103. $f(x) = \dfrac{ax + b}{cx + d}$

(a) Assume $bc - ad \neq 0$ and $f(x_1) = f(x_2)$. Then

$$\frac{ax_1 + b}{cx_1 + d} = \frac{ax_2 + b}{cx_2 + d}$$

$$acx_1x_2 + bcx_2 + adx_1 + bd = acx_1x_2 + adx_2 + bcx_1 + bd$$

$$(ad - bc)x_1 = (ad - bc)x_2$$

$$x_1 = x_2 \qquad (\text{because } ad - bc \neq 0)$$

So, f is one-to-one.

Now assume f is one-to-one. Suppose, on the contrary, that $ad = bc$. If $d = 0$, then either $b = 0$ or $c = 0$. In both cases, f is not one-to-one. Similarly, if $b = 0$, then $a = 0$ or $d = 0$ and f is not one-to-one. So consider

$$f(x) = \frac{ax + b}{cx + d} = \frac{adx + bd}{bcx + bd} \cdot \frac{b}{d} = \frac{bcx + bd}{bcx + bd} \cdot \frac{b}{d} = \frac{b}{d},$$

which is not one-to-one.

Alternate Solution:

$$f(x) = \frac{ax + b}{cx + d} \Rightarrow f'(x) = \frac{ad - bc}{(cx + d)^2}$$

f is monotonic (and therefore one-to-one) if and only if $ad - bc \neq 0$.

(b)

$$y = \frac{ax + b}{cx + d}$$

$$cyx + dy = ax + b$$

$$(cy - a)x = b - dy$$

$$x = \frac{b - dy}{cy - a}$$

$$f^{-1}(x) = y = \frac{b - dx}{cx - a}, \quad bc - ad \neq 0$$

(c)

$$\frac{ax + b}{cx + d} = \frac{b - dx}{cx - a}$$

$$acx^2 + bcx - a^2x - ab = bcx - cdx^2 + bd - d^2x$$

$$(ac + cd)x^2 + (d^2 - a^2)x - bd - ab = 0$$

$$c(a + d)x^2 + (d - a)(d + a)x - b(a + d) = 0$$

So, $f = f^{-1}$ if $a = -d$, or if $c = b = 0$ and $a = d$.

Section 5.4 Exponential Functions: Differentiation and Integration

1. $e^{\ln x} = 4$

$x = 4$

2. $e^{\ln 3x} = 24$

$3x = 24$

$x = 8$

3. $e^x = 12$

$x = \ln 12 \approx 2.485$

4. $5e^x = 36$

$e^x = \frac{36}{5}$

$x = \ln\left(\frac{36}{5}\right) \approx 1.974$

5. $9 - 2e^x = 7$

$2e^x = 2$

$e^x = 1$

$x = 0$

6. $8e^x - 12 = 7$

$8e^x = 19$

$e^x = \frac{19}{8}$

$x = \ln\left(\frac{19}{8}\right)$

≈ 0.865

7. $50e^{-x} = 30$

$e^{-x} = \frac{3}{5}$

$-x = \ln\left(\frac{3}{5}\right)$

$x = \ln\left(\frac{5}{3}\right)$

≈ 0.511

8. $100e^{-2x} = 35$

$e^{-2x} = \frac{35}{100} = \frac{7}{20}$

$-2x = \ln\left(\frac{7}{20}\right)$

$x = -\frac{1}{2}\ln\left(\frac{7}{20}\right) = \frac{1}{2}\ln\left(\frac{20}{7}\right)$

