5.4a Exercise Problems - Curve Sketching

p. 358-359 #1, ~~3~~, 9, 11, ~~13~~, ~~19~~, 25, 31, 39, ~~41~~, ~~43~~, 53, 55

Apply the following in graphing the curve:

i) Find domain and intercepts
ii) Identify asymptotes and end behavior
iii) Use 1st Derivative Test to find Interval Inc/Dec, Relative Extrema
iv) Use Test for Concavity to find Interval Concave Up/Down, POI
v) Use the info gathered above to graph the function curve

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1) $f(x) = x^4 - 6x^2 + 10$

D: $(-\infty, \infty)$ y-int: $(0, 10)$

$f'(x) = 4x^3 - 12x$

$0 = 4x(x^2 - 3)$

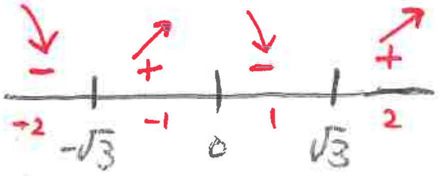
$0 = 4x$

$x = 0$

$x^2 - 3 = 0$

$x = \pm\sqrt{3}$

$f'(x)$



Rel. min $(-\sqrt{3}, \underline{1})$, $(\sqrt{3}, \underline{1})$

Rel. max: $(0, \underline{10})$

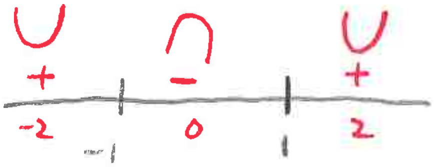
$f''(x) = 12x^2 - 12$

$0 = 12(x^2 - 1)$

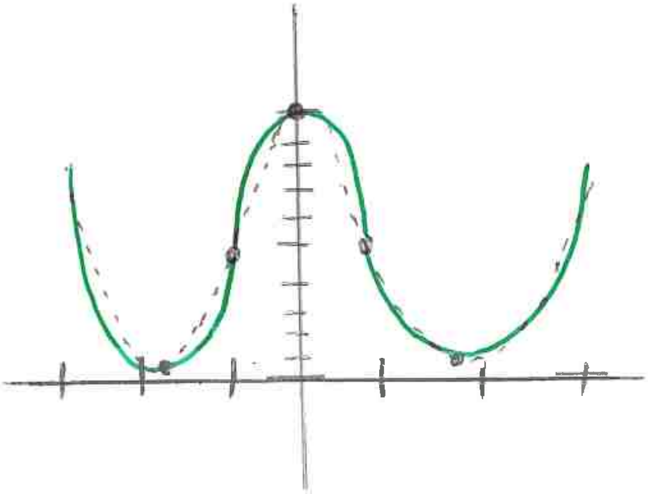
$0 = 12(x+1)(x-1)$

$x = 1, x = -1$

$f''(x)$



POI: $(-1, \underline{5})$ and $(1, \underline{5})$



$\boxed{5.4a}$

9) $f(x)=\frac{2x-1}{x+1}$

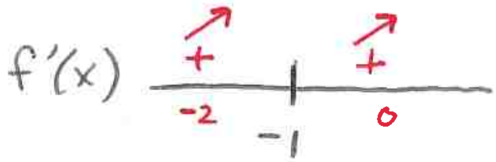
VA: $x=-1$, H.A: $y=\frac{2x}{1x} \rightarrow \overset{\text{H.A.}}{\boxed{y=2}}$, x-int: $2x-1=0$, $x=\frac{1}{2}$, $(\frac{1}{2},0)$ y-int: $(0,-1)$

Domain: $(-\infty,-1),(-1,\infty)$

$$f'(x)=\frac{(2)(x+1)-(2x-1)(1)}{(x+1)^2} \rightarrow \frac{2x+2-2x+1}{(x+1)^2} \rightarrow \frac{3}{(x+1)^2}$$

$$\boxed{f'(x)=\frac{3}{(x+1)^2}}$$

critical point:
$x+1=0$
$\underline{\underline{x=-1}}$



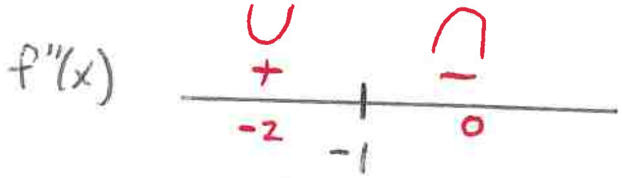
No relative extrema (Also, $x=-1$ is a vertical asymptote)

