5.4-5.5 Exponential Functions Derivatives Review WS #2

exponent properties: $e^a e^b = e^{a+b}$ $\quad \frac{e^a}{e^b} = e^{a-b}$ $\quad \frac{d}{dx}e^u = e^u * u'$ $\quad \frac{d}{dx}[\ln u] = \frac{u'}{u}$

$\ln(ab) = \ln a + \ln b$ | $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ | $\ln a^n = n\ln a$ | $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln e = 1$ | $\ln 1 = 0$

Solve the below Exponential or Logarithmic equation

1. $2\ln(x-4)^3 = 36$

2. $5 + 2e^{-3x} = 17$

Find the derivative of the following functions:

3. $y = 3e^{2(\sqrt[3]{x})}$

4. $y = (\ln x^3)e^{-2x}$

5. $f(x) = \frac{5}{\sqrt{(e^x - e^{2x})}}$

6. $f(x) = \ln\left(\frac{3-e^{2x}}{\sqrt{e^{3x}-1}}\right)$

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$\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$ $\frac{d}{dx}[a^u] = (\ln a)a^u u'$

Find the equation of a tangent line: $y - y_1 = m(x - x_1)$

7) $f(x) = e^{8-x^3}$ $\quad$ *at point* $(2,1)$

Use Implicit Differentiation to find dy/dx

8) $\ln\left(\frac{x}{y}\right) - e^{2y} - y - 2x = 12 - 3y^3$

Find the Derivative of the below functions: (Consider Expanding Log Expressions before Deriving if applicable)

9) $y = 3^{6e-3x^4}$

10) $f(x) = log_5\left(\frac{\sqrt{(6-\pi x)^5}}{5x^2}\right)$

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Key

exponent properties: $e^a e^b = e^{a+b}$ $\quad \frac{e^a}{e^b} = e^{a-b}$ $\quad \frac{d}{dx}e^u = e^u * u'$ $\quad \frac{d}{dx}[\ln u] = \frac{u'}{u}$

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Solve the below Exponential or Logarithmic equation

1. $2\ln(x-4)^3 = 36$

$2(3)\ln(x-4) = 36$

$\frac{6\ln(x-4)}{6} = \frac{36}{6}$

$\ln(x-4) = 6$

$e^{\ln(x-4)} = e^6$

$x - 4 = e^6$

$\boxed{x = 4 + e^6}$

2. $5 + 2e^{-3x} = 17$

$-5 \qquad -5$

$2e^{-3x} = 12$

$e^{-3x} = 6$

$\ln e^{-3x} = \ln 6$

$-3x \cdot \cancel{\ln e} = \ln 6$

$-3x = \ln 6$

$\boxed{x = -\frac{\ln 6}{3} \text{ or } -\frac{1}{3}\ln 6}$

Find the derivative of the following functions:

3. $y = 3e^{2(\sqrt[3]{x})}$

$y = 3e^{2x^{1/3}}$

$y' = \cancel{3}e^{2x^{1/3}} \cdot \frac{2}{\cancel{3}}x^{-2/3}$

$\boxed{y' = \frac{2e^{2x^{1/3}}}{x^{2/3}}}$

4. $y = (\ln x^3)e^{-2x}$

$y = \underbrace{3\ln x}_{f} \cdot \underbrace{e^{-2x}}_{g}$ * product rule

$y' = \underbrace{3\left(\frac{1}{x}\right)}_{f'} \cdot \underbrace{e^{-2x}}_{g} + \underbrace{3\ln x}_{f} \cdot \underbrace{e^{-2x}(-2)}_{g'}$

