5.4-5.5 Exponential Functions Derivatives Review WS #3

exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ $\frac{d}{dx}e^u = e^u * u'$ $\frac{d}{dx}[\ln u] = \frac{u'}{u}$

$\ln(ab) = \ln a + \ln b$ | $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ | $\ln a^n = n\ln a$ | $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln e = 1$ | $\ln 1 = 0$

Solve the below Exponential or Logarithmic equation

1. $2\ln\sqrt{(x^2 - 4)} - 5 = 13$

2. $11 - 5e^{3x-2} = 1$

Find the derivative of the following functions:

3. $y = 2e^{2\sqrt{x^7}}$

4. $y = (\ln\sqrt{3 - 4x})e^{5x}$

5. $f(x) = \frac{7}{\sqrt{(\ln x - e^{3x})^5}}$

6. $f(x) = \ln \sqrt[3]{\left(\frac{3 - e^{2x}}{\sqrt{e^{3x} - 1}}\right)^5}$

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$\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$ $\frac{d}{dx}[a^u] = (\ln a)a^u u'$

Find the equation of a tangent line: $y - y_1 = m(x - x_1)$

7) $f(x) = 2e^{4x-x^3}$ *at point* $(2,2)$

Use Implicit Differentiation to find dy/dx

8) $\ln\left(\frac{y\sqrt{y}}{x-3}\right) - e^y - y = 12x - 15$

Find the Derivative of the below functions: (Consider Expanding Log Expressions before Deriving if applicable)

9) $y = 2^{6e-3x^5}$

10) $f(x) = \log_3\left(x\sqrt{(7-\pi x)^3}\right)$

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Key

exponent properties: $e^a e^b = e^{a+b}$ $\quad \frac{e^a}{e^b} = e^{a-b}$ $\quad \frac{d}{dx}e^u = e^u * u'$ $\quad \frac{d}{dx}[\ln u] = \frac{u'}{u}$

$\ln(ab) = \ln a + \ln b$ | $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ | $\ln a^n = n\ln a$ | $e^{\ln x} = x$ | $\ln e^x = x$ | $\ln e = 1$ | $\ln 1 = 0$

Solve the below Exponential or Logarithmic equation

1. $2\ln\sqrt{(x^2-4)} - 5 = 13$

$2\ln(x^2-4)^{1/2} - 5 = 13$

$2 \cdot \frac{1}{2}\ln(x^2-4) - 5 = 13$

$\ln(x^2-4) = 18$

$e^{\ln(x^2-4)} = e^{18}$

$x^2 - 4 = e^{18}$

$x^2 = e^{18} + 4$

$\boxed{x = \pm\sqrt{e^{18}+4}}$

2. $11 - 5e^{3x-2} = 1$

$-11 \qquad -11$

$-5e^{3x-2} = -10$

$e^{3x-2} = 2$

$\ln e^{3x-2} = \ln 2$

$(3x-2)\cancel{\ln e} = \ln 2$

$3x - 2 = \ln 2$

$3x = 2 + \ln 2$

$\boxed{x = \frac{2+\ln 2}{3}}$

Find the derivative of the following functions:

3. $y = 2e^{2\sqrt{x^7}}$

$y = 2e^{2x^{7/2}}$ $\quad * \frac{d}{dx}e^u = e^u \cdot u'$

$y' = 2e^{2x^{7/2}} \cdot 2 \cdot \frac{7}{2}x^{5/2}$

$y' = 2e^{2x^{7/2}} \cdot 7x^{5/2}$

$\boxed{y' = 14x^{5/2}e^{2x^{7/2}}}$

4. $y = (\ln\sqrt{3-4x})e^{5x}$

$y = \ln(3-4x)^{1/2} \cdot e^{5x}$ $\quad$ *product rule

$y = \underbrace{\frac{1}{2}\ln(3-4x)}_{f} \cdot \underbrace{e^{5x}}_{g}$

$y' = \underbrace{\frac{1}{2}\cdot\frac{-4}{3-4x}}_{f'} \cdot \underbrace{e^{5x}}_{g} + \underbrace{\frac{1}{2}\ln(3-4x)}_{f} \cdot \underbrace{e^{5x}\cdot 5}_{g'}$