$f'(x)=3(x+1)^{-2}$

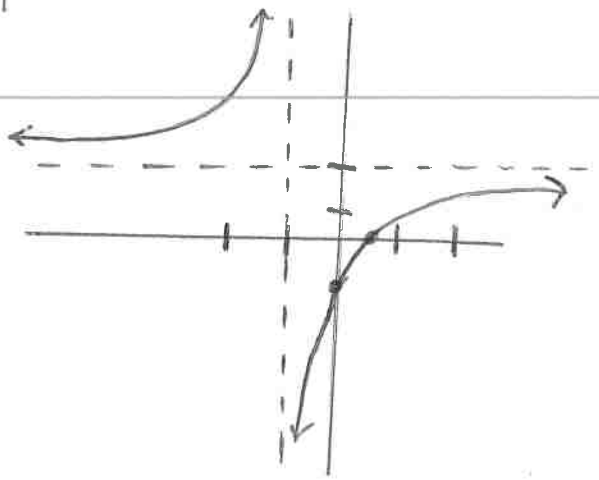
$f''(x)=-6(x+1)^{-3}(1)$

$f''(x)=\frac{-6}{(x+1)^3}$

critical pt: $x+1=0$, $x=-1$



No POI (since $x=-1$ is a VA)



$\boxed{5.4a}$

11) $f(x)=\frac{x}{x^2+1}$

No VA since $x^2+1\neq 0$

H.A: $y=0$

x-int: $(0,0)$

$\hookrightarrow x=0 \nearrow$

y-int: $(0,0)$

Domain: $(-\infty,\infty)$

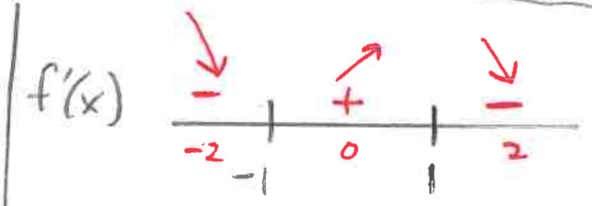
$$f'(x)=\frac{(1)(x^2+1)-x(2x)}{(x^2+1)^2}\rightarrow\frac{x^2+1-2x^2}{(x^2+1)^2}\rightarrow\boxed{f'(x)=\frac{1-x^2}{(x^2+1)^2}}$$

*find critical points:

$1-x^2=0$ | $x^2+1\neq 0$

$(1-x)(1+x)=0$ | none

$x=1, x=-1$



Relative minimum $(-1,\frac{-1}{2})$

Relative maximum $(1,\frac{1}{2})$

$$f''(x)=\frac{(-2x)(x^2+1)^2-(1-x^2)\cdot 2(x^2+1)(2x)}{(x^2+1)^4}\rightarrow\frac{\cancel{(x^2+1)}[-2x(x^2+1)-4x(1-x^2)]}{(x^2+1)^{\cancel{4}\,3}}$$

$$f''(x)=\frac{-2x^3-2x-4x+4x^3}{(x^2+1)^3}\rightarrow\boxed{f''(x)=\frac{2x^3-6x}{(x^2+1)^3}}$$

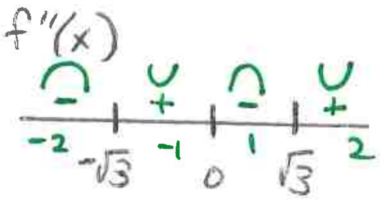
critical pts:

