = 2e^{4-x}$ $at\ point\ (4,2)$

Use Implicit Differentiation to find dy/dx

8) $\ln\left(\frac{\sqrt[3]{y}}{5-x}\right) - e^{3y} - 11x = y + 1$

Find the Derivative of the below functions: (Consider Expanding Log Expressions before Deriving if applicable)

9) $y = 7^{4e-2x^5}$

10) $f(x) = log_3\left(\frac{\sqrt{(1-\pi x)^5}}{x^3}\right)$

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Key

exponent properties: $e^a e^b = e^{a+b}$ $\quad \frac{e^a}{e^b} = e^{a-b}$ $\quad \frac{d}{dx}e^u = e^u * u'$ $\quad \frac{d}{dx}[\ln u] = \frac{u'}{u}$

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Solve the below Exponential or Logarithmic equation

1. $2\ln\sqrt{(x^3-6)^3} - 5 = 22$

$2\ln(x^3-6)^{3/2} - 5 = 22$

$2\cdot\frac{3}{2}\ln(x^3-6) = 27$

$3\ln(x^3-6) = 27$

$\ln(x^3-6) = 9$

$e^{\ln(x^3-6)} = e^9$

$x^3 - 6 = e^9$

$x^3 = 6 + e^9$

$\boxed{x = \sqrt[3]{6+e^9}}$

2. $13 - 4e^{1-6x} = 1$

$-13 \qquad -13$

$-4e^{1-6x} = -12$

$e^{1-6x} = 3$

$\ln e^{1-6x} = \ln 3$

$1 - 6x = \ln 3$

$1 - \ln 3 = 6x$

$\boxed{\frac{1-\ln 3}{6} = x}$

Find the derivative of the following functions:

3. $y = 2e^{5\sqrt{x^3}}$

$y = 2e^{5x^{3/2}}$

$y' = 2e^{5x^{3/2}} \cdot \frac{15}{2}x^{1/2}$

$\boxed{y' = 15e^{5x^{3/2}}x^{1/2}}$

4. $y = (\ln\sqrt[3]{(5-2x)^5})e^{5x}$

$y = \underbrace{\ln(5-2x)^{5/3}}_{f} \cdot \underbrace{e^{5x}}_{g}$

$y = \frac{5}{3}\ln(5-2x)\cdot e^{5x}$

$y' = \underbrace{\frac{5}{3}\cdot\frac{-2}{5-2x}}_{f'}\cdot \underbrace{e^{5x}}_{g} + \underbrace{\frac{5}{3}\ln(5-2x)}_{f}\cdot\underbrace{e^{5x}\cdot 5}_{g'}$

$\boxed{y' = \frac{-10e^{5x}}{3(5-2x)} + \frac{25e^{5x}\ln(5-2x)}{3}}$

5. $f(x) = \frac{-3}{\sqrt{(\ln x^2 - e^{4x})^3}}$

$f(x) = \frac{-3}{(\ln x^2 - e^{4x})^{3/2}} = -3(2\ln x - e^{4x})^{-3/2}$

*chain rule:

outside: $-3(\ )^{-3/2}$

inside: $2\ln x - e^{4x}$

$f'(x) = \frac{9}{2}(2\ln x - e^{4x})^{-5/2}\cdot\left(2\cdot\frac{1}{x} - e^{4x}\cdot 4\right)$

$\boxed{f'(x) = \frac{9\left(\frac{2}{x} - 4e^{4x}\right)}{2(2\ln x - e^{4x})^{5/2}}}$

6. $f(x) = \ln\sqrt[5]{\left(\frac{\sqrt{e^{4x}-x}}{3x-e^{2x}}\right)}$

$f(x) = \ln\left[\frac{(e^{4x}-x)^{1/2}}{3x-e^{2x}}\right]^{1/5}$

$f(x) = \frac{1}{5}\ln(e^{4x}-x)^{1/2} - \frac{1}{5}\ln(3x-e^{2x})$

$f(x) = \frac{1}{10}\ln(e^{4x}-x) - \frac{1}{5}\ln(3x-e^{2x})$

$f'(x) = \frac{1}{10}\cdot\frac{e^{4x}(4)-1}{e^{4x}-x} - \frac{1}{5}\cdot\frac{3-e^{2x}(2)}{3x-e^{2x}}$

$\boxed{f'(x) = \frac{4e^{4x}-1}{10(e^{4x}-x)} - \frac{3-2e^{2x}}{5(3x-e^{2x})}}$

exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ $\frac{d}{dx}e^u = e^u * u'$ $\frac{d}{dx}[\ln u] = \frac{u'}{u}$

$\ln(ab) = \ln a + \ln b$ | $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ | $\ln a^n = n\ln a$ | $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln e = 1$ | $\ln 1 = 0$

$\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$ $\frac{d}{dx}[a^u] = (\ln a)a^u u'$

Find the equation of a tangent line: $y - y_1 = m(x - x_1)$

7) $f(x) = 2e^{4-x}$ at point $(4,2)$

$f'(x) = 2e^{4-x}\cdot(-1)$

$f'(4) = 2e^{4-4}(-1) = 2e^0(-1) = -2$

point: $(4,2)$ slope: $m = -2$

$\boxed{y - 2 = -2(x-4)}$

Use Implicit Differentiation to find dy/dx

8) $\ln\left(\frac{\sqrt[3]{y}}{5-x}\right) - e^{3y} - 11x = y + 1$

$\ln y^{1/3} - \ln(5-x) - e^{3y} - 11x = y + 1$

$\frac{1}{3}\ln y - \ln(5-x) - e^{3y} - 11x = y + 1$

$\frac{1}{3}\cdot\frac{1}{y}\left(\frac{dy}{dx}\right) - \frac{-1}{5-x} - e^{3y}(3)\left(\frac{dy}{dx}\right) - 11 = 1\left(\frac{dy}{dx}\right)$

$\frac{1}{3y}\left(\frac{dy}{dx}\right) - 3e^{3y}\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) = 11 - \frac{1}{5-x}$

$\frac{dy}{dx}\left(\frac{1}{3y} - 3e^{3y} - 1\right) = 11 - \frac{1}{5-x}$

$$\boxed{\frac{dy}{dx} = \frac{11 - \frac{1}{5-x}}{\frac{1}{3y} - 3e^{3y} - 1}}$$

Find the Derivative of the below functions: (Consider Expanding Log Expressions before Deriving if applicable)

9) $y = 7^{4e-2x^5}$

$y' = \ln 7 . 7^{4e-2x^5} \cdot -10x^4$

$\boxed{y' = -10x^4(\ln 7)\cdot 7^{4e-2x^5}}$

10) $f(x) = \log_3\left(\frac{\sqrt{(1-\pi x)^5}}{x^3}\right)$

$f(x) = \log_3(1-\pi x)^{5/2} - \log_3(x^3)$

$f(x) = \frac{5}{2}\log_3(1-\pi x) - 3\log_3 x$

$f'(x) = \frac{5}{2}\cdot\frac{-\pi}{(\ln 3)(1-\pi x)} - 3\cdot\frac{1}{(\ln 3)(x)}$

$$\boxed{f'(x) = \frac{-5\pi}{2\ln 3(1-\pi x)} - \frac{3}{(\ln 3)(x)}}$$