≈ 0.525

9. $\frac{800}{100 - e^{x/2}} = 50$

$\frac{800}{50} = 100 - e^{x/2}$

$84 = e^{x/2}$

$\ln 84 = \frac{x}{2}$

$x = 2\ln 84 \approx 8.862$

10. $\frac{5000}{1 + e^{2x}} = 2$

$\frac{5000}{2} = 1 + e^{2x}$

$2499 = e^{2x}$

$\ln 2499 = 2x$

$x = \frac{1}{2}\ln 2499 \approx 3.912$

11. $\ln x = 2$

$x = e^2 \approx 7.389$

12. $\ln x^2 = 10$

$x^2 = e^{10}$

$x = \pm e^5 \approx \pm 148.413$

13. $\ln(x - 3) = 2$

$x - 3 = e^2$

$x = 3 + e^2 \approx 10.389$

14. $\ln 4x = 1$

$4x = e^1 = e$

$x = \frac{e}{4} \approx 0.680$

15. $\ln\sqrt{x + 2} = 1$

$\sqrt{x + 2} = e^1 = e$

$x + 2 = e^2$

$x = e^2 - 2 \approx 5.389$

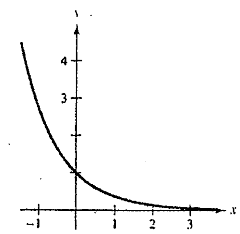
16. $\ln(x - 2)^2 = 12$

$(x - 2)^2 = e^{12}$

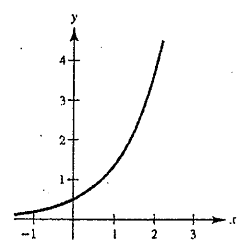
$x - 2 = e^6$

$x = 2 + e^6 \approx 405.429$

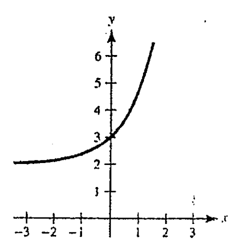
17. $y = e^{-x}$



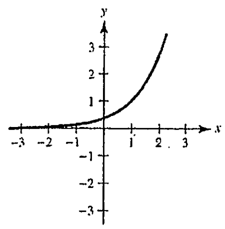
18. $y = \frac{1}{2}e^x$



19. $y = e^x + 2$



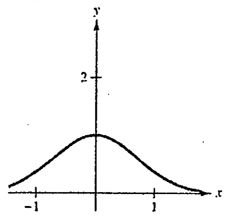
20. $y = e^{x-1}$



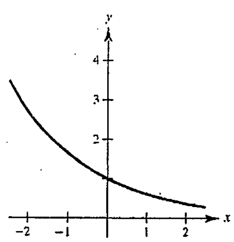
21. $y = e^{-x^2}$

Symmetric with respect to the y-axis

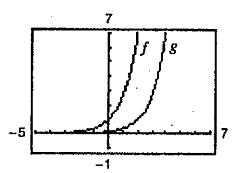
Horizontal asymptote: $y = 0$



22. $y = e^{-x/2}$

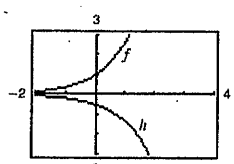


23. (a)



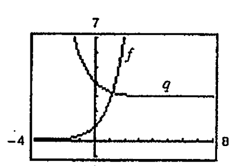
Horizontal shift 2 units to the right

(b)



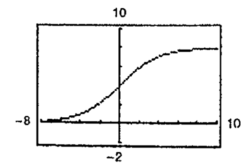
A reflection in the x-axis and a vertical shrink

(c)



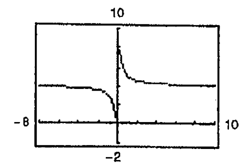
Vertical shift 3 units upward and a reflection in the y-axis

24. (a)



Horizontal asymptotes: $y = 0$ and $y = 8$

(b)



Horizontal asymptote: $y = 4$

25. $y = Ce^{ax}$

Horizontal asymptote: $y = 0$

Matches (c)

26. $y = Ce^{-ax}$

Horizontal asymptote: $y = 0$

Reflection in the y-axis

Matches (d)

27. $y = C(1 - e^{-ax})$

Vertical shift C units

Reflection in both the x- and y-axes

Matches (a)

28. $y = \dfrac{C}{1 + e^{-ax}}$

$$\lim_{x\to\infty} \frac{C}{1 + e^{-ax}} = C$$

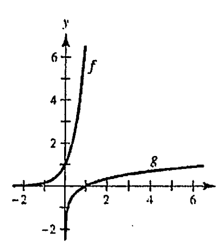
$$\lim_{x\to-\infty} \frac{C}{1 + e^{-ax}} = 0$$

Horizontal asymptotes: $y = C$ and $y = 0$

Matches (b)

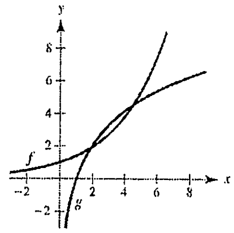
29. $f(x) = e^{2x}$

$g(x) = \ln\sqrt{x} = \dfrac{1}{2}\ln x$



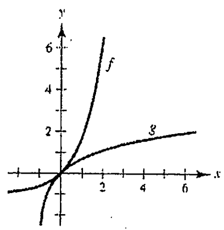
30. $f(x) = e^{x/3}$

$g(x) = \ln x^3 = 3 \ln x$



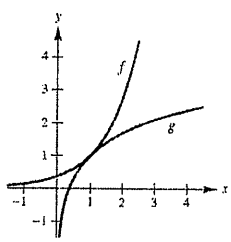
31. $f(x) = e^x - 1$

$g(x) = \ln(x + 1)$



32. $f(x) = e^{x-1}$

$g(x) = 1 + \ln x$



33. $f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

34. $y = e^{-8x}$

$y' = -8e^{-8x}$

35. $y = e^{\sqrt{x}}$

$\dfrac{dy}{dx} = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

36. $y = e^{-2x^3}$

$y' = -6x^2e^{-2x^3}$

37. $y = e^{x-4}$

$y' = e^{x-4}$

38. $y = 5e^{x^2+5}$

$y' = 5e^{x^2+5}(2x) = 10xe^{x^2+5}$

39. $y = e^x \ln x$

$y' = e^x\left(\dfrac{1}{x}\right) + e^x \ln x = e^x\left(\dfrac{1}{x} + \ln x\right)$

40. $y = xe^{4x}$

$y' = 4xe^{4x} + e^{4x} = e^{4x}(4x + 1)$

41. $y = x^3e^x$

$y' = x^3e^x + 3x^2(e^x) = x^2e^x(x + 3) = e^x(x^3 + 3x^2)$

42. $y = x^2e^{-x}$

$y' = x^2(-e^{-x}) + 2xe^{-x} = xe^{-x}(2 - x)$

43. $g(t) = (e^{-t} + e^t)^3$

$g'(t) = 3(e^{-t} + e^t)^2(e^t - e^{-t})$

44. $g(t) = e^{-3/t^2}$

$g'(t) = e^{-3/t^2}(6t^{-3}) = \dfrac{6}{t^3e^{3/t^2}}$

45. $y = \ln(1 + e^{2x})$

$\dfrac{dy}{dx} = \dfrac{2e^{2x}}{1 + e^{2x}}$

46. $y = \ln\left(\dfrac{1 + e^x}{1 - e^x}\right) = \ln(1 + e^x) - \ln(1 - e^x)$

$\dfrac{dy}{dx} = \dfrac{e^x}{1 + e^x} + \dfrac{e^x}{1 - e^x} = \dfrac{2e^x}{1 - e^{2x}}$