$2x^3-6x=0$ | $x^2+1\neq 0$

$2x(x^2-3)=0$ | none

$2x=0$ | $x^2-3=0$

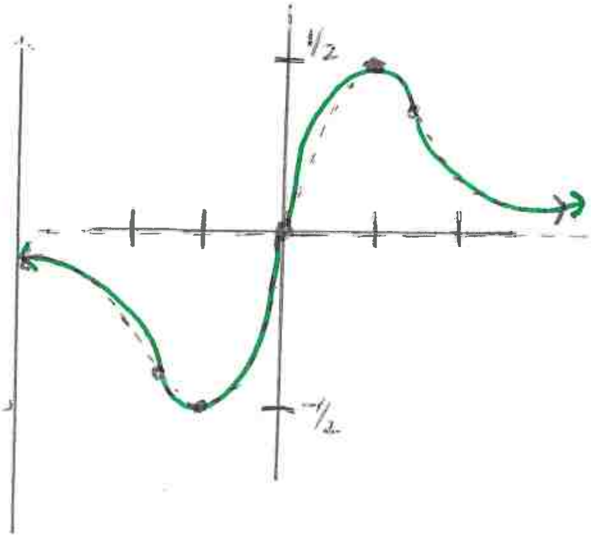
$x=0$ | $x=\pm\sqrt{3}$



POI:

$(-\sqrt{3},\frac{-\sqrt{3}}{4})$, $(0,0)$

and $(\sqrt{3},\frac{\sqrt{3}}{4})$



$\boxed{5.4a}$

25) $f(x)=\frac{x^2}{\sqrt{x+1}}=\frac{x^2}{(x+1)^{1/2}}$

VA: $x=-1$

HA: none

x-int: $x^2=0$

$(0,0)$

y-int: $(0,0)$

D: $\sqrt{x+1}\geq 0$

$x>-1$

D: $(-1,\infty)$

$$f'(x)=\frac{2x\cdot(x+1)^{1/2}-x^2\cdot\frac{1}{2}(x+1)^{-1/2}(1)}{\left[(x+1)^{1/2}\right]^2}\rightarrow\frac{\frac{2x(x+1)^{1/2}}{1}-\frac{x^2}{2(x+1)^{1/2}}}{x+1}$$

$$f'(x)=\frac{\frac{4x(x+1)^{1/2}(x+1)^{1/2}}{2(x+1)^{1/2}}-\frac{x^3}{2(x+1)^{1/2}}}{x+1}\rightarrow\frac{\frac{4x(x+1)-x^2}{2(x+1)^{1/2}}}{x+1}\rightarrow\frac{4x^2+4x-x^2}{2(x+1)^{1/2}}\cdot\frac{1}{x+1}$$

$$\boxed{f'(x)=\frac{3x^2+4x}{2(x+1)^{3/2}}}$$

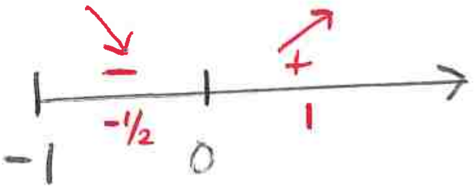
*critical points:

$3x(x+4)=0$ | $(x+1)^{3/2}=0$

$x=0,\ x=\cancel{-4}$ | $x=-1$

outside domain | (VA)

$f'(x)$



Relative min at $(0,\underline{0})$

$$f''(x)=\frac{(6x+4)\cdot2(x+1)^{3/2}-(3x^2+4x)\cdot3(x+1)^{+1/2}(1)}{\left[2(x+1)^{3/2}\right]^2}\rightarrow\frac{(x+1)^{1/2}\left[2(6x+4)(x+1)-3(3x^2+4x)\right]}{4(x+1)^3}$$

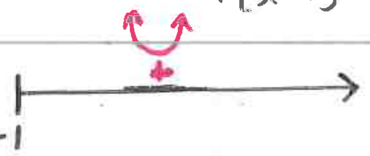
$$f''(x)=\frac{2(6x^2+6x+4x+4)-9x^2-12x}{4(x+1)^{5/2}}$$

$$f''(x)=\frac{12x^2+12x+8x+8-9x^2-12x}{4(x+1)^{5/2}}$$

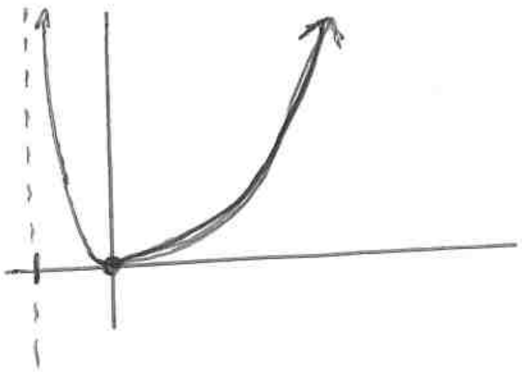
$$\boxed{f''(x)=\frac{3x^2+8x+8}{4(x+1)^{5/2}}}$$

