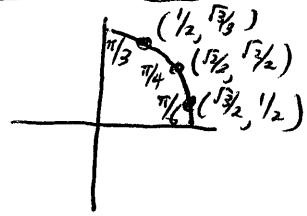
Ch. 5.6 Inverse Trig Derivatives

p. 372-373
#5, 7, 9, 11, 17-27 odd, 42, 43, 44, 49, 55, 59, 63

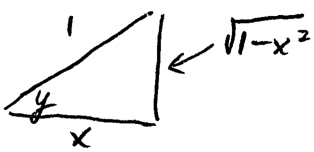
5) Evaluate inverse trig $\arccos \frac{1}{2} = \boxed{\pi/3}$



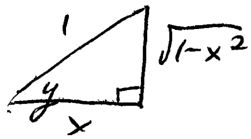
7) $\arctan \frac{\sqrt{3}}{3} = \arctan \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \boxed{\pi/6}$

9) $\arccsc(-\sqrt{2}) =$ 4th Quadrant $= \frac{7\pi}{4}$ or $\boxed{-\pi/4}$

17) $\tan y$ $\boxed{\tan y = \frac{\sqrt{1-x^2}}{x}}$

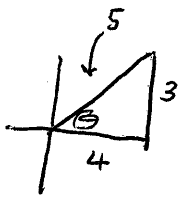


19) $\sec y$ $\boxed{\sec y = \frac{1}{x}}$



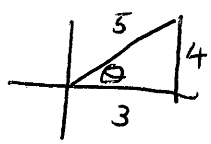
21) Evaluate expression:

a) $\sin\left(\arctan \frac{3}{4}\right)$



$\sin\left(\arctan \frac{3}{4}\right) = \boxed{\frac{3}{5}}$

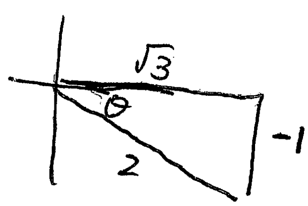
b) $\sec\left(\arcsin \frac{4}{5}\right)$



$\sec\left(\arcsin \frac{4}{5}\right) = \boxed{\frac{5}{3}}$

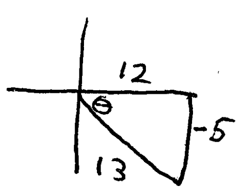
23)

a) $\cot\left[\arcsin\left(-\frac{1}{2}\right)\right]$



$\cot\left[\arcsin\left(-\frac{1}{2}\right)\right] = \frac{A}{O} = \frac{\sqrt{3}}{-1} = \boxed{-\sqrt{3}}$

b) $\csc\left[\arctan\left(-\frac{5}{12}\right)\right]$

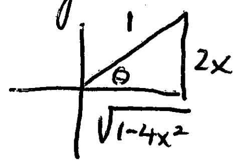


$\csc\left[\arctan\left(-\frac{5}{12}\right)\right] = \frac{H}{O} = \boxed{\frac{13}{-5}}$

$\boxed{5.6}$

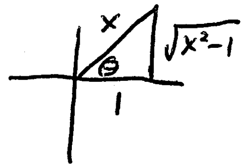
25) Simplify using right triangle

$\cos(\arcsin 2x)$



$$\cos(\arcsin(2x)) = \frac{\sqrt{1-4x^2}}{1} = \boxed{\sqrt{1-4x^2}}$$

27) $\sin(\operatorname{arcsec} x)$



$$\sin(\operatorname{arcsec} x) = \boxed{\frac{\sqrt{x^2-1}}{|x|}}$$

must be nonnegative

Find derivative:

Recall: $\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$ | $\frac{d}{dx}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$

$\frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$ | $\frac{d}{dx}\operatorname{arccot} u = \frac{-u'}{1+u^2}$

$\frac{d}{dx}\operatorname{arcsec} u = \frac{u'}{|u|\sqrt{u^2-1}}$ | $\frac{d}{dx}\operatorname{arccsc} u = \frac{-u'}{|u|\sqrt{u^2-1}}$

43) $f(x) = \arctan e^x$

$$f'(x) = \frac{e^x}{1+(e^x)^2} = \boxed{\frac{e^x}{1+e^{2x}}}$$

*Apply product rule, chain rule

49) $y = 2x\arccos x - 2\sqrt{1-x^2}$

$$y = \underbrace{2x}_{f}\underbrace{\arccos x}_{g} - 2(1-x^2)^{1/2}$$

$$y' = \underbrace{2}_{f'}\underbrace{(\arccos x)}_{g} + \underbrace{2x}_{f}\cdot\underbrace{\frac{-1}{\sqrt{1-x^2}}}_{g'} - 2\cdot\tfrac{1}{2}(1-x^2)^{-1/2}(-2x)$$

$$= 2\arccos x - \frac{2x}{\sqrt{1-x^2}} + \frac{2x}{\sqrt{1-x^2}} = \boxed{2\arccos x}$$

44) $f(x) = \arctan\sqrt{x}$

$f(x) = \arctan(x)^{1/2}$

$$f'(x) = \frac{\frac{1}{2}x^{-1/2}}{1+(\sqrt{x})^2} = \boxed{\frac{1}{2\sqrt{x}(1+x)}}$$