47. $y = \dfrac{2}{e^x + e^{-x}} = 2(e^x + e^{-x})^{-1}$

$\dfrac{dy}{dx} = -2(e^x + e^{-x})^{-2}(e^x - e^{-x}) = \dfrac{-2(e^x - e^{-x})}{(e^x + e^{-x})^2}$

48. $y = \dfrac{e^x - e^{-x}}{2}$

$\dfrac{dy}{dx} = \dfrac{e^x + e^{-x}}{2}$

49. $y = \dfrac{e^x + 1}{e^x - 1}$

$y' = \dfrac{(e^x - 1)e^x - (e^x + 1)e^x}{(e^x - 1)^2} = \dfrac{-2e^x}{(e^x - 1)^2}$

50. $y = \dfrac{e^{2x}}{e^{2x} + 1}$

$y' = \dfrac{(e^{2x} + 1)2e^{2x} - e^{2x}(2e^{2x})}{(e^{2x} + 1)^2} = \dfrac{2e^{2x}}{(e^{2x} + 1)^2}$

51. $y = e^x(\sin x + \cos x)$

$\dfrac{dy}{dx} = e^x(\cos x - \sin x) + (\sin x + \cos x)(e^x)$

$= e^x(2\cos x) = 2e^x \cos x$

52. $y = e^{2x} \tan 2x$

$y' = e^{2x}[2\sec^2 2x] + 2e^{2x}\tan 2x$

$= 2e^{2x}[\sec^2 2x + \tan 2x]$

53. $F(x) = \int_\pi^{\ln x} \cos e^t \, dt$

$F'(x) = \cos(e^{\ln x}) \cdot \dfrac{1}{x} = \dfrac{\cos(x)}{x}$

54. $F(x) = \int_0^{e^{2x}} \ln(t + 1)\, dt$

$F'(x) = \ln(e^{2x} + 1)2e^{2x} = 2e^{2x}\ln(e^{2x} + 1)$

55. $f(x) = e^{3x}$, $(0, 1)$

$f'(x) = 3e^{3x}$, $f'(0) = 3$

Tangent line: $y - 1 = 3(x - 0)$

$y = 3x + 1$

56. $f(x) = e^{-2x}$, $(0, 1)$

$f'(x) = -2e^{-2x}$, $f'(0) = -2$

Tangent line: $y - 1 = -2(x - 0)$

$y = -2x + 1$

57. $f(x) = e^{1-x}$, $(1, 1)$

$f'(x) = -e^{1-x}$, $f'(1) = -1$

Tangent line: $y - 1 = -1(x - 1)$

$y = -x + 2$

58. $y = e^{-2x+x^2}$, $(2, 1)$

$y' = (2x - 2)e^{-2x+x^2}$, $y'(2) = 2$

Tangent line: $y - 1 = 2(x - 2)$

$y = 2x - 3$

59. $f(x) = e^{-x}\ln x$, $(1, 0)$

$f'(x) = e^{-x}\left(\dfrac{1}{x}\right) - e^{-x}\ln x = e^{-x}\left(\dfrac{1}{x} - \ln x\right)$

$f'(1) = e^{-1}$

Tangent line: $y - 0 = e^{-1}(x - 1)$

$y = \dfrac{1}{e}x - \dfrac{1}{e}$

60. $y = \ln\dfrac{e^x + e^{-x}}{2}$, $(0, 0)$

$y' = \dfrac{1}{[(e^x + e^{-x})/2]}\left[\dfrac{e^x - e^{-x}}{2}\right]$

$y'(0) = 0$

Tangent line: $y = 0$

61. $y = x^2e^x - 2xe^x + 2e^x$, $(1, e)$

$y' = x^2e^x + 2xe^x - 2xe^x - 2e^x + 2e^x = x^2e^x$

$y'(1) = e$

Tangent line: $y - e = e(x - 1)$

$y = ex$

62. $y = xe^x - e^x$, $(1, 0)$

$y' = xe^x + e^x - e^x = xe^x$

$y'(1) = e$

Tangent line: $y - 0 = e(x - 1)$

$y = ex - e$

63. $xe^y - 10x + 3y = 0$

$xe^y\dfrac{dy}{dx} + e^y - 10 + 3\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}(xe^y + 3) = 10 - e^y$

$\dfrac{dy}{dx} = \dfrac{10 - e^y}{xe^y + 3}$

64. $e^{xy} + x^2 - y^2 = 10$

$\left(x\dfrac{dy}{dx} + y\right)e^{xy} + 2x - 2y\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}(xe^{xy} - 2y) = -ye^{xy} - 2x$

$\dfrac{dy}{dx} = -\dfrac{ye^{xy} + 2x}{xe^{xy} - 2y}$

65. $xe^y + ye^x = 1, \quad (0, 1)$

$xe^y y' + e^y + ye^x + y'e^x = 0$

At (0, 1): $e + 1 + y' = 0$

$y' = -e - 1$

Tangent line: $y - 1 = (-e - 1)(x - 0)$

$y = (-e - 1)x + 1$

66. $1 + \ln(xy) = e^{x-y}, \quad (1, 1)$

$\frac{1}{xy}[xy' + y] = e^{x-y}[1 - y']$

At (1, 1): $[y' + 1] = 1 - y'$

$y' = 0$

Tangent line: $y - 1 = 0(x - 1)$

$y = 1$

67. $f(x) = (3 + 2x)e^{-3x}$

$f'(x) = (3 + 2x)(-3e^{-3x}) + 2e^{-3x} = (-7 - 6x)e^{-3x}$

$f''(x) = (-7 - 6x)(-3e^{-3x}) - 6e^{-3x} = 3(6x + 5)e^{-3x}$

68. $g(x) = \sqrt{x} + e^x \ln x$

$g'(x) = \frac{1}{2\sqrt{x}} + \frac{e^x}{x} + e^x \ln x$

$g''(x) = -\frac{1}{4x^{3/2}} + \frac{xe^x - e^x}{x^2} + \frac{e^x}{x} + e^x \ln x$

