5.4-5.5 Exponential Functions Derivatives Review WS #1

exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ $\frac{d}{dx}e^u = e^u * u'$ $\frac{d}{dx}[\ln u] = \frac{u'}{u}$

$\ln(ab) = \ln a + \ln b$ | $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ | $\ln a^n = n \ln a$ | $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln e = 1$ | $\ln 1 = 0$

Solve the below Exponential or Logarithmic equation

1. $\ln \sqrt[3]{x - 4} = 2$

2. $5 + 3e^{4x} = 17$

Find the derivative of the following functions:

3. $y = e^{\sqrt[5]{x}}$

4. $y = 2x^3 e^{5x}$

5. $f(x) = (e^x - e^{3x})^4$

6. $f(x) = \ln\left(\frac{5 - e^{2x}}{e^{5x} + 5}\right)$

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Find the equation of a tangent line: $y - y_1 = m(x - x_1)$

7) $f(x) = e^{x^2-8}$ $\quad$ $at\ point\ (3, e)$

Use Implicit Differentiation to find dy/dx

8) $\ln xy - e^y - y + 8x^2 = 12 - 2y^3$

Find the Derivative of the below functions: (Consider Expanding Log Expressions before Deriving if applicable)

$\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$ $\qquad$ $\frac{d}{dx}[a^u] = (\ln a)a^u u'$

9) $y = 7^{5-4x^3}$

10) $f(x) = log_6\left(\frac{7x}{\sqrt{3-x}}\right)$

5.4-5.5 Exponential Functions Derivatives Review WS #1

Key

exponent properties: $e^a e^b = e^{a+b}$ $\quad \frac{e^a}{e^b} = e^{a-b}$ $\quad \frac{d}{dx}e^u = e^u * u'$ $\quad \frac{d}{dx}[\ln u] = \frac{u'}{u}$

$\ln(ab) = \ln a + \ln b$ | $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ | $\ln a^n = n\ln a$ | $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln e = 1$ | $\ln 1 = 0$

Solve the below Exponential or Logarithmic equation

1. $\ln \sqrt[3]{x-4} = 2$

$\ln(x-4)^{1/3} = 2$

$\frac{1}{3}\ln(x-4) = 2$

$3\left[\frac{1}{3}\ln(x-4) = 2\right]$

$\ln(x-4) = 6$

$e^{\ln(x-4)} = e^6$

$x-4 = e^6$

$\boxed{x = 4+e^6}$

2. $5 + 3e^{4x} = 17$

$3e^{4x} = 12$

$\frac{3e^{4x}}{3} = \frac{12}{3}$

$e^{4x} = 4$

$\ln e^{4x} = \ln 4$

$4x \cdot \cancel{\ln e} = \ln 4$

$\boxed{x = \frac{\ln 4}{4} \text{ or } \frac{1}{4}\ln 4}$

Find the derivative of the following functions:

3. $y = e^{\sqrt[5]{x}}$

$y = e^{x^{1/5}}$

$y' = e^{x^{1/5}} \cdot \frac{1}{5}x^{-4/5}$

$\boxed{y' = \frac{e^{x^{1/5}}}{5x^{4/5}}}$

4. $y = 2x^3e^{5x}$

$y = \overbrace{2x^3}^{f} \cdot \overbrace{e^{5x}}^{g}$ *product rule

$y' = \overbrace{6x^2}^{f'} \cdot \overbrace{e^{5x}}^{g} + \overbrace{2x^3}^{f} \cdot \overbrace{e^{5x} \cdot 5}^{g'}$

$\boxed{y' = 6x^2e^{5x} + 10x^3e^{5x}}$

5. $f(x) = (e^x - e^{3x})^4$

* chain rule

outside: $(\ )^4$

inside: $e^x - e^{3x}$

$f'(x) = 4(e^x - e^{3x})^3 \cdot (e^x - e^{3x}(3))$

$\boxed{f'(x) = 4(e^x - e^{3x})^3(e^x - 3e^{3x})}$

6. $f(x) = \ln\left(\frac{5-e^{2x}}{e^{5x}+5}\right)$

*expand term first:

$y = \ln(5 - e^{2x}) - \ln(e^{5x}+5)$

$f'(x) = \frac{-e^{2x}(2)}{5-e^{2x}} - \frac{e^{5x}(5)}{e^{5x}+5}$

$\boxed{f'(x) = \frac{-2e^{2x}}{5-e^{2x}} - \frac{5e^{5x}}{e^{5x}+5}}$

exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ $\frac{d}{dx}e^u = e^u * u'$ $\frac{d}{dx}[\ln u] = \frac{u'}{u}$

$\ln(ab) = \ln a + \ln b$ | $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ | $\ln a^n = n\ln a$ | $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln e = 1$ | $\ln 1 = 0$

Find the equation of a tangent line: $y - y_1 = m(x - x_1)$

7) $f(x) = e^{x^2-8}$ *at point* $(3, e)$

$f'(x) = e^{x^2-8} \cdot 2x$

$f'(x) = 2xe^{x^2-8}$

$f'(3) = 2(3) \cdot e^{3^2-8}$

$f'(3) = 6e^1$

point: $(3, e)$ slope: $m = 6e$

$y - y_1 = m(x - x_1)$

$\boxed{y - e = 6e(x-3)}$

Use Implicit Differentiation to find dy/dx

8) $\ln xy - e^y - y + 8x^2 = 12 - 2y^3$

*expand first if possible

$\ln x + \ln y - e^y - y + 8x^2 = 12 - 2y^3$

$\frac{1}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right) - e^y\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) + 16x = 0 - 6y^2\left(\frac{dy}{dx}\right)$

$\frac{dy}{dx}\left(\frac{1}{y} - e^y - 1 + 6y^2\right) = \frac{-1}{x} - 16x$

$$\boxed{\frac{dy}{dx} = \frac{\frac{-1}{x} - 16x}{\frac{1}{y} - e^y - 1 + 6y^2}}$$

Find the Derivative of the below functions: (Consider Expanding Log Expressions before Deriving if applicable)

$\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$ $\frac{d}{dx}[a^u] = (\ln a)a^u u'$

9) $y = 7^{5-4x^3}$

$y' = \ln 7 \cdot 7^{5-4x^3} \cdot -12x^2$

$\boxed{y' = -12x^2 \cdot \ln 7 \cdot 7^{5-4x^3}}$

10) $f(x) = log_6\left(\frac{7x}{\sqrt{3-x}}\right)$

$f(x) = \log_6(7x) - \log_6(3-x)^{1/2}$

$f(x) = \log_6(7x) - \frac{1}{2}\log_6(3-x)$

$f'(x) = \frac{1}{\ln 6}\frac{\cancel{7}}{\cancel{7}x} - \frac{1}{2} \cdot \frac{1}{\ln 6} \cdot \frac{-1}{3-x}$

$$\boxed{f'(x) = \frac{1}{x\ln 6} + \frac{1}{2\ln 6(3-x)}}$$