Key

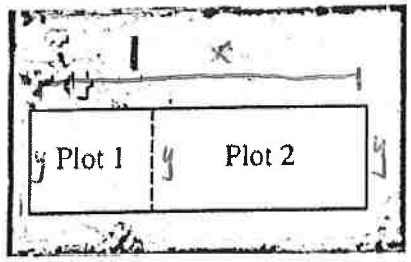
5.5 Optimization Homework

pg. 366-370 #5,6,7,9,12,14

5. Maximizing Area

$\frac{5000}{3}$

A gardener with 200 m of available fencing wishes to enclose a rectangular field and then divide it into two plots with a fence parallel to one of the sides, as shown in the figure. What is the largest area that can be enclosed?



* $A = xy$

$200 = 2x + 3y$

$3y = -2x + 200$

$y = \frac{-2x+200}{3}$

$A = x\left(\frac{-2x+200}{3}\right)$

$A = \frac{-2x^2}{3} + \frac{200x}{3}$

$A = -\frac{2}{3}x^2 + \frac{200}{3}x$

$A'(x) = -\frac{4}{3}x + \frac{200}{3}$

$0 = -\frac{4}{3}x + \frac{200}{3}$

$\frac{4}{3}x = \frac{200}{3}$

$4x = 200$

$x = 50$

$y = \frac{-2(50)+200}{3} = \frac{100}{3}$

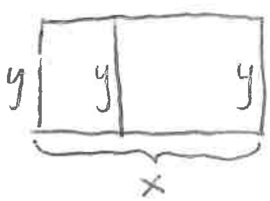
Largest Area is

$A = 50\left(\frac{100}{3}\right)$

$\boxed{A = \frac{5000}{3} m^2}$

6. Minimizing Fencing A realtor wishes to enclose 600 m^2 of land in a rectangular plot and then divide it into two plots with a fence parallel to one of the sides. What are the dimensions of the rectangular plot that require the least amount of fencing?

30×20



* $P = 2x + 3y$

$600 = xy$

$y = \frac{600}{x}$

$P = 2x + 3\left(\frac{600}{x}\right)$

$P = 2x + \frac{1800}{x}$

$P = 2x + 1800x^{-1}$

$P'(x) = 2 - 1800x^{-2}$

$0 = 2 - \frac{1800}{x^2}$

$\frac{1800}{x^2} = 2$

$2x^2 = 1800$

$x^2 = 900$

$x = 30$

$600 = xy$

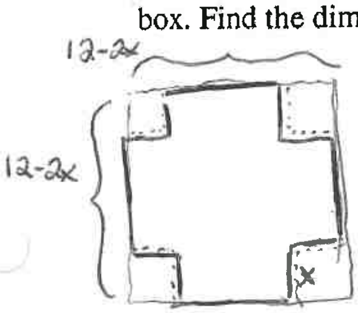
$600 = 30y$

$y = 20$

A 30m x 20m enclosure uses the least amount of fencing

7. Maximizing the Volume of a Box An open box with a square base is to be made from a square piece of cardboard that measures 12 cm on each side. A square will be cut out from each corner of the cardboard and the sides will be turned up to form the box. Find the dimensions that yield the maximum volume.

$8 \times 8 \times 2$



V = (height)(length)(width)

$V = x(12-2x)(12-2x)$

$V = x(144 - 48x + 4x^2)$

$V = 4x^3 - 48x^2 + 144x$

$V'(x) = 12x^2 - 96x + 144$

$V'(x) = 12(x^2 - 8x + 12)$

$0 = 12(x-6)(x-2)$

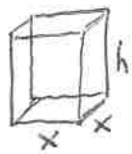
$\boxed{x = 2}$, $\cancel{x = 6}$ not enough material to cut this much out.

Dimensions are

2cm x 8cm x 8cm

$10\sqrt[3]{4} \times 10\sqrt[3]{4} \times 5\sqrt[3]{4}$

9. Minimizing the Surface Area of a Box An open box with a square base is to have a volume of 2000 cm^3. What should be the dimensions of the box if the amount of material used is to be a minimum?



$V = x^2 h$

$2000 = x^2 h$

$\boxed{\frac{2000}{x^2} = h}$

*Surface Area $= x^2 + 4xh$

$S = x^2 + 4xh$

$S = x^2 + 4x\left(\frac{2000}{x^2}\right)$

$S = x^2 + \frac{8000}{x}$

$S = x^2 + 8000x^{-1}$

$S'(x) = 2x - 8000x^{-2}$

$0 = 2x - \frac{8000}{x^2}$

$\frac{8000}{x^2} = \frac{2x}{1}$

$2x^3 = 8000$

$x^3 = 4000$

$x = \sqrt[3]{4000} = 10(\sqrt[3]{4})$

$h = \frac{2000}{x^2}$

$h = \frac{2000}{(10\sqrt[3]{4})^2} = \frac{2000}{100 \cdot \sqrt[3]{16}}$

$h = \frac{2000}{100 \cdot 2(\sqrt[3]{2})}$

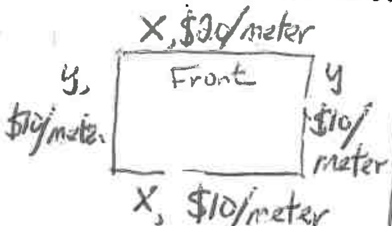
$h = \frac{10}{\sqrt[3]{2}}$

Dimensions are:

$\boxed{10(\sqrt[3]{4}) \times 10(\sqrt[3]{4}) \times \frac{10}{\sqrt[3]{2}} \text{ cm}}$

12. Minimizing the Cost of Fencing A builder wishes to fence in 60,000 m^2 of land in a rectangular shape. For security reasons, the fence along the front part of the land will cost \$20 per meter, while the fence for the other three sides will cost \$10 per meter. How much of each type of fence should the builder buy to minimize the cost of the fence? What is the minimum cost?

\$12,000



$A = xy$

$60{,}000 = xy$

$y = \frac{60{,}000}{x}$

Cost $= 20x + 10x + 10y + 10y$

$C = 30x + 20y$

$C = 30x + 20\left(\frac{60{,}000}{x}\right)$

$C = 30x + \frac{1200000}{x}$

$C = 30x + 1{,}200{,}000x^{-1}$

$C'(x) = 30 - 1{,}200{,}000x^{-2}$

$0 = 30 - \frac{1{,}200{,}000}{x^2}$

$\frac{1{,}200{,}000}{x^2} = 30$

$30x^2 = 1{,}200{,}000$

$x^2 = 40{,}000$

$x = 200$

$x = 200$

$y = 300$

$C = 30x + 20y$

$C = 30(200) + 20(300)$

$\boxed{C = \$12{,}000}$

14. Maximizing Revenue A charter flight club charges its members \$200 per year. But for each new member in excess of 60, the charge for every member is reduced by \$2. What number of members leads to a maximum revenue?

80 members

$R = (60 + x)(200 - 2x)$

*Revenue = (# of members) x (yearly charge)

$R(x) = (60 + x)(200 - 2x)$

$R(x) = 12000 + 80x - 2x^2$

$R'(x) = 0 + 80 - 4x$

$0 = 80 - 4x$

$4x = 80$

$x = 20$

*Revenue is maximized when $x = 20$

# of members is $60 + x$, so $60 + 20 = 80$ members

$\boxed{\text{80 members to maximize revenue}}$