$\boxed{5.6}$

42) Find $f'(x)$: $f(x) = \operatorname{arcsec} 2x$

$$f'(x) = \frac{2}{|2x|\sqrt{(2x)^2-1}} = \boxed{\frac{1}{|x|\sqrt{4x^2-1}}}$$

*Apply product rule
*chain rule

55) $y = 8\arcsin\left(\frac{x}{4}\right) - \frac{x\sqrt{16-x^2}}{2} = 8\arcsin\left(\frac{1}{4}x\right) - \frac{1}{2}x(16-x^2)^{1/2}$

$$y' = 8\left(\frac{\frac{1}{4}}{\sqrt{1-\left(\frac{x}{4}\right)^2}}\right) + \underbrace{-\frac{1}{2}}_{f'}\underbrace{(16-x^2)^{1/2}}_{g} + \underbrace{-\frac{1}{2}x}_{f} \cdot \underbrace{\frac{1}{2}(16-x^2)^{-1/2}(-2x)}_{g'}$$

$$\frac{2}{\sqrt{1-\frac{x^2}{16}}} = \frac{2}{\sqrt{\frac{16-x^2}{16}}} = \frac{2}{\frac{\sqrt{16-x^2}}{4}} \cdot \frac{4}{4} = \frac{8}{\sqrt{16-x^2}}$$

$$y' = \frac{8}{\sqrt{16-x^2}} - \frac{\sqrt{16-x^2}}{2} + \frac{x^2}{2\sqrt{16-x^2}} = \frac{16}{2\sqrt{16-x^2}} - \frac{16-x^2}{2\sqrt{16-x^2}} + \frac{x^2}{2\sqrt{16-x^2}}$$

$$y' = \frac{16-16+x^2+x^2}{2\sqrt{16-x^2}} = \frac{2x^2}{2\sqrt{16-x^2}} = \boxed{\frac{x^2}{\sqrt{16-x^2}}}$$

59) Find equation of tangent line

$y = 2\arcsin x$ at $\left(\frac{1}{2}, \frac{\pi}{3}\right)$

$$y' = 2\cdot\left(\frac{1}{\sqrt{1-x^2}}\right) = \frac{2}{\sqrt{1-x^2}}$$

$$y'\left(\tfrac{1}{2}\right) = \frac{2}{\sqrt{1-\left(\frac{1}{2}\right)^2}} = \frac{2}{\sqrt{1-\frac{1}{4}}} = \frac{2}{\sqrt{\frac{3}{4}}} = \frac{2}{\frac{\sqrt{3}}{2}} = \frac{4}{\sqrt{3}}$$

point: $\left(\frac{1}{2}, \frac{\pi}{3}\right)$

slope: $m = \frac{4}{\sqrt{3}}$

$$\boxed{y - \frac{\pi}{3} = \frac{4}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$$

63) Find equation of tangent line : $y = 4x\arccos(x-1)$ at $(1, 2\pi)$

$\nwarrow$ Apply product rule

$$y' = \underbrace{(4)}_{f'}\underbrace{\arccos(x-1)}_{g} + \underbrace{4x}_{f} \cdot \underbrace{\frac{-1}{\sqrt{1-(x-1)^2}}}_{g'}$$

$* \frac{d}{dx}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$

$$y' = 4\arccos(x-1) - \frac{4x}{\sqrt{1-(x-1)^2}}$$

$$y'(1) = 4\arccos(0) - \frac{4}{\sqrt{1-0^2}} = 4(\pi/2) - 4 = 2\pi - 4$$

point: $(1, 2\pi)$

slope: $m = 2\pi - 4$

$y - y_1 = m(x - x_1)$

$$\boxed{y - 2\pi = (2\pi - 4)(x-1)}$$

77) Use Implicit Differentiation to find equation of tangent line:

$x^2 + x\arctan y = y - 1$ at $(-\pi/4, 1)$

$*$ Apply implicit diff

$*$ product rule

$$2x + (1)\arctan y + x\left(\frac{\frac{dy}{dx}}{1+y^2}\right) = \frac{dy}{dx} - 0$$ $\swarrow$ $*$ plug in $(-\pi/4, 1)$ to find slope

$$2(-\pi/4) + \arctan(1) + (-\pi/4)\left(\frac{dy/dx}{1+1}\right) = \frac{dy}{dx}$$

$$-\pi/2 + \pi/4 - \frac{\pi}{8}\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx}\left(-\pi/8 - 1\right) = \pi/2 - \pi/4$$

$$\frac{dy}{dx}\left[\frac{-\pi-8}{8}\right] = \frac{2\pi}{4} - \frac{1\pi}{4} \swarrow = \pi/4$$

$$\frac{dy}{dx} = \frac{8}{-\pi-8} \cdot \frac{\pi}{4} = \frac{2\pi}{-\pi-8} = \frac{2\pi}{-(\pi+8)} = \frac{-2\pi}{\pi+8}$$

point: $(-\pi/4, 1)$ slope: $m = \frac{-2\pi}{\pi+8}$

$y - y_1 = m(x - x_1)$

$$\boxed{y - 1 = \frac{-2\pi}{\pi+8}\left(x + \pi/4\right)}$$