Ex. 9 Find relative extrema and points of inflection

$$g(t) = 1 + (2+t)e^{-t}$$

$$g'(t) = 0 + \underbrace{(1)}_{f'}\underbrace{(e^{-t})}_{g} + \underbrace{(2+t)}_{f}\underbrace{(e^{-t})(-1)}_{g'}$$

$$g'(t) = e^{-t} - 2e^{-t} - te^{-t}$$

$$g'(t) = \frac{1}{e^t} - \frac{2}{e^t} - \frac{t}{e^t}$$

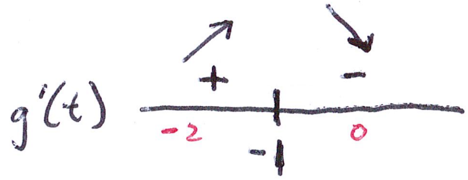
$$\boxed{g'(t) = \frac{-1-t}{e^t}}$$

*Find critical values:

$-1-t=0$ | $e^t = 0$

$t = -1$ | $\ln e^t = \ln 0$

no solution



Relative max at $(-1, 1+e)$ b/c $g'(t)$ changes from + to −

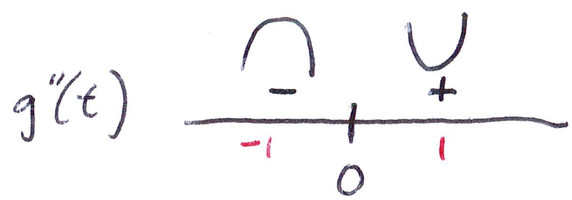
$g(-1) = 1 + (2-1)e^1 = 1 + 1e$

$$g''(t) = \frac{\underbrace{(-1)}_{f'}\underbrace{(e^t)}_{g} - \underbrace{(-1-t)}_{f}\underbrace{(e^t)}_{g'}}{\underbrace{(e^t)^2}_{g^2}} = \frac{-e^t + e^t + te^t}{(e^t)^2} \rightarrow g''(t) = \frac{te^t}{(e^t)^2} = \boxed{\frac{t}{e^t}}$$

*critical points:

$t = 0$ | $e^t = 0$

$t = 0$ | no solution



POI at $(0, 3)$ b/c $g''(t)$ change signs

$g(0) = 1 + (2+0)e^0 = 1 + 2 = 3$

Ex. 10 Find relative extrema and points of inflection for

$$f(x) = \frac{e^{x}-e^{-x}}{2} \qquad\Bigg|\qquad f(x) = \frac{e^{2x}-1}{2e^{x}}$$

$$f(x) = \frac{e^{x}-\frac{1}{e^{x}}}{2}\cdot(e^{x})$$

$$f'(x) = \frac{\overbrace{(e^{2x})(2)}^{f'}\cdot\overbrace{2e^{x}}^{g}-\overbrace{(e^{2x}-1)}^{f}\overbrace{(2e^{x})(1)}^{g'}}{\underbrace{(2e^{x})^{2}}_{g^{2}}} = \frac{4e^{3x}-2e^{3x}+2e^{x}}{4e^{2x}} = \frac{2e^{3x}+2e^{x}}{4e^{2x}}$$

$$f'(x) = \frac{2e^{x}(e^{2x}+1)}{4e^{2x}} \qquad \boxed{f'(x) = \frac{e^{2x}+1}{2e^{x}}}$$

*critical pts.

$e^{2x}+1=0$	$2e^{x}=0$
$e^{2x}=-1$	$e^{x}=0$
$\ln e^{2x} = \ln(-1)$	$\ln e^{x} \neq \ln 0$
no solution	no solution

$f'(x)$ $\longleftrightarrow$ $\nearrow$

$\boxed{f(x) \text{ is increasing } (-\infty, \infty), \text{ no relative extrema}}$

$$f''(x) = \frac{\overbrace{e^{2x}(2)}^{f'}\cdot\overbrace{2e^{x}}^{g}-\overbrace{(e^{2x}+1)}^{f}\cdot\overbrace{2e^{x}(1)}^{g'}}{\underbrace{(2e^{x})^{2}}_{g^{2}}} = \frac{4e^{3x}-2e^{3x}-2e^{x}}{4e^{2x}} = \frac{2e^{3x}-2e^{x}}{4e^{2x}}$$

$$f''(x) = \frac{2e^{x}(e^{2x}-1)}{4e^{2x}}$$

$$\boxed{f''(x) = \frac{e^{2x}-1}{2e^{x}}}$$

*critical pts:

$e^{2x}-1=0$	$2e^{x}=0$
$e^{2x}=1$	$e^{x}=0$
$\ln e^{2x} = \ln 1$	$\ln e^{x} \neq \ln 0$
$2x\cancel{\ln e}=0$	no solution
$2x=0$	
$x=0$	

$f''(x)$: $\cap$ $-$ | $\cup$ $+$; -1, 0, 1

POI at $(0, 0)$
b/c $f''(x)$ change signs

$$f(0) = \frac{e^{0}-1}{2e^{0}} = \frac{0}{2} = 0$$