$= -\frac{1}{4x\sqrt{x}} + \frac{e^x(2x - 1)}{x^2} + e^x \ln x$

69. $y = 4e^{-x}$

$y' = -4e^{-x}$

$y'' = 4e^{-x}$

$y'' - y = 4e^{-x} - 4e^{-x} = 0$

70. $y = e^{3x} + e^{-3x}$

$y' = 3e^{3x} - 3e^{-3x}$

$y'' = 9e^{3x} + 9e^{-3x}$

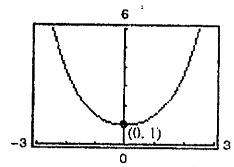
$y'' - 9y = (9e^{3x} + 9e^{-3x}) - 9(e^{3x} + e^{-3x}) = 0$

71. $f(x) = \frac{e^x + e^{-x}}{2}$

$f'(x) = \frac{e^x - e^{-x}}{2} = 0$ when $x = 0$.

$f''(x) = \frac{e^x + e^{-x}}{2} > 0$

Relative minimum: (0, 1)

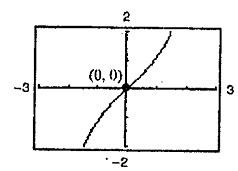


72. $f(x) = \frac{e^x - e^{-x}}{2}$

$f'(x) = \frac{e^x + e^{-x}}{2} > 0$

$f''(x) = \frac{e^x - e^{-x}}{2} = 0$ when $x = 0$.

Point of inflection: (0, 0)



73. $g(x) = \frac{1}{\sqrt{2\pi}}e^{-(x-2)^2/2}$

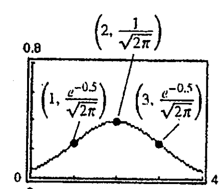
$g'(x) = \frac{-1}{\sqrt{2\pi}}(x - 2)e^{-(x-2)^2/2}$

$g''(x) = \frac{1}{\sqrt{2\pi}}(x - 1)(x - 3)e^{-(x-2)^2/2}$

Relative maximum: $\left(2, \frac{1}{\sqrt{2\pi}}\right) \approx (2, 0.399)$

Points of inflection:

$\left(1, \frac{1}{\sqrt{2\pi}}e^{-1/2}\right), \left(3, \frac{1}{\sqrt{2\pi}}e^{-1/2}\right) \approx (1, 0.242), (3, 0.242)$



74. $g(x) = \frac{1}{\sqrt{2\pi}}e^{-(x-3)^2/2}$

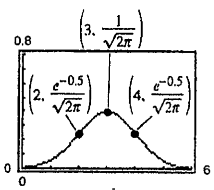
$g'(x) = \frac{-1}{\sqrt{2\pi}}(x-3)e^{-(x-3)^2/2}$

$g''(x) = \frac{1}{\sqrt{2\pi}}(x-2)(x-4)e^{-(x-3)^2/2}$

Relative maximum: $\left(3, \frac{1}{\sqrt{2\pi}}\right) \approx (3, 0.399)$

Points of inflection:

$\left(2, \frac{1}{\sqrt{2\pi}}e^{-1/2}\right), \left(4, \frac{1}{\sqrt{2\pi}}e^{-1/2}\right) \approx (2, 0.242), (4, 0.242)$



75. $f(x) = x^2e^{-x}$

$f'(x) = -x^2e^{-x} + 2xe^{-x}$

$= xe^{-x}(2-x) = 0$ when $x = 0, 2$.

$f''(x) = -e^{-x}(2x - x^2) + e^{-x}(2-2x)$

$= e^{-x}(x^2 - 4x + 2) = 0$ when $x = 2 \pm \sqrt{2}$.

Relative minimum: $(0, 0)$

Relative maximum: $(2, 4e^{-2})$

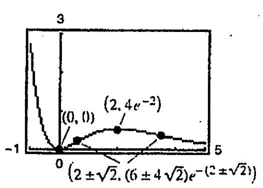
$x = 2 \pm \sqrt{2}$

$y = (2 \pm \sqrt{2})^2 e^{-(2\pm\sqrt{2})} = (6 \pm 4\sqrt{2})e^{-(2\pm\sqrt{2})}$

Points of inflection:

$\left(2 \pm \sqrt{2}, (6 \pm 4\sqrt{2})e^{-(2\pm\sqrt{2})}\right)$

$\approx (3.414, 0.384), (0.586, 0.191)$



76. $f(x) = xe^{-x}$

$f'(x) = -xe^{-x} + e^{-x}$

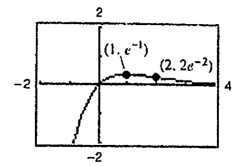
$= e^{-x}(1-x) = 0$ when $x = 1$.

$f''(x) = -e^{-x} + (-e^{-x})(1-x)$

$= e^{-x}(x-2) = 0$ when $x = 2$.

