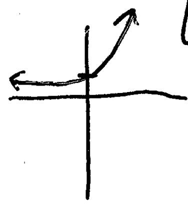
2017

Ch. 5.5 Other bases p. 362-363 # 1, 3, 15, 17, 21, 23, 27, 29, 37, 39, 43, 47, 49, 53, 55, 59, 63, 65

1) Evaluate log expression: $\log_2 \frac{1}{8} = \log_2 2^{-3} = \boxed{-3}$

3) $\log_7 1 = 0$

15) $f(x) = 3^x$



$\boxed{d}$

23) solve equation:

a) $x^2 - x = \log_5 25$ $\quad \log_5 (5)^2 = 2$

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\boxed{x = 2, -1}$

b) $3x + 5 = \log_2 64$ $\quad \log_2 2^6 = 6$

$3x + 5 = 6$

$3x = 1$

$\boxed{x = 1/3}$

27) $2^{3-z} = 625$

$\ln 2^{3-z} = \ln 625$

$(3-z)\ln 2 = \ln 625$

$3 - z = \frac{\ln 625}{\ln 2}$

$z = +3 - \frac{\ln 625}{\ln 2} \approx \boxed{-6.288}$

29) $\left(1 + \frac{0.09}{12}\right)^{12t} = 3$

$\ln\left(1 + \frac{0.09}{12}\right)^{12t} = \ln 3$

$12t \ln\left(1 + \frac{0.09}{12}\right) = \ln 3$

$t = \frac{\ln 3}{12 \ln\left(1 + \frac{0.09}{12}\right)} \approx 12.253$

$\boxed{t \approx 12.253}$

5.5

*Recall: $\frac{d}{dx} a^u = \ln a \cdot a^u \cdot u'$

$\frac{d}{dx} \log_a u = \frac{1}{\ln a} \cdot \frac{u'}{u}$

Find derivative:

37) $f(x) = 4^x$ $\quad f'(x) = \ln 4 \cdot 4^x \cdot 1 = \boxed{(\ln 4) 4^x}$

39) $y = 5^{-4x}$ $\quad y' = \ln 5 \cdot 5^{-4x}(-4) = -4 \ln 5 \cdot 5^{-4x} = \boxed{\dfrac{-4\ln 5}{5^{4x}}}$

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43) $g(t) = t^2 \cdot 2^t$ ← *product rule

$g'(t) = \underbrace{2t}_{f'} \cdot \underbrace{2^t}_{g} + \underbrace{t^2}_{f} \cdot \underbrace{\ln 2 \cdot 2^t(1)}_{g'}$

$g'(t) = 2t(2^t) + t^2 \ln 2 (2^t)$

$\boxed{g'(t) = 2^t(t)\left[2 + t \ln 2\right]}$

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47) $y = \log_4(5x+1)$

$y' = \frac{1}{\ln 4} \cdot \frac{5}{5x+1}$

$\boxed{y' = \dfrac{5}{\ln 4(5x+1)}}$

53) $f(x) = \log_2 \frac{x^2}{x-1}$ ← Expand log expression first!

$f(x) = \log_2 x^2 - \log_2(x-1)$

$f'(x) = \frac{1}{\ln 2}\left(\frac{2x}{x^2}\right) - \frac{1}{\ln 2}\left(\frac{1}{x-1}\right)$

$f'(x) = \frac{1}{\ln 2}\left[\frac{2}{x} - \frac{1}{x-1}\right] = \frac{1}{\ln 2}\left[\frac{2(x-1) - x}{x(x-1)}\right]$

$2x - 2 - x$

$\boxed{f'(x) = \dfrac{x-2}{(\ln 2) x(x-1)}}$

55) $h(x) = \log_3 \frac{x\sqrt{x-1}}{2}$ ← expand expression first!

$h(x) = \log_3 x + \log_3(x-1)^{1/2} - \log_3 2$

$h(x) = \log_3 x + \frac{1}{2}\log_3(x-1) = \log_3 2$

$h'(x) = \frac{1}{\ln 3}\left(\frac{1}{x}\right) + \frac{1}{2} \cdot \frac{1}{\ln 3}\left(\frac{1}{x-1}\right)$

$h'(x) = \frac{1}{\ln 3}\left[\frac{1}{x} + \frac{1}{2(x-1)}\right] = \frac{1}{\ln 3}\left[\frac{2(x-1) + x}{2x(x-1)}\right]$

$2x - 2 + x$

$\boxed{h'(x) = \dfrac{1}{\ln 3}\left[\dfrac{3x-2}{2x(x-1)}\right]}$

5.5 continued

59) Find equation of tangent line

$y = 2^{-x} \quad (-1, 2)$

$y' = \ln 2 \cdot 2^{-x}(-1)$

$y' = \dfrac{-\ln 2}{2^x}$

$y'(-1) = \dfrac{-\ln 2}{2^{-1}} = -2\ln 2$

slope: $m = -2\ln 2$ point: $(-1, 2)$

$y - y_1 = m(x - x_1)$

$\boxed{y - 2 = -2\ln 2(x+1)}$

63) Log Differentation: Steps: ① Take log of both sides
② expand expression (log)
③ Find derivative w/ implicit differentation

$y = x^{2/x}$

$\ln y = \ln x^{2/x}$

$\ln y = \frac{2}{x} \cdot \ln x$

Apply product rule

$\ln y = (2x^{-1})(\ln x)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \underbrace{(-2x^{-2})}_{f'}\underbrace{(\ln x)}_{g} + \underbrace{(2x^{-1})}_{f}\underbrace{\left(\frac{1}{x}\right)}_{g'} = \frac{-2\ln x}{x^2} + \frac{2}{x^2}$

$\frac{dy}{dx} = y\left[\frac{2 - 2\ln x}{x^2}\right]$

$\boxed{\frac{dy}{dx} = \frac{x^{2/x}(2 - 2\ln x)}{x^2} = 2x^{2/x - 2}[2 - 2\ln x]}$

65) Log differentiation

$y = (x-2)^{x+1}$

$\ln y = \ln(x-2)^{x+1}$

$\ln y = \underbrace{(x+1)}\,\underbrace{\ln(x-2)}$

Apply product rule

$\frac{1}{y}\left(\frac{dy}{dx}\right) = 1 \cdot \ln(x-2) + (x+1) \cdot \frac{1}{x-2} = \frac{\ln(x-2)}{1} + \frac{x+1}{x-2}$

$\frac{dy}{dx} = y\left[\frac{(x-2)\ln(x-2) + x + 1}{x-2}\right]$

$\boxed{\frac{dy}{dx} = (x-2)^{x+1}\left[\frac{(x-2)\ln(x-2) + x + 1}{x-2}\right]}$

~~Ch. 5.6 Inverse Trig~~