2017

$\boxed{5.4} \to e^x$ p.352-353 # 1, 5, 11, 19, 21, 33-51 odd, 59, 63, 65, 71, 75

Solve exponential/Log equations

1) $e^{\ln x} = 4$

$\boxed{x = 4}$

5) $9 - 2e^x = 7$

$-2e^x = -2$

$e^x = 1$

$\ln e^x = \ln 1$

$x = \ln 1$

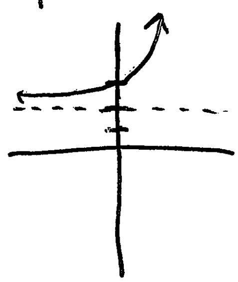
$\boxed{x = 0}$

11) $\ln x = 2$

$\log_e x = 2$

$e^2 = x$ $\boxed{x = e^2}$

19) $y = e^x + 2$



Find derivative: $* \frac{d}{dx} e^u = e^u \cdot u'$

35) $y = e^{\sqrt{x}}$ $y = e^{x^{1/2}}$ $y' = e^{x^{1/2}} \cdot \frac{1}{2} x^{-1/2} = \boxed{\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}}$

39) $y = e^x \ln x$

$y' = e^x \ln x + e^x \left(\frac{1}{x}\right)$

$$\boxed{\begin{aligned} y' &= e^x\left(\ln x + \frac{1}{x}\right) \\ &= \frac{e^x (x \ln x + 1)}{x} \end{aligned}}$$

*chain rule

43) $g(t) = \left(e^{-t} + e^{t}\right)^3$

$g'(t) = 3\left[e^{-t} + e^{t}\right]^2 \cdot \left(e^{-t}(-1) + e^{t}(1)\right)$

$\boxed{g'(t) = 3\left(e^{-t} + e^{t}\right)^2 \left(e^{t} - e^{-t}\right)}$

Find derivative

Apply $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ and $\frac{d}{dx} e^u = e^u \cdot u'$

45) $y = \ln(1+e^{2x})$

$$y' = \frac{e^{2x}(2)}{1+e^{2x}} = \boxed{\frac{2e^{2x}}{1+e^{2x}}}$$

*Apply chain rule

47) $y = \frac{2}{e^x+e^{-x}} = 2(e^x+e^{-x})^{-1}$

$$y' = 2\cdot -1(e^x+e^{-x})^{-2}(e^x+e^{-x}(-1)) = \boxed{\frac{-2(e^x-e^{-x})}{(e^x+e^{-x})^2}}$$

49) $y = \frac{e^x+1}{e^x-1}$ ← * quotient rule $\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{f'g-fg'}{g^2}$

$$y' = \frac{(e^x)(e^x-1)-(e^x+1)(e^x)}{(e^x-1)^2} = \frac{e^x[\cancel{e^x}-1-\cancel{e^x}-1]}{(e^x-1)^2} = \boxed{\frac{-2e^x}{(e^x-1)^2}}$$

Find equation of tangent line:

* Apply product rule

59) $f(x) = e^{-x}\ln x$ at $(1,0)$

$$f'(x) = e^{-x}(-1)\cdot \ln x + e^{-x}\cdot\left(\frac{1}{x}\right)$$

$$f'(x) = -\frac{\ln x}{e^x} + \frac{1}{xe^x}$$

$$f'(1) = -\frac{\ln(1)}{e^1} + \frac{1}{1(e^1)} = 0 + \frac{1}{e} = \frac{1}{e}$$

$$f'(1) = \frac{1}{e}$$

slope: $m = \frac{1}{e}$ point: $(1,0)$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{1}{e}(x-1)$$

$$\boxed{y = \frac{1}{e}(x-1)}$$

63) Find $\frac{dy}{dx}$ implicitly: $xe^y - 10x + 3y = 0$

* Apply product rule
* Implicit differentiation

$$\overbrace{1}^{f'}\overbrace{e^y}^{g} + \overbrace{x}^{f}\cdot \overbrace{e^y\left(\frac{dy}{dx}\right)}^{g'} - 10 + 3\left(\frac{dy}{dx}\right) = 0$$

$$xe^y\left(\frac{dy}{dx}\right) + 3\left(\frac{dy}{dx}\right) = 10 - e^y$$

$$\frac{dy}{dx}\left[xe^y + 3\right] = 10 - e^y$$

$$\boxed{\frac{dy}{dx} = \frac{10 - e^y}{xe^y + 3}}$$

65) Find equation of tangent line $xe^y + ye^x = 1 \quad (0,1)$

* Apply product rule
* Implicit differentiation

$$\overbrace{1}^{f'}\overbrace{e^y}^{g} + \overbrace{x}^{f}\overbrace{e^y\left(\frac{dy}{dx}\right)}^{g'} + \overbrace{1\left(\frac{dy}{dx}\right)}^{f'}\overbrace{e^x}^{g} + \overbrace{y}^{f}\cdot \overbrace{e^x}^{g'} = 0$$

$$xe^y\left(\frac{dy}{dx}\right) + e^x\left(\frac{dy}{dx}\right) = -ye^x - e^y$$

$$\frac{dy}{dx}\left[xe^y + e^x\right] = -ye^x - e^y$$

$$\frac{dy}{dx} = \frac{-ye^x - e^y}{xe^y + e^x}$$

$$\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{-1e^0 - e^1}{0e^1 + e^0} = \frac{-1-e}{1}$$

slope: $m = -e - 1$ point: $(0,1)$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 1 = (-e-1)(x - 0)}$$

71) $f(x) = \frac{e^x + e^{-x}}{2} = \frac{1}{2}(e^x + e^{-x})$ Find Rel extrema, POI

$$f'(x) = \frac{1}{2}(e^x + e^{-x}(-1)) = \frac{1}{2}(e^x - e^{-x}) = 0$$

$$e^x - \frac{1}{e^x} = 0 \quad e^x = \frac{1}{e^x} \quad \underline{\underline{x = 0}}$$

$$\underset{0}{\overset{\searrow \quad \nearrow}{- \mid +}}$$

$\boxed{\text{Rel. min at } (0,1)}$

$$f''(x) = \frac{1}{2}(e^x - e^{-x}(-1)) = \frac{1}{2}(e^x + e^{-x}) > 0$$

No critical pt, always concave up, no POI

75) Find Relative Extrema, POI.

$$f(x) = x^2e^{-x}$$

$$f'(x) = 2xe^{-x} + x^2 \cdot e^{-x}(-1)$$

$$0 = e^{-x}(2x - x^2)$$

$$0 = \frac{x(2-x)}{e^x}$$

$$\underline{x = 0, 2}$$

$$\underset{0 \qquad 2}{\overset{\searrow \quad \nearrow \quad \searrow}{- \mid + \mid -}}$$

Rel. min at $(0,0)$

Rel. max at $\left(2, \frac{4}{e^2}\right)$

$$f'(x) = e^{-x}(2x - x^2)$$

$$f''(x) = e^{-x}(-1)(2x - x^2) + e^{-x}(2 - 2x)$$

$$0 = e^{-x}\left[-2x + x^2 + 2 - 2x\right]$$

$$0 = e^{-x}\left[x^2 - 4x + 2\right]$$

$$x = \frac{4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

$$\underset{2-\sqrt{2} \qquad 2+\sqrt{2}}{\overset{\cup \qquad \cap \qquad \cup}{+ \mid - \mid +}}$$

$\boxed{\text{POI at } x = 2-\sqrt{2}, 2+\sqrt{2}}$