Relative maximum: $(1, e^{-1})$

Point of inflection: $(2, 2e^{-2})$



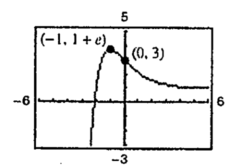
77. $g(t) = 1 + (2 + t)e^{-t}$

$g'(t) = -(1 + t)e^{-t}$

$g''(t) = te^{-t}$

Relative maximum: $(-1, 1 + e) \approx (-1, 3.718)$

Point of inflection: $(0, 3)$



78. $f(x) = -2 + e^{3x}(4 - 2x)$

$f'(x) = e^{3x}(-2) + 3e^{3x}(4 - 2x)$

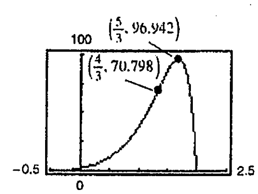
$= e^{3x}(10 - 6x) = 0$ when $x = \frac{5}{3}$.

$f''(x) = e^{3x}(-6) + 3e^{3x}(10 - 6x)$

$= e^{3x}(24 - 18x) = 0$ when $x = \frac{4}{3}$.

Relative maximum: $\left(\frac{5}{3}, 96.942\right)$

Point of inflection: $\left(\frac{4}{3}, 70.798\right)$

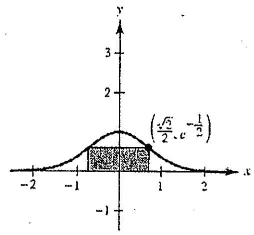


79. $A = (\text{base})(\text{height}) = 2xe^{-x^2}$

$$\frac{dA}{dx} = -4x^2e^{-x^2} + 2e^{-x^2}$$

$$= 2e^{-x^2}(1 - 2x^2) = 0 \text{ when } x = \frac{\sqrt{2}}{2}.$$

$$A = \sqrt{2}e^{-1/2}$$



80. (a) $f(c) = f(c + x)$

$$10ce^{-c} = 10(c + x)e^{-(c+x)}$$

$$\frac{c}{e^c} = \frac{c + x}{e^{c+x}}$$

$$ce^{c+x} = (c + x)e^c$$

$$ce^x = c + x$$

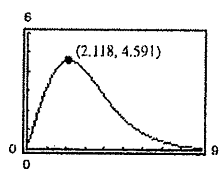
$$ce^x - c = x$$

$$c = \frac{x}{e^x - 1}$$

(b) $A(x) = xf(c) = x\left[10\left(\frac{x}{e^x - 1}\right)e^{-\left(x/\left(e^x - 1\right)\right)}\right]$

$$= \frac{10x^2}{e^x - 1}e^{x/\left(1 - e^x\right)}$$

(c) $A(x) = \frac{10x^2}{e^x - 1}e^{x/\left(1 - e^x\right)}$

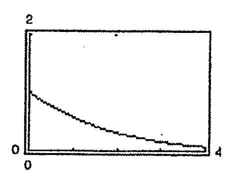


The maximum area is 4.591 for $x = 2.118$ and $f(x) = 2.547$.

(d) $c = \frac{x}{e^x - 1}$

$\lim_{x \to 0^+} c = 1$

$\lim_{x \to \infty} c = 0$



Answers will vary. *Sample answer*:

As x approaches 0 from the right, the height of the rectangle approaches 1.

As x approaches ∞, the height of the rectangle approaches 0.

81. $f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

Let $(x, y) = (x, e^{2x})$ be the point on the graph where the tangent line passes through the origin. Equating slopes,

$$2e^{2x} = \frac{e^{2x} - 0}{x - 0}$$

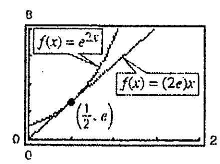
$$2 = \frac{1}{x}$$

$$x = \frac{1}{2}, \quad y = e, \quad y' = 2e.$$

Point: $\left(\frac{1}{2}, e\right)$

Tangent line: $y - e = 2e\left(x - \frac{1}{2}\right)$

$$y = 2ex$$

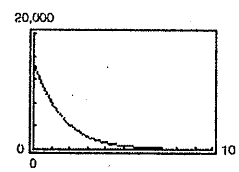


82. (a) f is increasing on $(-\infty, \infty)$. g is decreasing on $(-\infty, \infty)$.

(b) f and g are both concave upward on $(-\infty, \infty)$.

83. $V = 15{,}000e^{-0.6286t}, \quad 0 \le t \le 10$

(a)

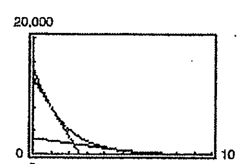


(b) $\frac{dV}{dt} = -9429e^{-0.6286t}$

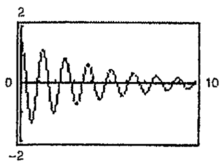
When $t = 1$, $\frac{dV}{dt} \approx -5028.84$.

When $t = 5$, $\frac{dV}{dt} \approx -406.89$.

(c)



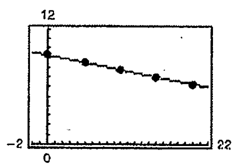
84. $1.56e^{-0.22t}\cos 4.9t \le 0.25$ (3 inches equals one-fourth foot.) Using a graphing utility or Newton' Method, you have $t \ge 7.79$ seconds.



85.

h	0	5	10	15	20
P	10,332	5583	2376	1240	517
$\ln P$	9.243	8.627	7.773	7.123	6.248

(a)



$y = -0.1499h + 9.3018$ is the regression line for data $(h, \ln P)$.