*No critical pts, $3x^2+8x+8>0$ for $(-1,\infty)$

$f''(x)$



No Point of Inflection



$\boxed{5.4a}$

31) $f(x) = \sin x - \cos x$

Domain: $(-\infty, \infty)$

VA: none

HA: none

$0 = \sin x - \cos x$

$\frac{\cos x}{\cos x} = \frac{\sin x}{\cos x}$

$1 = \tan x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

x-int: $(\frac{\pi}{4}, 0), (\frac{5\pi}{4}, 0)$

y-int: $(0, -1)$

$f'(x) = \cos x - (-\sin x)$

$f'(x) = \cos x + \sin x$

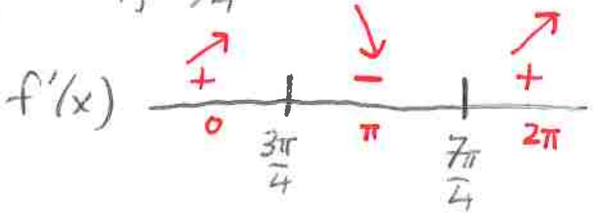
$0 = \cos x + \sin x$

$\frac{-\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$-\tan x = 1$

$\tan x = -1$

$x = \frac{3\pi}{4}, 7\pi/4$



Relative max $(\frac{3\pi}{4}, \underline{\sqrt{2}})$

Relative min $(\frac{7\pi}{4}, \underline{-\sqrt{2}})$

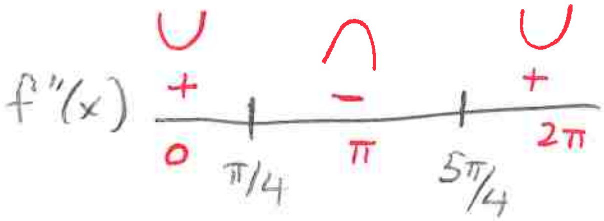
$f''(x) = -\sin x + \cos x$

$0 = -\sin x + \cos x$

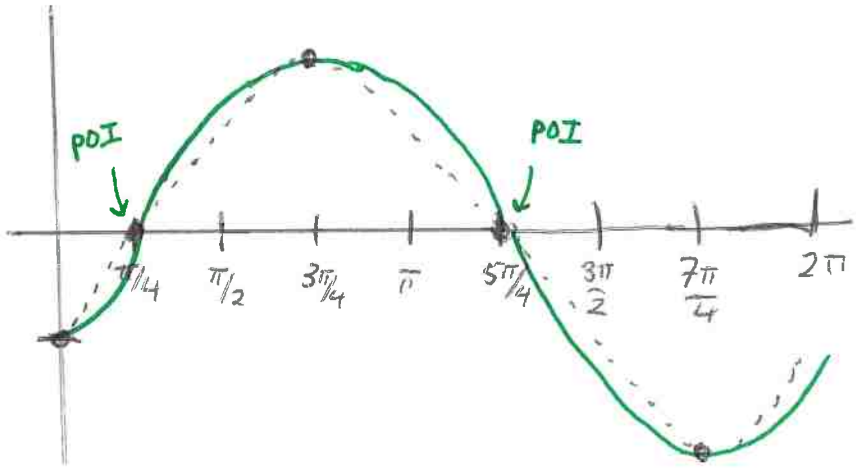
$\frac{\sin x}{\sin x} = \frac{\cos x}{\sin x}$

$1 = \cot x$

$x = \pi/4, 5\pi/4$



POI at $(\frac{\pi}{4}, \underline{0})$ and $(\frac{5\pi}{4}, \underline{0})$



$\boxed{5.4a}$

39) $f(x)=3e^{3x}(5-x)$

$\lim_{x\to-\infty} 3e^{3x}(5-x) \to \frac{3(5-x)}{e^{-3x}} \to 0$

D: $(-\infty,\infty)$

HA: none

x-int: $(5,0)$

y-int: $(0,15)$

$f'(x)=3e^{3x}\cdot3(5-x)+3e^{3x}\cdot(-1)$

$=3e^{3x}[15-3x-1]$

$f'(x)=3e^{3x}(14-3x)$

$0=14-3x$

$x=14/3$

$f'(x)$: $\nearrow$ $+$ (0) | $14/3$ | $\searrow$ $-$ (5)