$\boxed{y' = \frac{-2e^{5x}}{3-4x} + \frac{5e^{5x}\ln(3-4x)}{2}}$

5. $f(x) = \frac{7}{\sqrt{(\ln x - e^{3x})^5}}$

$f(x) = \frac{7}{(\ln x - e^{3x})^{5/2}}$

$f(x) = 7(\ln x - e^{3x})^{-5/2}$

*chain rule

outside: $7(\ )^{-5/2}$

inside: $\ln x - e^{3x}$

$f'(x) = \frac{-35}{2}(\ln x - e^{3x})^{-7/2}\left(\frac{1}{x} - e^{3x}(3)\right)$

$\boxed{f'(x) = \frac{-35\left(\frac{1}{x} - 3e^{3x}\right)}{2(\ln x - e^{3x})^{7/2}}}$

6. $f(x) = \ln\sqrt[3]{\left(\frac{3-e^{2x}}{\sqrt{e^{3x}-1}}\right)^5}$

$f(x) = \ln\left[\frac{3-e^{2x}}{(e^{3x}-1)^{1/2}}\right]^{5/3} = \frac{5}{3}\ln\left[\frac{3-e^{2x}}{(e^{3x}-1)^{1/2}}\right]$

$f(x) = \frac{5}{3}\ln(3-e^{2x}) - \frac{5}{3}\ln(e^{3x}-1)^{1/2}$

$f(x) = \frac{5}{3}\ln(3-e^{2x}) - \frac{5}{3}\cdot\frac{1}{2}\ln(e^{3x}-1)$

$f'(x) = \frac{5}{3}\cdot\frac{-e^{2x}(2)}{3-e^{2x}} - \frac{5}{\cancel{6}_2}\cdot\frac{e^{3x}(\cancel{3})}{e^{3x}-1}$

$\boxed{f'(x) = \frac{-10e^{2x}}{3(3-e^{2x})} - \frac{5e^{3x}}{2(e^{3x}-1)}}$

exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ $\frac{d}{dx}e^u = e^u * u'$ $\frac{d}{dx}[\ln u] = \frac{u'}{u}$

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$\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$ $\frac{d}{dx}[a^u] = (\ln a)a^u u'$

Find the equation of a tangent line: $y - y_1 = m(x - x_1)$

7) $f(x) = 2e^{4x-x^3}$ at point $(2,2)$

$f'(x) = 2e^{4x-x^3} \cdot (4-3x^2)$

$f'(2) = 2e^{8-2^3}(4-3(2)^2)$

$f'(2) = 2e^0(4-12)$

$f'(2) = 2(1)(-8)$

$f'(2) = -16$

point: $(2,2)$ slope: $m=-16$

$\boxed{y-2=-16(x-2)}$

Use Implicit Differentiation to find dy/dx

8) $\ln\left(\frac{y\sqrt{y}}{x-3}\right) - e^y - y = 12x - 15$

$\ln y + \ln y^{1/2} - \ln(x-3) - e^y - y = 12x-15$

$\frac{1}{y}\left(\frac{dy}{dx}\right) + \frac{1}{2}\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) - \frac{1}{x-3} - e^y\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) = 12$

$\frac{dy}{dx}\left[\frac{1}{y} + \frac{1}{2y} - e^y - 1\right] = 12 + \frac{1}{x-3}$

$$\boxed{\frac{dy}{dx} = \frac{12 + \frac{1}{x-3}}{\frac{1}{y} + \frac{1}{2y} - e^y - 1}}$$

Find the Derivative of the below functions: (Consider Expanding Log Expressions before Deriving if applicable)

9) $y = 2^{6e-3x^5}$

$* \ \frac{d}{dx}a^u = \ln a \cdot a^u \cdot u'$

$y' = \ln 2 \cdot 2^{6e-3x^5} \cdot -15x^4$

$\boxed{y' = -15x^4(\ln 2) \cdot 2^{6e-3x^5}}$

10) $f(x) = \log_3\left(x\sqrt{(7-\pi x)^3}\right)$

$f(x) = \log_3(x) + \log_3(7-\pi x)^{3/2}$

$f(x) = \log_3(x) + \frac{3}{2}\log_3(7-\pi x)$

$f'(x) = \frac{1}{\ln 3(x)} + \frac{3}{2} \cdot \frac{-\pi}{\ln 3(7-\pi x)}$

$$\boxed{f'(x) = \frac{1}{x\ln 3} - \frac{3\pi}{2\ln 3(7-\pi x)}}$$