(b) $\ln P = ah + b$

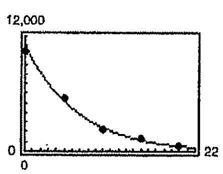
$$P = e^{ah+b} = e^b e^{ah}$$

$$P = Ce^{ah}, C = e^b$$

$a = -0.1499$ and $C = e^{9.3018} = 10{,}957.7.$

So, $P = 10{,}957.7e^{-0.1499h}$

(c)



(d) $\dfrac{dP}{dh} = (10{,}957.71)(-0.1499)e^{-0.1499h}$

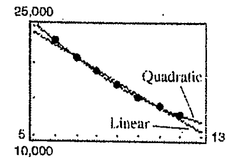
$$= -1642.56e^{-0.1499h}$$

For $h = 5, \dfrac{dP}{dh} = -776.3.$

For $h = 18, \dfrac{dP}{dh} \approx -110.6.$

86. (a) Linear model: $V = -1686.8t + 32{,}561$

Quadratic model: $V = 109.52t^2 - 3658.2t + 40{,}995$



(b) The slope represents the average loss in value per year.

(c) Exponential model:

$$V = 40{,}955.46(0.90724)^t = 40{,}955.46\,e^{-0.09735t}$$

(d) As $t \to \infty, V \to 0$ for the exponential model. The value of the car tends to zero.

(e) $V' = (40{,}955.46)(-0.09735)e^{-0.09735t}$

$$= -3987.01e^{-0.09735t}$$

When $t = 7, V' \approx -2017$ dollars/year.

When $t = 11, V' \approx -1366$ dollars/year.

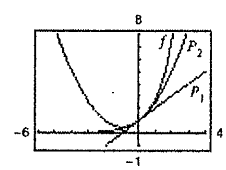
87. $f(x) = e^x \qquad f(0) = 1$

$f'(x) = e^x \qquad f'(0) = 1$

$f''(x) = e^x \qquad f''(0) = 1$

$P_1(x) = 1 + 1(x - 0) = 1 + x$

$P_2(x) = 1 + 1(x - 0) = \frac{1}{2}(1)(x - 0)^2 = 1 + x + \frac{x^2}{2}$



The values of f, P_1, and P_2 and their first derivatives agree at $x = 0$.

88. $f(x) = e^{x/2}$, $f(0) = 1$

$f'(x) = \frac{1}{2}e^{x/2}$, $f'(0) = \frac{1}{2}$

$f''(x) = \frac{1}{4}e^{x/2}$, $f''(0) = \frac{1}{4}$

$P_1(x) = 1 + \frac{1}{2}(x - 0) = \frac{x}{2} + 1$, $P_1(0) = 1$

$P_1'(x) = \frac{1}{2}$, $P_1'(0) = \frac{1}{2}$

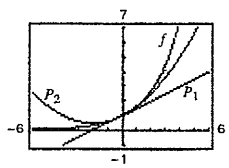
$P_2(x) = 1 + \frac{1}{2}(x - 0) + \frac{1}{8}(x - 0)^2$ $P_2(0) = 1$

$= \frac{x^2}{8} + \frac{x}{2} + 1$

$P_2'(x) = \frac{1}{4}x + \frac{1}{2}$, $P_2'(0) = \frac{1}{2}$

$P_2''(x) = \frac{1}{4}$, $P_2''(0) = \frac{1}{4}$

The values of f, P_1, P_2 and their first derivatives agree at $x = 0$. The values of the second derivatives of f and P_2 agree at $x = 0$.



89. $n = 12$

$12! = 12 \cdot 11 \cdot 10 \cdots 3 \cdot 2 \cdot 1 = 479{,}001{,}600$

Stirlings Formula:

$12! \approx \left(\frac{12}{e}\right)^{12}\sqrt{2\pi(12)} \approx 475{,}687{,}487$

90. $n = 15$

$15! = 15 \cdot 14 \cdots 3 \cdot 2 \cdot 1 = 1{,}307{,}674{,}368{,}000$

Stirlings Formula:

$15! \approx \left(\frac{15}{e}\right)^{15}\sqrt{2\pi(15)} \approx 1{,}300{,}430{,}722{,}200$

$\approx 1.3004 \times 10^{12}$

91. Let $u = 5x$, $du = 5\,dx$.

$\int e^{5x}(5)\,dx = e^{5x} + C$

92. Let $u = -x^4$, $du = -4x^3dx$.

$\int e^{-x^4}(-4x^3)\,dx = e^{-x^4} + C$

93. Let $u = 2x - 1$, $du = 2\,dx$.

$\int e^{2x-1}\,dx = \frac{1}{2}\int e^{2x-1}(2)\,dx = \frac{1}{2}e^{2x-1} + C$

94. Let $u = 1 - 3x$, $du = -3\,dx$.

$\int e^{1-3x}\,dx = -\frac{1}{3}\int e^{1-3x}(-3)\,dx = -\frac{1}{3}e^{1-3x} + C$

95. Let $u = x^3$, $du = 3x^2\,dx$.

$\int x^2e^{x^3}\,dx = \frac{1}{3}\int e^{x^3}(3x^2)\,dx = \frac{1}{3}e^{x^3} + C$

96. Let $u = e^x + 1$, $du = e^x\,dx$.

$\int e^x(e^x + 1)^2\,dx = \int (e^x + 1)^2(e^x)\,dx = \frac{(e^x + 1)^3}{3} + C$

97. Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\,dx$.