Relative Max at $\left(\frac{14}{3}, \underline{e^{14}}\right)$

$f''(x)=3e^{3x}\cdot3(14-3x)+3e^{3x}(-3)$

$=9e^{3x}[14-3x-1]$

$f''(x)=9e^{3x}(13-3x)$

*critical point

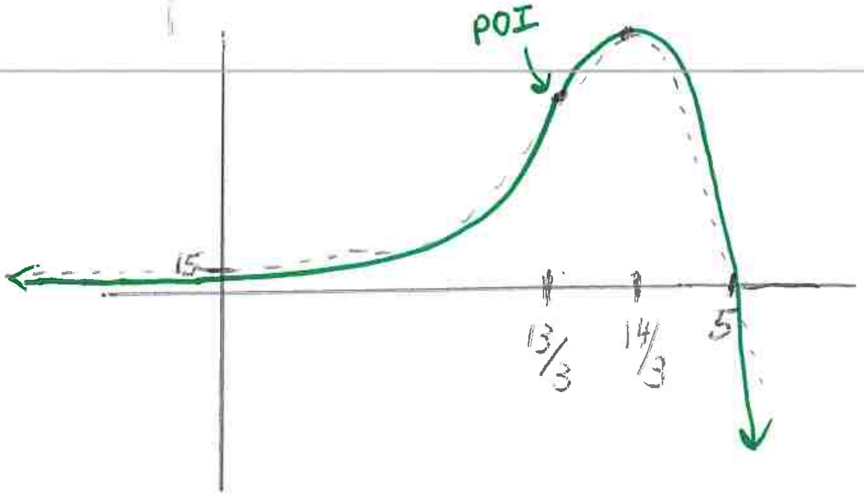
$13-3x=0$

$3x=13$

$x=13/3$

$f''(x)$: $\cup$ $+$ (0) | $13/3$ | $\cap$ $-$ (5)

Point of Inflection $\left(\frac{13}{3}, \underline{2e^{13}}\right)$



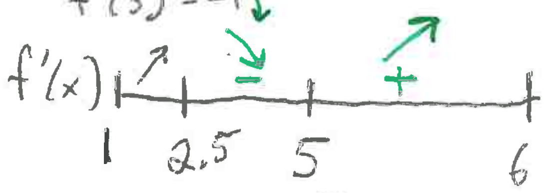
$\boxed{5.4}$ Graph function f that is continuous on [1,6] and satisfies given conditions

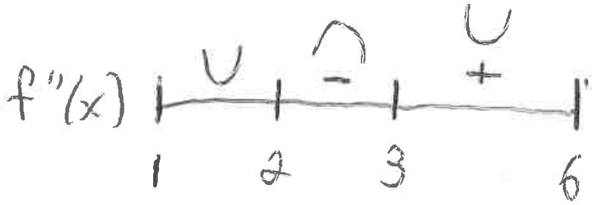
53) $f'(2) = dne$ | $f''(x) < 0$, $2 < x < 3$

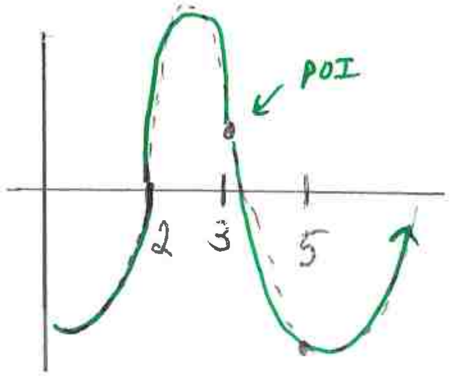
$f''(3) = 0$ | $f''(x) > 0$, $x > 3$

$f'(5) = 0$

$f'(3) = -1$







55) $f'(2) = 0$ | $f'(5) = 0$ | interval [1,6]

$\lim_{x \to 3^-} f'(x) = \infty$ | $f''(x) > 0$, $x < 3$

$\lim_{x \to 3^+} f'(x) = \infty$ | $f''(x) < 0$, $x > 3$