$\boxed{y' = \frac{3}{xe^{2x}} - \frac{6\ln x}{e^{2x}}}$

5. $f(x) = \frac{5}{\sqrt{(e^x - e^{2x})}}$

$f(x) = \frac{5}{(e^x - e^{2x})^{1/2}} = 5(e^x - e^{2x})^{-1/2}$

*chain rule

outside: $5(\ )^{-1/2}$

inside: $e^x - e^{2x}$

$f'(x) = -\frac{5}{2}(e^x - e^{2x})^{-3/2}(e^x - e^{2x}(2))$

$\boxed{f'(x) = \frac{-5(e^x - 2e^{2x})}{2(e^x - e^{2x})^{3/2}}}$

6. $f(x) = \ln\left(\frac{3 - e^{2x}}{\sqrt{e^{3x} - 1}}\right)$

$f(x) = \ln(3 - e^{2x}) - \ln(e^{3x} - 1)^{1/2}$

$f(x) = \ln(3 - e^{2x}) - \frac{1}{2}\ln(e^{3x} - 1)$

$f'(x) = \frac{-2e^{2x}}{3 - e^{2x}} - \frac{1}{2} \cdot \frac{e^{3x}(3)}{e^{3x} - 1}$

$\boxed{f'(x) = \frac{-2e^{2x}}{3 - e^{2x}} - \frac{3e^{3x}}{2(e^{3x} - 1)}}$

exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ $\frac{d}{dx}e^u = e^u * u'$ $\frac{d}{dx}[\ln u] = \frac{u'}{u}$

$\ln(ab) = \ln a + \ln b$ | $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ | $\ln a^n = n\ln a$ | $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln e = 1$ | $\ln 1 = 0$

Find the equation of a tangent line: $y - y_1 = m(x - x_1)$

7) $f(x) = e^{8-x^3}$ at point (2,1)

$f'(x) = e^{8-x^3}\cdot(-3x^2)$

$f'(x) = -3x^2 e^{8-x^3}$

$f'(2) = -3(2)^2 e^{8-2^3}$

$f'(2) = -12e^{8-8}$

$f'(2) = -12e^0 = -12$

point: (2,1) slope: $m = -12$

$y - y_1 = m(x - x_1)$

$\boxed{y - 1 = -12(x-2)}$

Use Implicit Differentiation to find dy/dx

8) $\ln\left(\frac{x}{y}\right) - e^{2y} - y - 2x = 12 - 3y^3$

$\ln x - \ln y - e^{2y} - y - 2x = 12 - 3y^3$

$\frac{1}{x} - \frac{1}{y}\left(\frac{dy}{dx}\right) - e^{2y}\cdot(2)\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) - 2 = 0 - 9y^2\left(\frac{dy}{dx}\right)$

$\frac{dy}{dx}\left(9y^2 - \frac{1}{y} - 2e^{2y} - 1\right) = 2 - \frac{1}{x}$

$\boxed{\frac{dy}{dx} = \frac{2 - \frac{1}{x}}{9y^2 - \frac{1}{y} - 2e^{2y} - 1}}$

Find the Derivative of the below functions: (Consider Expanding Log Expressions before Deriving if applicable)

$\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$ $\frac{d}{dx}[a^u] = (\ln a)a^u u'$

9) $y = 3^{6e-3x^4}$

$y' = \ln 3 \cdot 3^{6e-3x^4} \cdot -12x^3$

$\boxed{y' = -12x^3 \ln 3 \cdot 3^{6e-3x^4}}$

10) $f(x) = \log_5\left(\frac{\sqrt{(6-\pi x)^5}}{5x^2}\right)$

$f(x) = \log_5(6-\pi x)^{5/2} - \log_5(5x^2)$

$f(x) = \frac{5}{2}\log_5(6-\pi x) - \log_5(5x^2)$

$f'(x) = \frac{5}{2}\cdot\frac{1}{\ln 5}\cdot\frac{-\pi}{6-\pi x} - \frac{1}{\ln 5}\cdot\frac{\cancel{10}^{2}x}{\cancel{5}x^2}$

$\boxed{f'(x) = \frac{-5\pi}{2\ln 5(6-\pi x)} - \frac{2}{\ln 5(x)}}$