$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right)dx = 2e^{\sqrt{x}} + C$

98. Let $u = \frac{1}{x^2}$, $du = \frac{-2}{x^3}\,dx$.

$\int \frac{e^{1/x^2}}{x^3}\,dx = -\frac{1}{2}\int e^{1/x^2}\left(\frac{-2}{x^3}\right)dx = -\frac{1}{2}e^{1/x^2} + C$

99. Let $u = 1 + e^{-x}$, $du = -e^{-x}\,dx$.

$\int \frac{e^{-x}}{1 + e^{-x}}\,dx = -\int \frac{-e^{-x}}{1 + e^{-x}}\,dx = -\ln(1 + e^{-x}) + C$

$= \ln\left(\frac{e^x}{e^x + 1}\right) + C$

$= x - \ln(e^x + 1) + C$

100. Let $u = 1 + e^{2x}$, $du = 2e^{2x}\,dx$.

$\int \frac{e^{2x}}{1 + e^{2x}}\,dx = \frac{1}{2}\int \frac{2e^{2x}}{1 + e^{2x}}\,dx = \frac{1}{2}\ln(1 + e^{2x}) + C$

101. Let $u = 1 - e^x$, $du = -e^x\,dx$.

$\int e^x\sqrt{1 - e^x}\,dx = -\int (1 - e^x)^{1/2}(-e^x)\,dx$

$= -\frac{2}{3}(1 - e^x)^{3/2} + C$

102. Let $u = e^x + e^{-x}$, $du = (e^x - e^{-x})\,dx$.

$\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx = \ln(e^x + e^{-x}) + C$

103. Let $u = e^x - e^{-x}$, $du = (e^x + e^{-x})\,dx$.

104. Let $u = e^x + e^{-x}$, $du = (e^x - e^{-x})\,dx$.

$$\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2}\,dx = 2\int (e^x + e^{-x})^{-2}(e^x - e^{-x})\,dx$$
$$= \frac{-2}{e^x + e^{-x}} + C$$

105. $$\int \frac{5 - e^x}{e^{2x}}\,dx = \int 5e^{-2x}\,dx - \int e^{-x}\,dx$$
$$= -\frac{5}{2}e^{-2x} + e^{-x} + C$$

106. $$\int \frac{e^{2x} + 2e^x + 1}{e^x}\,dx = \int (e^x + 2 + e^{-x})\,dx$$
$$= e^x + 2x - e^{-x} + C$$

107. $$\int e^{-x}\tan(e^{-x})\,dx = -\int [\tan(e^{-x})](-e^{-x})\,dx$$
$$= \ln\left|\cos(e^{-x})\right| + C$$

108. $$\int e^{2x}\csc(e^{2x})\,dx = \frac{1}{2}\int \csc(e^{2x})(2e^{2x})\,dx$$
$$= -\frac{1}{2}\ln\left|\csc(e^{2x}) + \cot(e^{2x})\right| + C$$

109. $$\int_0^1 e^{-2x}\,dx = -\frac{1}{2}\int_0^1 e^{-2x}(-2)\,dx = \left[-\frac{1}{2}e^{-2x}\right]_0^1$$
$$= \frac{1}{2}(1 - e^{-2}) = \frac{e^2 - 1}{2e^2}$$

110. $$\int_1^2 e^{5x-3}\,dx = \left[\frac{1}{5}e^{5x-3}\right]_1^2$$
$$= \frac{1}{5}(e^7 - e^{-2})$$

111. $$\int_0^1 xe^{-x^2}\,dx = -\frac{1}{2}\int_0^1 e^{-x^2}(-2x)\,dx$$
$$= -\frac{1}{2}\left[e^{-x^2}\right]_0^1$$
$$= -\frac{1}{2}\left[e^{-1} - 1\right]$$
$$= \frac{1 - (1/e)}{2} = \frac{e - 1}{2e}$$

112. $$\int_{-2}^0 x^2 e^{x^3/2}\,dx = \frac{2}{3}\int_{-2}^0 e^{x^3/2}\left(\frac{3}{2}x^2\right)dx$$
$$= \frac{2}{3}\left[e^{x^3/2}\right]_{-2}^0$$
$$= \frac{2}{3}\left[1 - e^{-4}\right]$$
$$= \frac{2}{3}\left[1 - \frac{1}{e^4}\right] = \frac{2(e^4 - 1)}{3e^4}$$

113. Let $u = \frac{3}{x}$, $du = -\frac{3}{x^2}\,dx$.

$$\int_1^3 \frac{e^{3/x}}{x^2}\,dx = -\frac{1}{3}\int_1^3 e^{3/x}\left(-\frac{3}{x^2}\right)dx$$
$$= \left[-\frac{1}{3}e^{3/x}\right]_1^3 = \frac{e}{3}(e^2 - 1)$$

114. Let $u = \frac{-x^2}{2}$, $du = -x\,dx$.

$$\int_0^{\sqrt{2}} xe^{-x^2/2}\,dx = -\int_0^{\sqrt{2}} e^{-x^2/2}(-x)\,dx$$
$$= \left[-e^{-x^2/2}\right]_0^{\sqrt{2}} = 1 - e^{-1} = \frac{e - 1}{e}$$

115. Let $u = 1 + e^{2x}$, $du = 2e^{2x}\,dx$.

$$\int_0^3 \frac{2e^{2x}}{1 + e^{2x}}\,dx = \left[\ln(1 + e^{2x})\right]_0^3$$
$$= \ln(1 + e^6) - \ln 2 = \ln\left(\frac{1 + e^6}{2}\right)$$

116. Let $u = 5 - e^x$, $du = -e^x\,dx$.

$$\int_0^1 \frac{e^x}{5 - e^x}\,dx = -\int_0^1 \frac{1}{5 - e^x}(-e^x)\,dx$$
$$= \left[-\ln\left|5 - e^x\right|\right]_0^1$$
$$= -\ln(5 - e) + \ln 4$$
$$= \ln\left(\frac{4}{5 - e}\right)$$

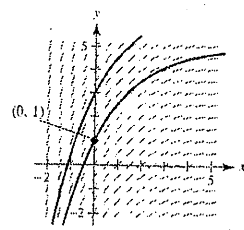
117. Let $u = \sin \pi x$, $du = \pi \cos \pi x\,dx$.

$$\int_0^{\pi/2} e^{\sin \pi x}\cos \pi x\,dx = \frac{1}{\pi}\int_0^{\pi/2} e^{\sin \pi x}(\pi \cos \pi x)\,dx$$
$$= \frac{1}{\pi}\left[e^{\sin \pi x}\right]_0^{\pi/2}$$
$$= \frac{1}{\pi}\left[e^{\sin(\pi^2/2)} - 1\right]$$

118. Let $u = \sec 2x$, $du = 2\sec 2x \tan 2x\,dx$.

$$\int_{\pi/3}^{\pi/2} e^{\sec 2x}\sec 2x \tan 2x\,dx = \frac{1}{2}\int_{\pi/3}^{\pi/2} e^{\sec 2x}(2\sec 2x \tan 2x)\,dx$$
$$= \frac{1}{2}\left[e^{\sec 2x}\right]_{\pi/3}^{\pi/2}$$
$$= \frac{1}{2}\left[e^{-1} - e^{-2}\right]$$
$$= \frac{1}{2}\left[\frac{1}{e} - \frac{1}{e^2}\right] = \frac{e - 1}{2e^2}$$

119. (a)



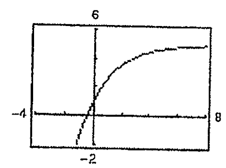
(b) $\frac{dy}{dx} = 2e^{-x/2}, \ (0, 1)$

$$y = \int 2e^{-x/2}\,dx = -4\int e^{-x/2}\left(-\frac{1}{2}\,dx\right)$$

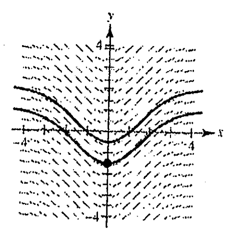
$$= -4e^{-x/2} + C$$

$(0, 1)$: $1 = -4e^0 + C = -4 + C \Rightarrow C = 5$

$y = -4e^{-x/2} + 5$



120. (a)



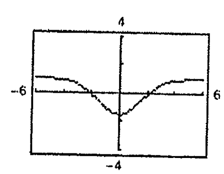
(b) $\frac{dy}{dx} = xe^{-0.2x^2}, \ \left(0, -\frac{3}{2}\right)$

$$y = \int xe^{-0.2x^2}\,dx = \frac{1}{-0.4}\int e^{-0.2x^2}(-0.4x)\,dx$$

$$= -\frac{1}{0.4}e^{-0.2x^2} + C = -2.5e^{-0.2x^2} + C$$

$\left(0, -\frac{3}{2}\right)$: $-\frac{3}{2} = -2.5e^0 + C = -2.5 + C \Rightarrow C = 1$

$y = -2.5e^{-0.2x^2} + 1$



121. Let $u = ax^2$, $du = 2ax\,dx$. (Assume $a \neq 0$.)

$$y = \int xe^{ax^2}\,dx$$

$$= \frac{1}{2a}\int e^{ax^2}(2ax)\,dx = \frac{1}{2a}e^{ax^2} + C$$

122. $y = \int\left(e^x - e^{-x}\right)^2 dx$

$$= \int\left(e^{2x} - 2 + e^{-2x}\right)dx$$

$$= \frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x} + C$$

123. $f'(x) = \int\frac{1}{2}\left(e^x + e^{-x}\right)dx = \frac{1}{2}\left(e^x - e^{-x}\right) + C_1$

$f'(0) = C_1 = 0$

$f(x) = \int\frac{1}{2}\left(e^x - e^{-x}\right)dx = \frac{1}{2}\left(e^x + e^{-x}\right) + C_2$

$f(0) = 1 + C_2 = 1 \Rightarrow C_2 = 0$

$f(x) = \frac{1}{2}\left(e^x + e^{-x}\right)$

124. $f'(x) = \int\left(\sin x + e^{2x}\right)dx = -\cos x + \frac{1}{2}e^{2x} + C_1$

$f'(0) = -1 + \frac{1}{2} + C_1 = \frac{1}{2} \Rightarrow C_1 = 1$

$f'(x) = -\cos x + \frac{1}{2}e^{2x} + 1$

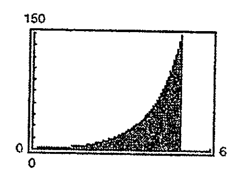
$f(x) = \int\left(-\cos x + \frac{1}{2}e^{2x} + 1\right)dx$

$= -\sin x + \frac{1}{4}e^{2x} + x + C_2$

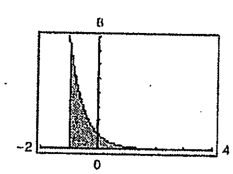
$f(0) = \frac{1}{4} + C_2 = \frac{1}{4} \Rightarrow C_2 = 0$

$f(x) = x - \sin x + \frac{1}{4}e^{2x}$

125. $\int_0^5 e^x dx = \left[e^x\right]_0^5 = e^5 - 1 \approx 147.413$



126. $A = \int_{-1}^{3} e^{-2x}dx = \left[-\frac{1}{2}e^{-2x}\right]_{-1}^{3} = -\frac{1}{2}\left(e^{-6} - e^2\right) \approx 3.693$



127. $\int_0^{\sqrt{6}} xe^{-x^2/4}dx = \left[-2e^{-x^2/4}\right]_0^{\sqrt{6}}$

$= -2e^{-3/2} + 2 \approx 1.554$